Find The Equation Of The Linethrough The OriginO(0,0,0)which Perpendicularly Meets The Line:x=1+t,y=2+t,z=1+t.

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Introduction

In the realm of three-dimensional coordinate geometry, understanding the relationships between lines and points is fundamental. When dealing with problems involving perpendicularity, lines passing through specific points, and the equations of lines in 3D space, a systematic approach is essential. This article aims to guide you through the process of finding the equation of a line passing through the origin (0,0,0) that perpendicularly intersects a given line parametrized as $x=1+t, y=2+t, z=1+t$.

This problem is a classic example often encountered in vector calculus and analytical geometry, and solving it involves concepts such as direction vectors, point-line distance, and the conditions for perpendicularity. We will explore these concepts in detail and provide a step-by-step methodology to derive the required line's equation.

Understanding the Given Line

The line given is expressed in parametric form as:

$$
x = 1 + t, \quad y = 2 + t, \quad z = 1 + t
$$

Interpretation:

- The parameter t varies over all real numbers.
- The line passes through the point $(1, 2, 1)$ when $t=0$.
- The direction vector of this line, denoted as $\vec{d}$, can be read directly from the parametric equations:

$$
\boxed{
\vec{d} = \langle 1, 1, 1 \rangle
}
$$

This vector indicates the direction in which the line extends in space.

Objective

Find the equation of a line L that:

- Passes through the origin $O(0,0,0)$,
- Intersects the given line L' at some point P,
- Is perpendicular to L', meaning the direction vector of L is orthogonal to the direction vector $\vec{d}$ of L',
- Perpendicularly meets L', i.e., the shortest distance between the two lines is along the line L, which is perpendicular to L'.

Conceptual Framework

To solve this problem, we need to understand several key concepts:

1. Direction Vectors and Line Equations

- Line in vector form:
Any line can be expressed as:

$$
\text{Line}:\quad \vec{r} = \vec{r_0} + \lambda \vec{d}
$$

where $\vec{r_0}$ is a point on the line, and $\vec{d}$ is its direction vector.

- Equation of the line through the origin:
Since the line passes through $(0,0,0)$, its equation simplifies to:

$$
\vec{r} = \lambda \vec{d}
$$

with $\lambda \in \mathbb{R}$.

2. Perpendicularity Conditions

- The line L that passes through the origin and is perpendicular to L' must have a direction vector $\vec{n}$ such that:

$$
\vec{n} \cdot \vec{d} = 0
$$

where $\vec{d} = \langle 1, 1, 1 \rangle$.

- The line L must intersect L' at some point P, and at this point, the line L is perpendicular to L'.

3. The Shortest Distance Between Two Skew Lines

- The minimal distance between two skew lines occurs along the common perpendicular, which is perpendicular to both lines.
- When the line L passes through the origin and intersects L' perpendicularly, the point of intersection P lies on L', and the connecting segment OP is perpendicular to L'.

Step-by-Step Solution

Let's now proceed step-by-step to find the required line's equation.

Step 1: Determine the General Form of the Line Through the Origin

Since the line passes through $(0,0,0)$, its equation in parametric form is:

$$
L:\quad \vec{r} = \lambda \vec{n}
$$

where $\vec{n}$ is the direction vector of the line L. Our goal is to find $\vec{n}$.

Step 2: Establish the Perpendicularity Condition

Because L must be perpendicular to L', the direction vector $\vec{n}$ must satisfy:

$$
\boxed{
\vec{n} \cdot \vec{d} = 0
}
$$

Given $\vec{d} = \langle 1, 1, 1 \rangle$, this condition becomes:

$$
n_x + n_y + n_z = 0
$$

This is the first key equation, which constrains the possible $\vec{n}$.

Step 3: Find the Point of Intersection P on L'

Suppose the point of intersection P on L' corresponds to some parameter $t = t_0$:

$$
P = (1 + t_0,\, 2 + t_0,\, 1 + t_0)
$$

The line L passes through the origin $O(0,0,0)$ and intersects L' at P. The vector from O to P is:

$$
\vec{OP} = \langle 1 + t_0,\, 2 + t_0,\, 1 + t_0 \rangle
$$

Since L passes through O with direction $\vec{n}$, the point P must lie on L, i.e.,

$$
\vec{OP} = \mu \vec{n}
$$

for some scalar μ.

Therefore,

$$
\langle 1 + t_0,\, 2 + t_0,\, 1 + t_0 \rangle = \mu \vec{n}
$$

Step 4: Perpendicularity Between $\vec{OP}$ and $\vec{d}$

Since the line L intersects L' perpendicularly, the vector $\vec{OP}$ connecting O to P must be perpendicular to the direction vector $\vec{d}$ of L'. Mathematically,

$$
\vec{OP} \cdot \vec{d} = 0
$$

Plugging in the vectors:

$$
(1 + t_0) \times 1 + (2 + t_0) \times 1 + (1 + t_0) \times 1 = 0
$$

Simplify:

$$
(1 + t_0) + (2 + t_0) + (1 + t_0) = 0
$$

$$
(1 + 2 + 1) + (t_0 + t_0 + t_0) = 0
$$

$$
4 + 3 t_0 = 0
$$

Solve for t_0:

$$
3 t_0 = -4 \quad \Rightarrow \quad t_0 = -\frac{4}{3}
$$

Step 5: Find the Point P

Substitute $t_0 = -\frac{4}{3}$ into the parametric equations of L':

$$
P = \left(1 - \frac{4}{3},\, 2 - \frac{4}{3},\, 1 - \frac{4}{3} \right)
$$

Calculate:

$$
\begin{aligned}
x_P &= 1 - \frac{4}{3} = \frac{3}{3} - \frac{4}{3} = -\frac{1}{3} \\
y_P &= 2 - \frac{4}{3} = \frac{6}{3} - \frac{4}{3} = \frac{2}{3} \\
z_P &= 1 - \frac{4}{3} = -\frac{1}{3}
\end{aligned}
$$

Thus,

$$
P = \left(-\frac{1}{3},\, \frac{2}{3},\, -\frac{1}{3} \right)
$$

Step 6: Find the Direction Vector $\vec{n}$ of Line L

Recall that $\vec{OP}$ is proportional to $\vec{n}$:

$$

\vec{OP} = \left\l

Frequently Asked Questions

What is the given line equation in parametric form?

The given line is defined by $x=1+t$, $y=2+t$, $z=1+t$.

What is the point through which the required line passes?

The line passes through the origin point $O(0,0,0)$.

What is the geometric condition for the new line to be perpendicular to the given line?

The new line must have a direction vector that is orthogonal (perpendicular) to the direction vector of the given line.

What is the direction vector of the given line?

The direction vector of the given line is $(1, 1, 1)$.

How do you find a direction vector for the line passing through the origin and perpendicular to the given line?

Any vector orthogonal to $(1, 1, 1)$ can serve as the direction vector; for example, $(1, -1, 0)$ or $(1, 0, -1)$.

What is the general equation of the line passing through the origin with a specified direction vector?

The line can be written as $r = t\,d$, where d is the direction vector, so for example, $r = t(1, -1, 0)$.

How do you verify the perpendicularity condition between the two lines?

Verify that the dot product of the two direction vectors is zero; for example, $(1, 1, 1) \cdot (a, b, c) = 0$.

What is the equation of the line passing through the

origin and perpendicular to the given line?

One possible line is x = t, y = -t, z = 0, which has direction vector (1, -1, 0).

How do you write the symmetric form of the found line?

The line can be written as x/y = -1, z=0, or in parametric form as x = t, y = -t, z=0.

Additional Resources

Find The Equation Of The Line Through The Origin Which Perpendicularly Meets The Line: x=1+t, y=2+t, z=1+t

Understanding how to find the equation of the line through the origin which perpendicularly meets the line: x=1+t, y=2+t, z=1+t is a fundamental problem in three-dimensional coordinate geometry. This problem involves spatial reasoning, vector algebra, and the application of perpendicularity conditions in 3D space. Whether you're a student preparing for exams or someone interested in the geometric properties of lines in space, mastering this type of problem enhances your overall understanding of vector calculus and spatial relationships.

In this guide, we will explore this problem systematically, breaking down each step with detailed explanations, drawing on vector operations, and illustrating the geometric intuition behind the solution.

Understanding the Problem

Before diving into the solution, let's clarify what the problem entails:

- Given Line:
The parametric equations of the given line are
$$
\begin{cases}
x=1+t \\
y=2+t \\
z=1+t
\end{cases}
$$
which describes a line passing through the point $(1,2,1)$ with a direction vector $\vec{d} = (1,1,1)$.

- Desired Line:
Find the equation of a line that:
1. Passes through the origin $(0,0,0)$.
2. Is perpendicular (i.e., makes a right angle) with the given line.

Geometric Intuition and Approach

The key geometric insight is:

- The line we seek passes through the origin $(0,0,0)$.
- It is perpendicular to the given line, which has direction vector $\vec{d} = (1,1,1)$.
- The line through the origin can be represented parametrically as
$$
\vec{r} = s \vec{n}
$$
where $\vec{n}$ is the direction vector of our sought line.

Objective:
Determine the vector $\vec{n}$ such that:

1. The line passing through the origin with direction $\vec{n}$ is perpendicular to the given line.
2. The line intersects the given line perpendicularly—meaning the shortest distance between lines is along the shortest line segment, perpendicular to both lines.

Step-by-Step Solution

Step 1: Express the Given Line in Vector Form

The given line L is:

$$
x=1+t, \quad y=2+t, \quad z=1+t
$$

which can be written as:

$$
\vec{r}_L(t) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}
$$

where:

- Point through which the line passes: $\vec{r}_0 = (1, 2, 1)$,
- Direction vector: $\vec{d} = (1, 1, 1)$.

Step 2: Define the Unknown Line Through the Origin

Let the line passing through the origin have direction vector $\vec{n} = (a, b, c)$. Its parametric form:

$$
\vec{r}_N(s) = s \begin{bmatrix} a \\ b \\ c \end{bmatrix}
$$

We want this line to perpendicularly meet the given line L, i.e., the shortest distance between the two lines occurs at their closest points, and the segment joining those points is perpendicular to both lines.

Step 3: Find the Condition for Perpendicularity

The shortest line segment connecting the two skew lines is along a line perpendicular to both. The vector connecting any point on the first line to any point on the second line should be orthogonal to both direction vectors.

Suppose:

- Point on line L: $\vec{r}_L(t) = \vec{r}_0 + t \vec{d}$,
- Point on the line through origin: $\vec{r}_N(s) = s \vec{n}$.

The vector connecting these points is:

$$
\vec{R} = \vec{r}_L(t) - \vec{r}_N(s) = (\vec{r}_0 + t \vec{d}) - s \vec{n}
$$

The key is that at the closest approach, the vector $\vec{R}$ is perpendicular to both $\vec{d}$ and $\vec{n}$:

$$
\boxed{
\begin{cases}
\vec{R} \cdot \vec{d} = 0 \\
\vec{R} \cdot \vec{n} = 0
\end{cases}
}
$$

Step 4: Derive Equations for t and s

From the conditions:

1. $\vec{R} \cdot \vec{d} = 0$:

$$
(\vec{r}_0 + t \vec{d} - s \vec{n}) \cdot \vec{d} = 0
$$

2. $\vec{R} \cdot \vec{n} = 0$:

$$
(\vec{r}_0 + t \vec{d} - s \vec{n}) \cdot \vec{n} = 0
$$

Step 5: Write the System of Equations

Given data:

$$
\vec{r}_0 = (1, 2, 1), \quad \vec{d} = (1, 1, 1)
$$

Let $\vec{n} = (a, b, c)$. The equations become:

1. Perpendicular to $\vec{d}$:

$$
(1, 2, 1) \cdot (1, 1, 1) + t (1, 1, 1) \cdot (1, 1, 1) - s (a, b, c) \cdot (1, 1, 1) = 0
$$

which simplifies to:

$$
(1 + 2 + 1) + t (1 + 1 + 1) - s (a + b + c) = 0
$$
$$
4 + 3t - s (a + b + c) = 0 \quad \text{(Equation 1)}
$$

2. Perpendicular to $\vec{n}$:

$$
(1, 2, 1) \cdot (a, b, c) + t (1, 1, 1) \cdot (a, b, c) - s (a, b, c) \cdot (a, b, c) = 0
$$

which simplifies to:

$$
(1a + 2b + 1c) + t (a + b + c) - s (a^2 + b^2 + c^2) = 0
$$

or:

$$
(a + 2b + c) + t (a + b + c) - s (a^2 + b^2 + c^2) = 0 \quad \text{(Equation 2)}
$$

Step 6: Choose $\vec{n}$ to Simplify

Since the problem asks for the line through the origin that perpendicularly meets the given line, the direction vector $\vec{n}$ must be perpendicular to $\vec{d} = (1,1,1)$.

This is a key insight: the line through the origin that is perpendicular to the given line must have a direction vector orthogonal to $\vec{d}$.

Hence, the condition:

$$
\vec{n} \cdot \vec{d} = a + b + c = 0
$$

This simplifies our previous equations.

Summary of Conditions:

- $\vec{n}$ is perpendicular to $\vec{d}$:

$$
a + b + c = 0
$$

- The minimal distance condition reduces to:

$$
4 + 3t = 0 \Rightarrow t = -\frac{4}{3}
$$

- The vector $\vec{R}$ simplifies to:

$$
\vec{R} = (1, 2, 1) + t (1, 1, 1) - s (a, b, c)
$$

Final Step: Find the Direction Vector $\vec{n}$

Since $\vec{n}$ must be orthogonal to $\vec{d}$, and the line through the origin must meet the given line perpendicularly, the direction vector $\vec{n}$ must be orthogonal to $\vec{d}$.

Let's find all $\vec{n} = (a, b, c)$ satisfying:

$$

a + b + c = 0
\]

Choose arbitrary values that satisfy this:

- For example, pick

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