Find The Maximum Rate Of Change Of F At The Given Point And The Direction In Which It Occursf(x,y,z)=(3x+9y)/z,

Find The Maximum Rate Of Change Of F At The Given Point And The Direction In Which It Occursf(x,y,z)=(3x+9y)/z,

Understanding how functions change at specific points is fundamental in multivariable calculus, with applications spanning physics, engineering, economics, and beyond. One key concept in this context is the rate of change of a function, particularly the maximum rate of change, which indicates the steepest ascent or descent at a particular point. This measure is crucial for optimizing functions, modeling real-world phenomena, and understanding the behavior of multivariable systems.

In this article, we explore how to determine the maximum rate of change of the function $f(x, y, z) = \frac{3x + 9y}{z}$ at a given point, and the specific direction in which this maximum occurs. We will delve into the mathematical tools involved, such as the gradient vector, and provide a comprehensive, step-by-step explanation. Whether you're a student studying multivariable calculus or a professional applying these concepts, this guide aims to clarify the process and deepen your understanding.

Understanding the Concept of Rate of Change in Multivariable Functions

Before diving into calculations, it’s essential to understand what the rate of change means in the context of functions with multiple variables.

The Gradient Vector and Its Significance

For a function $f(x, y, z)$, the gradient vector, denoted by ∇f, encapsulates the direction and rate of the steepest ascent of the function at a specific point. It is composed of the partial derivatives of f with respect to each variable:

$$
\nabla f(x, y, z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)
$$

The magnitude of the gradient vector at a point indicates the maximum rate of change of the function at that point. Moreover, the direction of the gradient vector points in the direction where the function increases most rapidly.

Maximum Rate of Change and Direction

- The maximum rate of change of \(f\) at a point is given by \(\|\nabla f\|\), the magnitude of the gradient vector.
- The direction in which this maximum occurs is along the direction of \(\nabla f\).

By calculating the gradient and its magnitude at a specified point, we can determine both the maximum rate and the direction of the steepest ascent.

Given Function and Objective

The function under consideration is:

\[
f(x, y, z) = \frac{3x + 9y}{z}
\]

Our goal is:

1. To compute the maximum rate of change of \(f\) at a specific point \((x_0, y_0, z_0)\).
2. To identify the direction in which this maximum rate occurs.

Since the user has not specified a particular point, we'll demonstrate the process with a general point \((x_0, y_0, z_0)\). The methodology can then be applied to any specific coordinates.

Calculating the Gradient of \(f(x, y, z)\)

The first step involves computing the partial derivatives of \(f\) with respect to each variable.

Partial derivatives of \(f\)

Given:

\[
f(x, y, z) = \frac{3x + 9y}{z}
\]

which can be rewritten as:

\[

$$
f(x, y, z) = (3x + 9y) \cdot z^{-1}
$$

Using the product rule and power rule:

- $\frac{\partial f}{\partial x}$:

$$
\frac{\partial}{\partial x} \left[(3x + 9y) \cdot z^{-1}\right] = 3 \cdot z^{-1} = \frac{3}{z}
$$

- $\frac{\partial f}{\partial y}$:

$$
\frac{\partial}{\partial y} \left[(3x + 9y) \cdot z^{-1}\right] = 9 \cdot z^{-1} = \frac{9}{z}
$$

- $\frac{\partial f}{\partial z}$:

$$
\frac{\partial}{\partial z} \left[(3x + 9y) \cdot z^{-1}\right] = (3x + 9y) \cdot (-1) z^{-2} = - \frac{3x + 9y}{z^2}
$$

Thus, the gradient vector is:

$$
\nabla f(x, y, z) = \left(\frac{3}{z}, \frac{9}{z}, - \frac{3x + 9y}{z^2} \right)
$$

Evaluating the Gradient at a Specific Point

To find the maximum rate of change at a point, substitute the coordinates (x_0, y_0, z_0) into the gradient:

$$
\nabla f(x_0, y_0, z_0) = \left(\frac{3}{z_0}, \frac{9}{z_0}, - \frac{3x_0 + 9y_0}{z_0^2} \right)
$$

The magnitude of this vector gives the maximum rate of change:

$$
\|\nabla f(x_0, y_0, z_0)\| = \sqrt{ \left(\frac{3}{z_0} \right)^2 + \left(\frac{9}{z_0} \right)^2 + \left(- \frac{3x_0 + 9y_0}{z_0^2} \right)^2 }
$$

Simplify:

$$
\|\nabla f\| = \sqrt{ \frac{9}{z_0^2} + \frac{81}{z_0^2} + \frac{(3x_0 + 9y_0)^2}{z_0^4} }
$$

$$
= \sqrt{ \frac{90}{z_0^2} + \frac{(3x_0 + 9y_0)^2}{z_0^4} }
$$

This expression provides the maximum rate of change at the chosen point.

Applying the Process to a Specific Point

To illustrate, assume a specific point:

$$
(x_0, y_0, z_0) = (1, 2, 3)
$$

Calculate each component:

- $\frac{3}{z_0} = \frac{3}{3} = 1$
- $\frac{9}{z_0} = \frac{9}{3} = 3$
- $3x_0 + 9y_0 = 3(1) + 9(2) = 3 + 18 = 21$

Compute the third component:

$$
- \frac{3x_0 + 9y_0}{z_0^2} = - \frac{21}{9} = - \frac{7}{3}
$$

Now, the gradient vector at this point:

$$
\nabla f(1, 2, 3) = \left(1, 3, - \frac{7}{3} \right)
$$

Magnitude:

$$
\|\nabla f\| = \sqrt{1^2 + 3^2 + \left(- \frac{7}{3}\right)^2} = \sqrt{1 + 9 + \frac{49}{9}} = \sqrt{10 + \frac{49}{9}}
$$

Expressed with a common denominator:

$$
= \sqrt{ \frac{90}{9} + \frac{49}{9} } = \sqrt{ \frac{139}{9} } = \frac{\sqrt{139}}{3}
$$

\]

Therefore, the maximum rate of change of \(f\) at \((1, 2, 3)\) is \(\frac{\sqrt{139}}{3}\).

The direction of this maximum change is along the gradient vector:

\[
\left(1, 3, - \frac{7}{3} \right)
\]

which can be normalized to a unit vector:

\[
\hat{\mathbf{u}} = \frac{1}{\|\nabla f\|} \left(1, 3, - \frac{7}{3} \right)
\]

Summary of the Process

To find the maximum rate of change of \(f(x, y, z)\) at a given point and the direction in which it occurs, follow these steps:

1. Compute the gradient \(\nabla f\):

- Find the partial derivatives \(\frac{\partial f}{\partial x}\), \(\frac{\partial f}{\partial y}\), \(\frac{\partial f}{\partial z}\

Frequently Asked Questions

How do you find the maximum rate of change of the function f(x, y, z) = (3x + 9y) / z at a given point?

To find the maximum rate of change, compute the gradient vector of f at the point, then find its magnitude. The maximum rate of change equals the magnitude of the gradient vector, and it occurs in the direction of the gradient.

What is the gradient vector of f(x, y, z) = (3x + 9y) / z?

The gradient vector is $\nabla f = (\partial f/\partial x, \partial f/\partial y, \partial f/\partial z)$. Computing partial derivatives gives $\nabla f = (3 / z, 9 / z, -(3x + 9y) / z^2)$.

How do you determine the direction in which the maximum

rate of change occurs for the given function?

The maximum rate of change occurs in the direction of the gradient vector at the point. Normalize the gradient vector to find the unit direction vector.

Given a specific point (x_0, y_0, z_0), how do you compute the maximum rate of change for f at that point?

Calculate the gradient vector at (x_0, y_0, z_0) using the partial derivatives, then find its magnitude. The maximum rate of change is equal to this magnitude.

Are there any special considerations when computing the maximum rate of change for functions involving division, such as f(x, y, z) = (3x + 9y) / z?

Yes, ensure that $z \neq 0$ to avoid division by zero. Also, be cautious with the domain restrictions and evaluate the gradient carefully at the point of interest.

Additional Resources

Find The Maximum Rate Of Change Of F At The Given Point And The Direction In Which It Occursf(x,y,z)=(3x+9y)/z

Understanding how functions change at specific points is a fundamental aspect of multivariable calculus, particularly when analyzing scalar fields. The maximum rate of change of $f(x, y, z) = \frac{3x + 9y}{z}$ at a given point reveals the steepest ascent direction on the surface defined by the function. This concept is central to fields such as physics, engineering, and data analysis, where understanding the rate and direction of change can inform decision-making, optimization, and modeling. In this comprehensive review, we will break down the process of computing the maximum rate of change (gradient magnitude) and the corresponding direction vector, elucidating the underlying mathematical principles involved.

Understanding the Function and Its Context

Before delving into calculations, it’s crucial to grasp the nature of the given function:

$$
f(x, y, z) = \frac{3x + 9y}{z}
$$

This function can be viewed as a scalar field in three-dimensional space, where the output depends on the spatial coordinates x, y, z. The numerator $3x + 9y$ suggests a linear combination of x and y, scaled by constants, while the denominator z introduces a division that affects the function's behavior significantly, especially near $z=0$.

Key features:
- Linear numerator: $3x + 9y$ indicates the function increases in the direction where this linear combination grows.
- Division by z: As z approaches zero, the function tends toward infinity or negative infinity, indicating a vertical asymptote.
- Domain considerations: The function is defined for all points where $z \neq 0$.

Calculating the Gradient ∇f

The maximum rate of change of a scalar function at a point is given by the magnitude of its gradient vector at that point. The gradient vector, denoted ∇f, points in the direction of the steepest ascent, and its magnitude indicates how steep that ascent is.

Definition:

$$
\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)
$$

Step-by-step calculation:

1. Partial derivative with respect to x:

$$
\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left(\frac{3x + 9y}{z} \right) = \frac{1}{z} \cdot \frac{\partial}{\partial x}(3x + 9y) = \frac{3}{z}
$$

2. Partial derivative with respect to y:

$$
\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left(\frac{3x + 9y}{z} \right) = \frac{1}{z} \cdot \frac{\partial}{\partial y}(3x + 9y) = \frac{9}{z}
$$

3. Partial derivative with respect to z:

Since f is a quotient, apply the quotient rule or treat as $f = (3x + 9y) \cdot z^{-1}$:

$$
\frac{\partial f}{\partial z} = (3x + 9y) \cdot \frac{\partial}{\partial z} (z^{-1}) = (3x + 9y) \cdot (-z^{-2}) = -\frac{3x + 9y}{z^2}
$$

Gradient vector:

$$
\nabla f(x, y, z) = \left(\frac{3}{z}, \frac{9}{z}, -\frac{3x + 9y}{z^2} \right)
$$

Evaluating the Gradient at a Specific Point

To find the maximum rate of change at a particular point, say (x_0, y_0, z_0), substitute these coordinates into the gradient:

$$
\nabla f(x_0, y_0, z_0) = \left(\frac{3}{z_0}, \frac{9}{z_0}, -\frac{3x_0 + 9y_0}{z_0^2} \right)
$$

Example:

Suppose the point is $(x_0, y_0, z_0) = (1, 2, 3)$.

- Compute the x-component:

$$
\frac{3}{3} = 1
$$

- Compute the y-component:

$$
\frac{9}{3} = 3
$$

- Compute the z-component:

$$
-\frac{3 \times 1 + 9 \times 2}{(3)^2} = -\frac{3 + 18}{9} = -\frac{21}{9} = -\frac{7}{3}
$$

Thus, the gradient at $(1, 2, 3)$:

$$
\nabla f = (1, 3, -\frac{7}{3})
$$

Maximum Rate of Change and Its Direction

The maximum rate of change at a point corresponds to the magnitude of the gradient vector:

$$
\text{Maximum Rate} = |\nabla f| = \sqrt{\left(\frac{3}{z}\right)^2 + \left(\frac{9}{z}\right)^2 + \left(-\frac{3x + 9y}{z^2}\right)^2}
$$

Using the general form, the magnitude at (x_0, y_0, z_0):

$$
|\nabla f| = \sqrt{\left(\frac{3}{z_0}\right)^2 + \left(\frac{9}{z_0}\right)^2 + \left(-\frac{3x_0 + 9y_0}{z_0^2}\right)^2}
$$

which simplifies to:

$$
|\nabla f| = \sqrt{\frac{9}{z_0^2} + \frac{81}{z_0^2} + \frac{(3x_0 + 9y_0)^2}{z_0^4}} = \sqrt{\frac{90}{z_0^2} + \frac{(3x_0 + 9y_0)^2}{z_0^4}}
$$

The direction in which this maximum rate occurs is along the gradient vector ∇f. To find the unit vector in this direction:

$$
\hat{\mathbf{u}} = \frac{\nabla f}{|\nabla f|}
$$

This unit vector indicates the precise direction of the steepest ascent at the given point.

Implications:
- Moving in the direction of ∇f results in the fastest increase of f.
- Moving in the opposite direction (i.e., $-\nabla f$) results in the fastest decrease.

Discussion: Features, Pros, and Cons

Features:

- The gradient provides both magnitude and direction, essential for optimization.
- The calculation directly ties into practical applications like gradient ascent/descent algorithms.
- It highlights how the function's behavior depends heavily on the point's coordinates, especially z.

Pros:

- Straightforward computation using partial derivatives.
- Clear geometric interpretation: the gradient points in the steepest ascent.
- Applicable to a broad class of functions, including ratios and non-linear functions.

Cons:

- The function's domain excludes $z=0$, where the function is undefined.
- Near the asymptote $z=0$, the gradient magnitude becomes very large, indicating steep changes but also potential numerical instability.
- For more complicated functions, gradient calculations may become complex.

Conclusion and Summary

In summary, determining the maximum rate of change of the function $f(x, y, z) = \frac{3x + 9y}{z}$ involves computing its gradient vector, evaluating its magnitude at the point of interest, and identifying the direction along which the gradient points. The gradient encapsulates the essence of the function's local behavior, offering insights into the steepest ascent and descent directions.

Key takeaways:

- The gradient of f is $\nabla f = \left(\frac{3}{z}, \frac{9}{z}, -\frac{3x + 9y}{z^2} \right)$.
- The maximum rate of change at a point is the magnitude of this vector.
- The direction of maximum increase corresponds to the unit vector in the direction of ∇f.

By mastering these concepts, one gains powerful tools to analyze and optimize multivariable functions, essential across scientific and engineering disciplines.

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