Find The (perpendicular) Distance From The Line Given By The Parametric Equationsx(t)= 10ty(t)=-3+7tz(t)=-2+9tto

Find The (perpendicular) Distance From The Line Given By The Parametric Equations x(t)= 10t, y(t)=-3+7t, z(t)=-2+9t is a fundamental problem in analytical geometry, often encountered in vector calculus and multivariable calculus. Understanding how to determine the shortest distance from a point to a line helps in various fields such as physics, engineering, computer graphics, and navigation systems. This article provides a comprehensive guide to calculating the perpendicular distance from a point to a line defined by parametric equations, with step-by-step explanations, mathematical formulas, and practical examples to enhance your understanding.

Understanding the Parametric Equations of a Line

What Are Parametric Equations?

Parametric equations describe a line or curve in space by expressing the coordinates as functions of a parameter, typically denoted as t. For a line in three-dimensional space, these equations take the form:

- $x(t) = x_0 + a_1 t$
- $y(t) = y_0 + a_2 t$
- $z(t) = z_0 + a_3 t$

where \((x_0, y_0, z_0)\) is a point on the line, and \((a_1, a_2, a_3)\) is the direction vector indicating the line's orientation.

Analyzing the Given Equations

The parametric equations provided are:

- $x(t) = 10t$
- $y(t) = -3 + 7t$
- $z(t) = -2 + 9t$

These describe a line passing through the point when \(t=0\), which is:

\[
(x(0), y(0), z(0)) = (0, -3, -2)
\]

and directed along the vector:

$$
\vec{d} = (10, 7, 9)
$$

This vector indicates the line's direction in 3D space.

Calculating the Perpendicular Distance From a Point to a Line

The General Formula

To find the perpendicular (shortest) distance D from a point $P_0(x_1, y_1, z_1)$ to a line defined by a point P on the line and a direction vector $\vec{d}$, the following formula is used:

$$
D = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}
$$

where:
- $\vec{AP} = \vec{P_0} - \vec{P}$ is the vector from a point on the line to the point P_0.
- $\times$ denotes the vector cross product.
- $|\cdot|$ indicates the magnitude of a vector.

This formula computes the perpendicular distance as the magnitude of the cross product of $\vec{AP}$ and $\vec{d}$, divided by the magnitude of the direction vector.

Steps to Find the Distance

To accurately compute the perpendicular distance, follow these steps:

1. Identify a point P on the line, typically at $t=0$: $P = (0, -3, -2)$.
2. Determine the point P_0 from which the distance is measured.
3. Calculate the vector $\vec{AP} = \vec{P_0} - \vec{P}$.
4. Compute the cross product $\vec{AP} \times \vec{d}$.
5. Find the magnitude of the cross product $|\vec{AP} \times \vec{d}|$.
6. Calculate the magnitude of the direction vector $|\vec{d}|$.
7. Divide the magnitude of the cross product by the magnitude of the direction vector to find the perpendicular distance D.

Example Calculation: Finding the Distance From a Specific Point

Suppose the Point is $P_0(5, 2, 3)$

Let's apply the steps to find the perpendicular distance from the point $P_0(5, 2, 3)$ to the line given by the parametric equations.

Step 1: Point on the Line

At $t=0$, the point on the line is:

$$
P = (0, -3, -2)
$$

Step 2: Vector $\vec{AP}$

Calculate:

$$
\vec{AP} = (x_0 - x_P, y_0 - y_P, z_0 - z_P) = (5 - 0, 2 - (-3), 3 - (-2)) = (5, 5, 5)
$$

Step 3: Direction Vector $\vec{d}$

From the parametric equations, the direction vector is:

$$
\vec{d} = (10, 7, 9)
$$

Step 4: Cross Product $\vec{AP} \times \vec{d}$

Calculating the cross product:

$$
\vec{AP} \times \vec{d} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
5 & 5 & 5 \\
10 & 7 & 9 \\
\end{vmatrix}
$$

Expanding the determinant:

$$
\mathbf{i} (5 \times 9 - 5 \times 7) - \mathbf{j} (5 \times 9 - 5 \times 10) + \mathbf{k} (5 \times 7 - 5 \times 10)
$$

Calculations:

$$
\mathbf{i} (45 - 35) - \mathbf{j} (45 - 50) + \mathbf{k} (35 - 50)
$$

$$
= \mathbf{i} (10) - \mathbf{j} (-5) + \mathbf{k} (-15)
$$

$$
= (10, 5, -15)
$$

Step 5: Magnitude of the Cross Product

Compute:

$$
|\vec{AP} \times \vec{d}| = \sqrt{10^2 + 5^2 + (-15)^2} = \sqrt{100 + 25 + 225} = \sqrt{350} \approx 18.71
$$

Step 6: Magnitude of $\vec{d}$

Calculate:

$$
|\vec{d}| = \sqrt{10^2 + 7^2 + 9^2} = \sqrt{100 + 49 + 81} = \sqrt{230} \approx 15.17
$$

Step 7: Final Distance Calculation

Apply the formula:

$$
D = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|} = \frac{18.71}{15.17} \approx 1.23
$$

Thus, the perpendicular distance from the point $(5, 2, 3)$ to the line is approximately 1.23 units.

Additional Tips for Accurate Calculation

Use of Vector Algebra Tools

For complex calculations, consider using scientific calculators, computer algebra systems (CAS), or software like MATLAB, WolframAlpha, or GeoGebra to perform vector operations accurately.

Verifying the Result

To ensure correctness, double-check the steps, especially the cross product and magnitudes. Small errors in calculation can significantly affect the final distance.

Handling Different Points and Lines

The method outlined applies to any point and line in 3D space. Just ensure the correct point on the line and the point from which the distance is measured are used.

Practical Applications of Finding the Distance From a Line

- **Navigation and GPS:** Calculating the shortest distance from a location to a route or path.
- **Physics:** Determining the shortest distance between particles or objects moving along lines.
- **Engineering:** Structural analysis where distances between elements affect stability.
- **Computer Graphics:** Calculating proximity between objects and lines for rendering or collision detection.

Conclusion

Finding the perpendicular distance from a point to a line defined by parametric equations like \(x(t) = 10t\), \(y(t) = -3 + 7t\), and \(z(t) = -2 + 9t\) involves understanding the geometry and applying vector

Frequently Asked Questions

How do you find the perpendicular distance from a point to a line given in parametric form?

To find the perpendicular distance from a point to a line given parametrically, first identify a point on the line and its direction vector. Then, use the formula for the distance from a point to a line: the magnitude of the cross product of the vector from the point to the line's point and the line's direction vector, divided by the magnitude of the direction vector.

What is the general formula for the perpendicular distance from a point to a parametric line?

Given a point P and a line with a point A and direction vector v, the perpendicular distance d is calculated as: d = |(AP × v)| / |v|, where AP is the vector from A to P.

How do you identify a point on the line from the parametric equations x(t)=10t, y(t)=-3+7t, z(t)=-2+9t?

Any point on the line can be found by choosing a specific value of t. For example, at t=0, the point is (0, -3, -2). At t=1, the point is (10, 4, 7). These points are on the line.

What is the direction vector of the line given by x(t)=10t, y(t)=-3+7t, z(t)=-2+9t?

The direction vector v is obtained from the coefficients of t: v = (10, 7, 9).

How do you calculate the perpendicular distance from a point to the line when given a specific point and the parametric equations?

Select a point on the line (e.g., at t=0), form the vector from this point to the given point, compute the cross product with the line's direction vector, then divide its magnitude by the magnitude of the direction vector to get the perpendicular distance.

Can you demonstrate finding the perpendicular distance from the point (0,0,0) to the line x(t)=10t, y(t)=-3+7t, z(t)=-2+9t?

Yes. Take point P=(0,0,0). The point on the line at t=0 is A=(0, -3, -2). The vector AP = (0-0, 0-(-3), 0-(-2)) = (0, 3, 2). The direction vector v = (10, 7, 9). Then, compute |AP × v| and divide by |v| to find the perpendicular distance.

What is the step-by-step process to compute the perpendicular distance from a given point to the parametric line in this problem?

1. Choose a specific t (e.g., t=0) to find a point on the line, A. 2. Form the vector from A to the given point P. 3. Calculate the cross product of this vector and the line's direction vector v. 4. Find the magnitude of this cross product. 5. Divide this magnitude by the magnitude of v to get the perpendicular distance.

Why is the cross product used in calculating the perpendicular distance from a point to a line?

The cross product of the vector from the line point to the external point and the line's direction vector gives a vector whose magnitude equals the area of the parallelogram formed, which relates directly to the perpendicular distance. Dividing by the magnitude of the direction vector normalizes this to the shortest distance.

Additional Resources

Find The (Perpendicular) Distance From The Line Given By The Parametric Equations x(t) = 10t, y(t) = -3 + 7t, z(t) = -2 + 9t

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Introduction

In the realm of analytical geometry, understanding the spatial relationships between lines and points is fundamental. One key concept is calculating the perpendicular distance from a point to a line in three-dimensional space. This measure tells us how far a point lies from a line in the shortest possible way—along a line perpendicular to it.

In this article, we delve into a thorough exploration of how to find the perpendicular distance from a point to a line specified by parametric equations, specifically:

- Line given by parametric equations:

\[
\begin{cases}
x(t) = 10t \\
y(t) = -3 + 7t \\
z(t) = -2 + 9t
\end{cases}

\]

- Point of interest: Let's assume a point $P(x_0, y_0, z_0)$ in space. For illustrative purposes, we will choose P as a specific point, but the methodology applies generally.

Understanding the Parametric Equations of the Line

The given line can be expressed in parametric form:

$$
\mathbf{r}(t) = \mathbf{r}_0 + t \mathbf{v}
$$

where:

- $\mathbf{r}_0$ is a point on the line (the position vector at $t=0$)
- $\mathbf{v}$ is the direction vector of the line

Extracting Key Components

Given the parametric equations:

$$
x(t) = 10t \Rightarrow \text{direction component in } x: 10
$$
$$
y(t) = -3 + 7t \Rightarrow \text{direction component in } y: 7
$$
$$
z(t) = -2 + 9t \Rightarrow \text{direction component in } z: 9
$$

Thus:

- Point on the line ($t=0$):

$$
\mathbf{r}_0 = (0, -3, -2)
$$

- Direction vector:

$$
\mathbf{v} = (10, 7, 9)
$$

The Goal: Find the Perpendicular Distance from a Point to the Line

Suppose the point P is at (x_0, y_0, z_0). The perpendicular distance d from P to the line is given by a well-known vector formula:

$$
d = \frac{\left| (\mathbf{P} - \mathbf{r}_0) \times \mathbf{v} \right|}{|\mathbf{v}|}
$$

\]

where:

- $\mathbf{P} = (x_0, y_0, z_0)$
- $\mathbf{r}_0$ is a point on the line
- $\times$ denotes the cross product
- $|\cdot|$ denotes the vector magnitude

This formula computes the magnitude of the cross product between the vector from a point on the line to the external point and the line's direction vector, divided by the length of the direction vector.

Choosing a Specific Point P

To illustrate the procedure explicitly, let's pick:

$$
P(4, 2, 3)
$$

Our goal is to find the shortest distance from $P(4, 2, 3)$ to the line.

Step-by-Step Calculation

1. Compute $\mathbf{P} - \mathbf{r}_0$:

$$
\mathbf{P} - \mathbf{r}_0 = (4 - 0, 2 - (-3), 3 - (-2)) = (4, 5, 5)
$$

2. Compute the cross product $(\mathbf{P} - \mathbf{r}_0) \times \mathbf{v}$:

Given:

$$
\mathbf{A} = (4, 5, 5)
$$
$$
\mathbf{v} = (10, 7, 9)
$$

The cross product:

$$
\mathbf{A} \times \mathbf{v} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
4 & 5 & 5 \\
10 & 7 & 9
\end{vmatrix}
$$

Calculating component-wise:

- $\mathbf{i}$: $(5)(9) - (5)(7) = 45 - 35 = 10$

- $\mathbf{j}$: $-[(4)(9) - (5)(10)] = -[36 - 50] = -(-14) = 14$
- $\mathbf{k}$: $(4)(7) - (5)(10) = 28 - 50 = -22$

Thus,

$$
\mathbf{A} \times \mathbf{v} = (10, 14, -22)
$$

3. Compute the magnitude of the cross product:

$$
|\mathbf{A} \times \mathbf{v}| = \sqrt{10^2 + 14^2 + (-22)^2} = \sqrt{100 + 196 + 484} = \sqrt{780}
$$

$$
|\mathbf{A} \times \mathbf{v}| \approx \sqrt{780} \approx 27.928
$$

4. Compute the magnitude of the direction vector $|\mathbf{v}|$:

$$
|\mathbf{v}| = \sqrt{10^2 + 7^2 + 9^2} = \sqrt{100 + 49 + 81} = \sqrt{230} \approx 15.1658
$$

5. Calculate the perpendicular distance:

$$
d = \frac{|\mathbf{A} \times \mathbf{v}|}{|\mathbf{v}|} \approx \frac{27.928}{15.1658} \approx 1.841
$$

Result:

The shortest (perpendicular) distance from the point $P(4, 2, 3)$ to the line defined by the parametric equations is approximately 1.84 units.

Generalized Approach for Any Point $P(x_0, y_0, z_0)$

The method illustrated is adaptable:

1. Identify a point $\mathbf{r}_0$ on the line (here, $(0, -3, -2)$)
2. Determine the direction vector $\mathbf{v}$ (here, $(10, 7, 9)$)
3. Compute the vector from the point on the line to P: $\mathbf{A} = \mathbf{P} - \mathbf{r}_0$
4. Calculate the cross product $\mathbf{A} \times \mathbf{v}$
5. Find the magnitude of this cross product
6. Divide by the magnitude of $\mathbf{v}$

This process yields a quick and reliable measure of the shortest distance.

Additional Considerations

Variations in the Point P

If the point P is not specified, the same process applies for any arbitrary point. The key is to substitute the coordinates of P into the calculations.

Alternative Point Choices on the Line

While in this example, we chose $t=0$ to find a specific point $\mathbf{r}_0$, any other point on the line can be used, provided the calculations are consistent.

Importance and Applications

Calculating the perpendicular distance from a point to a line has significant applications across various fields:

- Engineering: Precise measurements in structural analysis
- Physics: Determining shortest paths or minimal distances in space
- Computer Graphics: Collision detection and object modeling
- Navigation and Robotics: Path planning and obstacle avoidance

Understanding the geometric relationships in three-dimensional space facilitates accurate modeling, simulation, and problem-solving.

Conclusion

Finding the perpendicular distance from a point to a line given by parametric equations is a fundamental skill in analytical geometry. The vector cross product method provides an elegant and efficient solution, applicable in both academic and practical contexts.

By recognizing the components of the parametric line and applying vector algebra, one can determine the shortest distance with clarity and precision. The example elucidated here demonstrates the process step-by-step, empowering readers to apply the methodology to their own spatial problems with confidence.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Lay, D. (2012). Linear Algebra and Its Applications. Pearson.
- Larson, R., & Edwards, B. H. (2016). Precalculus with Limits. Cengage Learning.

Note: For different points P, simply replace the coordinates in the calculations. The approach remains consistent, making this a versatile tool

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