Find The Relative Extrema, If Any, Of The Function. Use The Second Derivative Test If Applicable.g(x)=x^2+2/x.

Find The Relative Extrema, If Any, Of The Function. Use The Second Derivative Test If Applicable. g(x) = x^2 + 2/x.

Understanding the behavior of a function, particularly identifying its relative maxima and minima, is fundamental in calculus. These points, known as relative extrema, are critical in analyzing the function's overall shape, optimizing real-world problems, and understanding the nature of the graph. In this article, we will thoroughly explore how to find the relative extrema of the function $g(x) = x^2 + \frac{2}{x}$ using the first and second derivative tests, with detailed steps and explanations.

Analyzing the Function g(x) = x^2 + 2/x

Before diving into the process of finding extrema, it’s essential to understand the structure and domain of the function.

Function Overview

- Expression: $g(x) = x^2 + \frac{2}{x}$
- Domain: All real numbers $x \neq 0$, since division by zero is undefined.
- Behavior at the boundaries:
- As $x \to 0^+$, $g(x) \to +\infty$ because $2/x \to +\infty$.
- As $x \to 0^-$, $g(x) \to -\infty$ because $2/x \to -\infty$.
- As $x \to \pm \infty$, $g(x) \to \infty$ due to the x^2 term dominating.

Understanding these behaviors helps anticipate where extrema might occur, typically near critical points where derivatives change signs.

Step 1: Find the First Derivative g'(x)

The first derivative indicates where the function's slope is zero or undefined, which are potential candidates for local maxima or minima.

Calculating g'(x)

Given:

$$
g(x) = x^2 + \frac{2}{x}
$$

Differentiate term-by-term:

$$
g'(x) = \frac{d}{dx} (x^2) + \frac{d}{dx} \left(\frac{2}{x} \right)
$$

$$
g'(x) = 2x + 2 \cdot \frac{d}{dx} (x^{-1})
$$

$$
g'(x) = 2x - 2x^{-2}
$$

$$
g'(x) = 2x - \frac{2}{x^2}
$$

Critical points occur where $g'(x) = 0$ or $g'(x)$ is undefined.

Find Critical Points

Set:

$$
2x - \frac{2}{x^2} = 0
$$

Multiply through by x^2 (noting $x \neq 0$):

$$
2x^3 - 2 = 0
$$

$$
2x^3 = 2
$$

$$
x^3 = 1
$$

$$
x = 1
$$

Since $x = 0$ is not in the domain, it cannot be a critical point.

Summary of critical points:
- $x = 1$

Step 2: Determine the Nature of Critical Points Using the First Derivative Test

Now, examine the sign of $g'(x)$ around $x=1$ to determine whether it is a maximum, minimum, or neither.

Sign Analysis of g'(x)

- For $x < 1$:
- Pick $x = 0.5$:

$$
g'(0.5) = 2(0.5) - \frac{2}{(0.5)^2} = 1 - \frac{2}{0.25} = 1 - 8 = -7 < 0
$$

- So, $g'(x) < 0$ for $x < 1$.

- For $x > 1$:
- Pick $x = 2$:

$$
g'(2) = 2(2) - \frac{2}{(2)^2} = 4 - \frac{2}{4} = 4 - 0.5 = 3.5 > 0
$$

- So, $g'(x) > 0$ for $x > 1$.

Interpretation:
- The derivative changes from negative to positive at $x=1$.
- Therefore, $g(x)$ has a local minimum at $x=1$.

Step 3: Find the Corresponding y-value of the Critical Point

Calculate $g(1)$:

$$
g(1) = (1)^2 + \frac{2}{1} = 1 + 2 = 3
$$

Thus, the function has a local minimum at $(1, 3)$.

Step 4: Confirm the Nature of the Critical Point Using the Second Derivative Test

The second derivative test provides a straightforward way to classify critical points.

Calculate g''(x)

Recall:
$$
g'(x) = 2x - \frac{2}{x^2}
$$
Differentiate again:
$$
g''(x) = 2 - \frac{d}{dx} \left(\frac{2}{x^2} \right)
$$
$$
g''(x) = 2 - 2 \cdot \frac{d}{dx} (x^{-2})
$$
$$
g''(x) = 2 - 2 (-2) x^{-3} = 2 + 4x^{-3}
$$
$$
g''(x) = 2 + \frac{4}{x^3}
$$

Evaluate at $x=1$:
$$
g''(1) = 2 + \frac{4}{1^3} = 2 + 4 = 6 > 0
$$

Since $g''(1) > 0$, the critical point at $x=1$ is a local minimum.

Summary of Findings

- Critical point: $x=1$
- Function value at critical point: $g(1) = 3$
- Nature of critical point: Local minimum (confirmed by second derivative test)

Additional Analysis: Behavior Near Domain Boundaries and Other Points

While we identified one critical point, it's valuable to analyze the function's behavior across its domain to understand the overall shape and potential for other extrema.

Behavior as $x \to 0^+$ and $x \to 0^-$

- As $x \to 0^+$, $g(x) \to +\infty$

- As $x \to 0^-$, $g(x) \to -\infty$

Behavior as $x \to \pm \infty$

- $g(x) \to +\infty$

This indicates the function has no other critical points where the derivative is zero, and the end behavior suggests a single minimum point.

Conclusion: Identifying and Classifying the Relative Extrema of $g(x) = x^2 + 2/x$

Through calculus techniques, we have determined:

- The only critical point in the domain is at $x=1$.
- The first derivative changes from negative to positive at $x=1$, indicating a local minimum.
- The second derivative at $x=1$ confirms this, as it is positive.
- The function's value at this point is $g(1) = 3$.

Therefore, the function $g(x) = x^2 + \frac{2}{x}$ has a single relative minimum at the point $(1, 3)$. There are no relative maxima in its domain.

Practical Applications of Finding Extrema

Understanding how to find the relative extrema of functions like $g(x) = x^2 + 2/x$ has numerous applications:

- Optimization problems: Finding the minimum cost, maximum profit, or minimal resource usage.
- Physics: Determining equilibrium points or points of maximum/minimum energy.
- Engineering: Designing systems with optimal performance characteristics.
- Economics: Identifying optimal pricing or production levels.

Knowing how to apply derivatives effectively allows professionals across various fields to make informed decisions based on the behavior of mathematical models.

Summary of Key Steps in Finding Relative Extrema

1. Compute the first derivative \(g'(x) \).
2. Find critical points by solving \(g'(x) = 0 \) or where \(g'(x) \) is undefined.
3. Use the First Derivative Test to classify the critical points:
- Sign change from negative to positive indicates a local minimum.

Frequently Asked Questions

What is the first step to find the relative extrema of g(x) = x^2 + 2/x?

The first step is to find the first derivative g'(x) to identify critical points where g'(x) = 0 or is undefined.

How do you compute the first derivative g'(x) of g(x) = x^2 + 2/x?

Using differentiation rules, g'(x) = 2x - 2/x^2.

What are the critical points of g(x) = x^2 + 2/x?

Critical points occur where g'(x) = 0 or is undefined. Solving 2x - 2/x^2 = 0 gives x = ±1.

How do you apply the second derivative test to determine the nature of the critical points?

Calculate the second derivative g''(x). If g''(x) > 0 at a critical point, it is a local minimum; if g''(x) < 0, it is a local maximum.

What is the second derivative g''(x) of g(x) = x^2 + 2/x?

Differentiating g'(x) = 2x - 2/x^2 gives g''(x) = 2 + 4/x^3.

What are the values of g''(x) at the critical points x = 1 and x = -1?

At x=1, g''(1) = 2 + 4/1 = 6 > 0, indicating a local minimum. At x=-1, g''(-1) = 2 + 4/(-1)^3 = 2 - 4 = -2 < 0, indicating a local maximum.

What are the relative extrema of g(x) = x^2 + 2/x?

The function has a local maximum at x = -1 and a local minimum at x = 1.

Additional Resources

Finding the Relative Extrema of the Function \(g(x) = x^2 + \frac{2}{x} \)

Understanding the behavior of a function in terms of its relative (local) maxima and minima is fundamental in calculus, offering insights into the function’s graph and its critical points. In this comprehensive review, we will explore how to find the relative extrema of the function \(g(x) = x^2 + \frac{2}{x} \), utilizing calculus principles such as derivatives, critical points, and the Second Derivative Test. This analysis will cover each step meticulously, ensuring clarity in the process and insights into the function's behavior.

Introduction to the Function \(g(x) = x^2 + \frac{2}{x} \)

Before diving into the calculus operations, it's essential to understand the nature and domain of the function:

- Function Definition: \(g(x) = x^2 + \frac{2}{x} \)
- Domain: \(x \neq 0 \) because of the term \(\frac{2}{x} \). So, the domain is \((-\infty, 0) \cup (0, \infty) \).

This function combines a quadratic term \(x^2 \), which grows large for large \(|x| \), with a reciprocal term \(\frac{2}{x} \), which introduces a vertical asymptote at \(x=0 \) and influences the shape of the graph significantly near zero.

Step 1: Calculating the First Derivative \(g'(x) \)

The first step in locating relative extrema is to compute the first derivative of \(g(x) \), which indicates the slope of the tangent line at any point:

\[

g'(x) = \frac{d}{dx} \left(x^2 + \frac{2}{x} \right).
\]

Applying basic differentiation rules:

- Derivative of \(x^2 \) is \(2x \).
- Derivative of \(\frac{2}{x} \) is \(-\frac{2}{x^2} \).

Therefore,

\[
g'(x) = 2x - \frac{2}{x^2}.
\]

This derivative function tells us where the slope of \(g(x) \) is zero (possible extrema) or undefined (possible vertical tangents or points of interest).

Step 2: Finding Critical Points

Critical points occur where the derivative is zero or undefined, provided those points are within the domain.

Set \(g'(x) = 0 \):

\[
2x - \frac{2}{x^2} = 0.
\]

Solve for \(x \):

\[
2x = \frac{2}{x^2}.
\]

Multiply both sides by \(x^2 \) (note: \(x \neq 0 \)):

\[
2x \cdot x^2 = 2,
\]

which simplifies to:

\[
2x^3 = 2,
\]

divide both sides by 2:

$$
x^3 = 1,
$$

and take the cube root:

$$
x = 1.
$$

Check for points where $g'(x)$ is undefined:

- $g'(x)$ is undefined at $x=0$, but since $x=0$ is outside the domain, it is not a critical point.

Summary:

- The only critical point in the domain is at $x=1$.

Step 3: Classifying Critical Points Using the Second Derivative Test

To determine whether $x=1$ corresponds to a local maximum, minimum, or neither, we employ the Second Derivative Test.

Step 3.1: Compute the second derivative $g''(x)$:

From

$$
g'(x) = 2x - \frac{2}{x^2},
$$

we differentiate term-by-term:

- Derivative of $2x$ is 2.
- Derivative of $-\frac{2}{x^2}$ is:

$$
-\frac{d}{dx} \left(\frac{2}{x^2} \right) = -2 \cdot \frac{d}{dx} \left(x^{-2} \right) = -2 \cdot (-2) x^{-3} = 4x^{-3} = \frac{4}{x^3}.
$$

Hence,

\[
g''(x) = 2 + \frac{4}{x^3}.
\]

Step 3.2: Evaluate \(g''(x) \) at the critical point \(x=1 \):

\[
g''(1) = 2 + \frac{4}{1^3} = 2 + 4 = 6.
\]

Since \(g''(1) > 0 \), the Second Derivative Test indicates that \(g \) has a local minimum at \(x=1 \).

Step 4: Finding the Coordinates of the Extrema

Having identified the critical point, we now find the corresponding function value:

\[
g(1) = (1)^2 + \frac{2}{1} = 1 + 2 = 3.
\]

Conclusion:

- The function has a relative (local) minimum at the point \((1, 3) \).

Step 5: Analyzing the Behavior at the Domain Boundaries and Infinity

While the primary focus is on local extrema, understanding the behavior of the function at the boundaries and as \(x \to \pm \infty \) provides a complete picture:

- As \(x \to \infty \):

\[
g(x) = x^2 + \frac{2}{x} \to \infty,
\]

since \(x^2 \to \infty \) dominates.

- As \(x \to 0^+ \):

$$
g(x) \to +\infty,
$$

because $\frac{2}{x} \to +\infty$.

- As $x \to 0^-$:

$$
g(x) \to -\infty,
$$

since $\frac{2}{x} \to -\infty$.

- As $x \to -\infty$:

$$
g(x) \to \infty,
$$

again dominated by x^2.

This analysis confirms the function’s graph has a vertical asymptote at $x=0$, with the function tending to $+\infty$ or $-\infty$, depending on the side.

Step 6: Summary and Final Conclusions

Based on the detailed calculus analysis:

- Critical Point: $x=1$ (within the domain).

- Second Derivative at $x=1$: $g''(1) = 6 > 0$, indicating a local minimum.

- Function value at the minimum: $g(1) = 3$.

- Nature of the extremum: The point $(1, 3)$ is a local minimum.

- No other critical points: The derivative does not vanish elsewhere, and the only point of interest in the domain for extrema is at $x=1$.

Additional Insights and Practical Applications

Understanding the relative extrema of functions like $g(x) = x^2 + \frac{2}{x}$ has practical applications in optimization problems, physics, economics, and engineering where such functions model real-world phenomena.

- For instance, in physics, the function could model potential energy with components representing different forces, and finding extrema could signify equilibrium points.

- In economics, such functions could represent cost functions with variable components, where minima indicate optimal production levels.

Summary of Key Steps for Finding Relative Extrema:

1. Compute the first derivative $g'(x)$.
2. Find critical points by solving $g'(x) = 0$ and checking where $g'(x)$ is undefined.
3. Use the Second Derivative Test by computing $g''(x)$ at critical points.
4. Determine the nature of each critical point (max, min, or saddle).
5. Evaluate the function at critical points to find the actual extremal values.
6. Analyze the behavior at the domain boundaries and at infinity to understand the overall graph.

In the case of $g(x) = x^2 + \frac{2}{x}$:

- The function has a local minimum at $(1, 3)$.
- No local maxima are present within the domain.
- The function exhibits asymptotic behavior near zero and unbounded growth as $x \to \pm \infty$.

Final Remarks

The process of finding relative extrema involves a systematic approach rooted in calculus. The key is understanding the behavior of derivatives and their signs. The Second Derivative Test provides a quick way to classify critical points but must be supplemented with a thorough analysis of the function

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