# Find The Root(s) Of F (x) = (x + 5)3(x - 9)2(x + 1). 50 POINTS!!!!!!! -5 With Multiplicity 3 5 With Multiplicity

**Find The Root(s) Of F (x) = (x + 5)3(x - 9)2(x + 1). 50 POINTS!!!!!!! -5 With Multiplicity 3 5 With Multiplicity**

Understanding how to find the roots of a polynomial function is a fundamental skill in algebra. In particular, analyzing the roots of the function $F(x) = (x + 5)^3 (x - 9)^2 (x + 1)$ involves identifying the values of $x$ where the function equals zero, as well as understanding the multiplicities of these roots. This process not only helps in graphing the function but also provides insight into its behavior near each root. This comprehensive guide will walk you through the steps to find the roots of the function, interpret their multiplicities, and understand the significance of these multiplicities in the context of polynomial functions.

## Understanding Polynomial Roots and Multiplicity

### What Are Roots of a Polynomial?

A root (or zero) of a polynomial function $F(x)$ is any value of $x$ that makes $F(x) = 0$. These are the points where the graph of the polynomial intersects the x-axis. For the given function, roots are the solutions to the equation:

$$(x + 5)^3 (x - 9)^2 (x + 1) = 0$$

## What Is Multiplicity?

The multiplicity of a root refers to the exponent of the corresponding factor in the factored form of the polynomial. In the function \( F(x) \), each factor's exponent indicates the multiplicity of that root:

- A root with an odd multiplicity results in the graph crossing the x-axis at that point.
- A root with an even multiplicity results in the graph touching the x-axis and turning around (tangent to the axis) at that point.

In our given function:

- The root at \( x = -5 \) has multiplicity 3 (since the factor is \( (x + 5)^3 \))
- The root at \( x = 9 \) has multiplicity 2 (since the factor is \( (x - 9)^2 \))
- The root at \( x = -1 \) has multiplicity 1 (since the factor is \( (x + 1) \), which is \( (x + 1)^1 \))

# Step-by-Step Process to Find the Roots of \( F(x) = (x + 5)^3 (x - 9)^2 (x + 1) \)

## Step 1: Factor the Polynomial Completely

The polynomial is already factored:

\[ F(x) = (x + 5)^3 (x - 9)^2 (x + 1) \]

This form makes it straightforward to identify the roots and their multiplicities.

## Step 2: Set Each Factor Equal to Zero

To find the roots, set each factor equal to zero:

\[

\begin{aligned}

x + 5 &= 0 \\

x - 9 &= 0 \\

x + 1 &= 0

\end{aligned}

\]

### Step 3: Solve for \( x \)

Solve each equation:

- \( x + 5 = 0 \Rightarrow x = -5 \)

- \( x - 9 = 0 \Rightarrow x = 9 \)

- \( x + 1 = 0 \Rightarrow x = -1 \)

These are the roots of the polynomial.

### Step 4: Determine the Multiplicity of Each Root

From the factored form:

- \( x = -5 \) has multiplicity 3

- \( x = 9 \) has multiplicity 2

- \( x = -1 \) has multiplicity 1

Understanding multiplicity helps predict the behavior of the graph at each root:

- Roots with odd multiplicity (such as \( -5 \) and \( -1 \)) will have the graph crossing the x-axis.

- Roots with even multiplicity (such as \( 9 \)) will have the graph touching but not crossing the x-axis.

## Implications of Root Multiplicity on Graph Behavior

## Graphical Behavior at Roots

The multiplicity affects how the graph interacts with the x-axis at each root:

- Odd multiplicity (1, 3, 5, ...): The graph crosses the x-axis at the root.
- Even multiplicity (2, 4, 6, ...): The graph touches the x-axis and turns around at the root, creating a tangent point.

For our roots:

- At $x = -5$ (multiplicity 3): The graph crosses the x-axis, and the crossing is relatively smooth but may have a point of inflection.
- At $x = 9$ (multiplicity 2): The graph touches the x-axis and turns back, creating a tangent point.
- At $x = -1$ (multiplicity 1): The graph crosses the x-axis with a linear behavior.

## Summary of Root Behavior

| Root $x$ | Multiplicity | Behavior on Graph |
|---|---|---|
| $-5$ | 3 | Crosses the x-axis (smooth crossing) |
| $9$ | 2 | Touches and turns around |
| $-1$ | 1 | Crosses the x-axis |

# Additional Insights and Applications

## Using Roots to Sketch the Graph

Knowing the roots and their multiplicities allows you to sketch a rough graph:

- Plot the roots on the x-axis.
- Determine whether the graph crosses or just touches the x-axis at each root.
- Consider the end behavior of the polynomial to understand how the graph extends to infinity.

## End Behavior of the Polynomial

The degree of the polynomial influences the end behavior:

- Total degree = sum of multiplicities = 3 + 2 + 1 = 6

- Since the highest degree is even (6), the ends of the graph will both go in the same direction:

- As $x \to \pm \infty$, $F(x) \to +\infty$, assuming the leading coefficient is positive.

# Conclusion: Final Roots and Their Significance

By analyzing the factored form $F(x) = (x + 5)^3 (x - 9)^2 (x + 1)$, we have:

- Root at $x = -5$ with multiplicity 3: The graph crosses the x-axis at this point, with a smooth but inflected crossing.
- Root at $x = 9$ with multiplicity 2: The graph touches and turns back at this point, not crossing the x-axis.
- Root at $x = -1$ with multiplicity 1: The graph crosses the x-axis with a simple linear crossing.

Understanding these roots and their multiplicities provides critical insight into the shape and behavior of the polynomial's graph, enabling better visualization and analysis. Whether you're solving for roots in algebra, sketching graphs, or analyzing polynomial behavior, mastering the process of finding roots with their multiplicities is an essential skill in mathematics.

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Key Takeaways:

- Roots are found by setting each factor equal to zero.
- The multiplicity of a root determines how the graph behaves at that point.
- The degree of the polynomial influences its end behavior.
- Factoring simplifies the process of identifying roots and their multiplicities.

This comprehensive approach ensures you are well-equipped to handle similar problems in algebra and calculus, enhancing your mathematical proficiency.

## Frequently Asked Questions

### What are the roots of the function F(x) = (x + 5)^3 (x - 9)^2 (x + 1)?

The roots are x = -5 (multiplicity 3), x = 9 (multiplicity 2), and x = -1 (multiplicity 1).

### How do the multiplicities of roots affect the graph of F(x)?

Roots with odd multiplicities (like -5 and -1) cause the graph to cross the x-axis at those points, while roots with even multiplicities (like 9) cause the graph to touch and bounce off the x-axis.

### What is the significance of the root at x = -5 with multiplicity 3?

Since -5 has an odd multiplicity (3), the graph crosses the x-axis at x = -5, and the crossing is relatively flat due to the multiplicity being greater than 1.

### Calculate the total degree of the polynomial F(x).

The degree is 3 + 2 + 1 = 6, since the exponents are 3, 2, and 1 respectively.

### If asked to find the roots for a point value of 50 points, what is the approach?

Identify the roots and their multiplicities from the factorization, and verify their correctness. The point value indicates the importance of understanding the roots thoroughly.

### What does the root at x = 9 with multiplicity 2 tell us about the behavior of F(x) near that point?

The graph touches the x-axis at x = 9 and bounces off, since the root has an even multiplicity, creating a local minimum or maximum at that point.

### How can you verify the roots of $F(x) = (x + 5)^3 (x - 9)^2 (x + 1)$?

Set F(x) = 0 and solve each factor for zero: x + 5 = 0  x = -5, x - 9 = 0  x = 9, x + 1 = 0  x = -1, confirming the roots and their multiplicities.

## Additional Resources

Find The Root(s) Of F (x) = $(x + 5)^3(x - 9)^2(x + 1)$. 50 POINTS!!!!!!! -5 With Multiplicity 3 5 With Multiplicity

Understanding the roots of a polynomial function is fundamental in algebra and calculus, as it provides insights into the function's behavior, graph intersections with the x-axis, and solutions to equations. In this article, we will explore how to determine the roots of the given polynomial function:

$$F(x) = (x + 5)^3 \cdot (x - 9)^2 \cdot (x + 1)$$

This function is a product of factors raised to various powers, each indicating the multiplicity of the corresponding root. The process involves identifying the roots, their multiplicities, and understanding their implications on the graph of $F(x)$.

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## Breaking Down the Polynomial Function

Before diving into root identification, it's crucial to understand the structure of the polynomial:

- The polynomial is written in factored form, which explicitly shows the roots and their multiplicities.
- The factors are:
- $(x + 5)^3$

- $(x - 9)^2$
- $(x + 1)$

Each factor corresponds to a root of the polynomial, with the exponent indicating its multiplicity.

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## Identifying the Roots and Their Multiplicities

1. Roots from the Factors

- Root 1: $x = -5$
- Root 2: $x = 9$
- Root 3: $x = -1$

2. Multiplicities of the Roots

- Root at $x = -5$: The factor is $(x + 5)^3$, so the multiplicity is 3.

- Root at $x = 9$: The factor is $(x - 9)^2$, so the multiplicity is 2.

- Root at $x = -1$: The factor is $(x + 1)^1$, so the multiplicity is 1.

3. Summary Table

| Root | Factor | Multiplicity | Explanation |
|--------|------------|--------------|--------------|
| $-5$ | $(x + 5)^3$ | 3 | Cubic multiplicity |
| $9$ | $(x - 9)^2$ | 2 | Quadratic multiplicity |
| $-1$ | $(x + 1)$ | 1 | Simple root |

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## Understanding Multiplicity and Its Impact

The multiplicity of a root influences the shape of the graph at the point where the root occurs:

- Multiplicity 1 (Simple Root): The graph crosses the x-axis at this root, changing sign.
- Odd Multiplicity (>1): The graph still crosses the x-axis, but with a flatter tangent at the root, leading to a more gentle crossing.
- Even Multiplicity: The graph touches the x-axis and turns around (tangent) at the root, without crossing it.

Let's analyze each root in detail:

1. Root at $x = -5$ (Multiplicity 3)

- Since the multiplicity is odd (3), the graph will cross the x-axis at $x = -5$.
- The crossing will be relatively gentle, with the graph passing through the axis and changing sign.
- The higher the odd multiplicity, the flatter the crossing appears.

2. Root at $x = 9$ (Multiplicity 2)

- With an even multiplicity (2), the graph will touch the x-axis at $x = 9$ without crossing.
- It will "bounce off" the axis, creating a tangent point where the slope is zero.
- The graph will be tangent to the x-axis at this root.

3. Root at $x = -1$ (Multiplicity 1)

- Being a simple root, the graph crosses the x-axis at $x = -1$.
- The crossing will be steep relative to roots with higher multiplicity.

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## Graphical Implications and Behavior Near Roots

Understanding the behavior of $F(x)$ near each root helps in sketching or analyzing the graph:

- At $x = -5$: The graph crosses from negative to positive or vice versa, with a relatively flat crossing.
- At $x = 9$: The graph touches the x-axis and turns back, indicating a local minimum or maximum at that point.
- At $x = -1$: The graph crosses the axis sharply, indicating a change in sign.

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## Verifying Roots and Multiplicities: A Step-by-Step Approach

While the factored form directly reveals roots and their multiplicities, in practical scenarios, you might need to verify these roots through other methods such as polynomial division, synthetic division, or using the Rational Root Theorem.

Step 1: Confirm the roots by substitution.

- Substitute $x = -5$:
$$F(-5) = ((-5)+5)^3 \cdot ((-5)-9)^2 \cdot ((-5)+1) = 0^3 \times (-14)^2 \times (-4) = 0$$
So, $x = -5$ is a root.

- Substitute $x = 9$:
$$F(9) = (9+5)^3 \times (9-9)^2 \times (9+1) = 14^3 \times 0^2 \times 10 = 0$$
Confirmed as a root.

- Substitute $x = -1$:

$$F(-1) = (-1+5)^3 \times (-1-9)^2 \times (-1+1) = 4^3 \times (-10)^2 \times 0 = 0$$

Confirmed as a root.

Step 2: Use polynomial division to find factors if starting from expanded form.

Step 3: Confirm multiplicities by analyzing the powers of each factor.

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## Calculating the Roots' Contributions to the Polynomial Degree

The degree of the polynomial is the sum of the multiplicities:

- $(x + 5)^3$ contributes degree 3.
- $(x - 9)^2$ contributes degree 2.
- $(x + 1)$ contributes degree 1.

Total degree:

$$3 + 2 + 1 = 6$$

Thus, $F(x)$ is a polynomial of degree 6, which is consistent with the highest power sum.

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## Implications for the Polynomial's End Behavior

Knowing the roots and their multiplicities helps determine how the polynomial behaves as $x \to \pm \infty$:

- The leading coefficient is positive (since all factors have positive leading terms when expanded).
- The degree (6) is even, so the polynomial will tend toward $+\infty$ as $x \to \pm \infty$.

Summary:

- As $x \to -\infty$, $F(x) \to +\infty$.
- As $x \to +\infty$, $F(x) \to +\infty$.

The roots cause the graph to cross or touch the x-axis, depending on their multiplicity, shaping the overall graph's appearance.

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## Conclusion: The Roots and Their Significance

By analyzing the factored form:

$$F(x) = (x + 5)^3 (x - 9)^2 (x + 1)$$

we have identified the roots as:

- $x = -5$ with multiplicity 3
- $x = 9$ with multiplicity 2
- $x = -1$ with multiplicity 1

Understanding these roots and their multiplicities provides critical insights into the polynomial's behavior:

- The nature of crossings at each root.

- The shape of the graph near the roots.

- The overall end behavior of the polynomial.

This process exemplifies how algebraic factorizations translate directly into graphical features, making root analysis an essential skill in both theoretical and applied mathematics.

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Final Note: Whether you're solving polynomial equations, sketching graphs, or analyzing functions, recognizing the roots and their multiplicities transforms complex algebraic expressions into visual and conceptual understandings, bridging the gap between abstract formulas and tangible insights.

## Find The Root S Of F X X 5 3 X 9 2 X 1 50 Points 5 With Multiplicity 3 5 With Multiplicity

Find The Root S Of F X X 5 3 X 9 2 X 1 50 Points 5 With Multiplicity 3 5 With Multiplicity

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