Find The Sum-of-products Expansions Of The The Following Boolean Functions:a) F(x,y,z)=x+y+zb) F(x,y,z)=(x+z)yc)

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Introduction

Boolean algebra is a fundamental branch of algebra dealing with variables that have two distinct values: true (1) and false (0). It forms the backbone of digital logic design, enabling engineers and computer scientists to simplify and implement logical expressions efficiently. One of the key techniques in Boolean algebra is the sum-of-products (SOP) expansion, which expresses a Boolean function as a sum (OR operation) of products (AND operations) of literals.

Understanding how to derive the SOP form of Boolean functions is crucial for designing optimized digital circuits, such as combinational logic circuits, multiplexers, and programmable logic devices. The SOP form simplifies the process of implementing Boolean functions using AND, OR, and NOT gates, making it a cornerstone concept in digital electronics.

In this article, we will explore how to find the sum-of-products expansions for two specific Boolean functions:

a) $F(x, y, z) = x + y + z$

b) $F(x, y, z) = (x + z) y c$

(Note: The original functions are provided with some notation issues; assuming the intended functions are as above based on standard Boolean notation. If the functions are different, please clarify. For the purpose of this article, we'll interpret the functions as standard Boolean expressions.)

Understanding Boolean Functions and SOP Expansions

Before delving into the specific functions, let's clarify the key concepts:

What Is a Sum-of-Products (SOP) Expression?

A sum-of-products expression is a Boolean function written as a logical OR (sum) of multiple AND (product) terms. Each product term is a conjunction (AND) of literals, where literals are variables or their complements.

Example:

$$
F(A, B, C) = AB + A'C + BC'
$$

This is an SOP form where each term is a product (e.g., AB) and the entire function is a sum (OR) of these products.

Why Use SOP Forms?

- They are straightforward to implement using AND and OR gates.
- They simplify the process of designing combinational circuits.
- They facilitate Boolean simplification and optimization.

Part A: Finding SOP Expansion for $F(x, y, z) = x + y + z$

Step 1: Understanding the Function

The function:

$$
F(x, y, z) = x + y + z
$$

is a simple OR of three variables. It outputs 1 whenever at least one of x, y, or z is 1.

Step 2: Constructing the Truth Table

To find the SOP form, start by constructing the truth table for F:

x	y	z	$F(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Step 3: Identify the Minterms

The function outputs 1 for all input combinations except when all variables are 0. The minterms where $F=1$:

- When $x=0, y=0, z=1$ → minterm $m_1 = \overline{x} \overline{y} z$

- When $x=0, y=1, z=0$ → minterm $m_2 = \overline{x} y \overline{z}$
- When $x=0, y=1, z=1$ → minterm $m_3 = \overline{x} y z$
- When $x=1, y=0, z=0$ → minterm $m_4 = x \overline{y} \overline{z}$
- When $x=1, y=0, z=1$ → minterm $m_5 = x \overline{y} z$
- When $x=1, y=1, z=0$ → minterm $m_6 = x y \overline{z}$
- When $x=1, y=1, z=1$ → minterm $m_7 = x y z$

Step 4: Write the SOP Expression

The SOP form is a sum of all minterms where the function is 1:

$$
F(x, y, z) = m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7
$$

Expressed explicitly:

$$
F(x, y, z) = \overline{x} \overline{y} z + \overline{x} y \overline{z} + \overline{x} y z + x \overline{y} \overline{z} + x \overline{y} z + x y \overline{z} + x y z
$$

Step 5: Simplify the Expression

Notice that the original function is a simple OR of three variables. The minimal SOP expression is just:

$$
F(x, y, z) = x + y + z
$$

which is already in SOP form. Hence, the SOP expansion of F is simply:

$$
\boxed{
F(x, y, z) = x + y + z
}
$$

This is both the minimal sum and the most straightforward SOP form.

Part B: Finding SOP Expansion for $F(x, y, z) = (x + z) y c$

Step 1: Clarify the Function

Assuming the function:

\[

$$
F(x, y, z) = (x + z) \cdot y \cdot c
$$

where c is an additional variable, making it a four-variable function $F(x, y, z, c)$. If the original function is only in three variables, please specify. For this example, we'll treat c as a variable, and the function is:

$$
F(x, y, z, c) = (x + z) \cdot y \cdot c
$$

Step 2: Expand the Function

Distribute the expression:

$$
F = (x + z) y c = (x y c) + (z y c)
$$

This is already in SOP form, as it is a sum (OR) of product terms:

$$
F = x y c + z y c
$$

Step 3: Write the Complete SOP Form

The minimal SOP expression is:

$$
\boxed{
F(x, y, z, c) = x y c + z y c
}
$$

which is a sum of two product terms involving all variables in each term.

Step 4: Express as a Sum of Minterms

To further express this in canonical SOP form, identify the minterms:

- For $x y c$:

- $x=1, y=1, c=1$, z can be 0 or 1.

- For $z y c$:

- $z=1, y=1, c=1$, x can be 0 or 1.

Thus, the minterms where $F=1$:

x	y	z	c	Minterm Expression	Minterm Number
1	1	0	1	$x=1, y=1, z=0, c=1$	$\overline{z} x y c$
1	1	1	1	$x=1, y=1, z=1, c=1$	$x y z c$
0	1	1	1	$x=0, y=1, z=1, c=1$	$z y c \overline{x}$

Expressed as minterms:

$$
F = \overline{z} x y c + x y z c + z y c \overline{x}
$$

Alternatively, the canonical SOP form is the sum of all minterms where the function evaluates to 1.

Summary and Key Takeaways

- The SOP form simplifies Boolean functions into a sum (OR) of ANDed literals, facilitating digital circuit design.
- For simple functions like $x + y + z$, the minimal SOP form is the same as the original expression.
- For more complex functions, constructing the truth table and identifying minterms helps derive the canonical SOP form.
- Distribution and Boolean algebra simplification help reduce the number of terms and literals, optimizing circuit implementation.

Conclusion

Mastering the process of finding sum-of-products expansions of Boolean functions is

Frequently Asked Questions

What is the sum-of-products (SOP) form of the Boolean function F(x,y,z) = x + y + z?

The SOP form of F(x,y,z) = x + y + z is expressed as the sum (OR) of product (AND) terms: F = x + y + z. Since this is already in a sum form, it can be written as F = x + y + z, which is minimal. Alternatively, representing explicitly: F = (x) + (y) + (z).

How do you find the sum-of-products expansion for

the Boolean function F(x,y,z) = (x + z) y?

First, distribute y over the expression: F = (x + z) y = x y + z y. Therefore, the sum-of-products form is F = x y + y z.

What is the difference between the given functions in terms of their SOP expansions?

Function a) simplifies directly to a sum of literals, while function b) involves distributing a product over a sum, resulting in two product terms: x y + y z.

Can the SOP form of F(x,y,z) = x + y + z be minimized further?

No, since the function is a sum of literals and cannot be simplified further; it is already in its minimal SOP form.

What Boolean algebra rules are used to derive the SOP form of F(x,y,z) = (x + z) y?

The distribution law: A(B + C) = AB + AC, is used to expand (x + z) y to x y + y z.

Is the function F(x,y,z) = x + y + z equivalent to the function F(x,y,z) = x y + y z?

No, these functions are not equivalent. x + y + z is a sum of literals, while x y + y z is a sum of product terms. The first is true when any of x, y, or z is true; the second is true only when either x and y are both true or y and z are both true.

How do you verify the correctness of the SOP expansion for the given functions?

By constructing truth tables for the original functions and their SOP forms and confirming that they produce identical outputs for all input combinations.

What is the significance of converting Boolean functions into SOP form?

Converting to SOP form facilitates implementation using logic gates and simplifies analysis, optimization, and design of digital circuits.

Are there any common simplifications for the functions provided in the question?

Yes, function a) x + y + z is already in its simplest form. Function b) x y + y z can sometimes be simplified further using Boolean algebra, but in this case, it is already minimized.

How can Karnaugh maps assist in deriving the SOP form of these Boolean functions?

Karnaugh maps visually group minterms to identify common expressions, making it easier to derive the minimal SOP form of the Boolean functions.

Additional Resources

Find The Sum-of-products Expansions Of The The Following Boolean Functions: a) F(x,y,z)=x + y + zb) F(x,y,z)=(x+z) y c

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Introduction

Boolean algebra forms the backbone of digital logic design, enabling engineers and computer scientists to simplify and implement complex logical expressions efficiently. A fundamental task in this realm is converting Boolean functions into their Sum-of-Products (SOP) forms. The SOP form is especially useful because it directly maps to electronic circuits, making it straightforward to realize functions using AND, OR, and NOT gates.

In this article, we focus on two specific Boolean functions:

- Function a: F(x, y, z) = x + y + zb
- Function b: F(x, y, z) = (x + z) y c

Our goal is to derive the SOP expressions for each, providing a detailed, step-by-step explanation suitable for learners and practitioners alike. Understanding these expansions not only deepens comprehension of Boolean algebra but also enhances practical skills in digital design and logic optimization.

Understanding Sum-of-Products (SOP) Form

Before diving into the specific functions, it is crucial to understand what SOP form entails:

- Sum: Logical OR operation (+)

- Product: Logical AND operation (implicit in concatenation or explicitly via AND gates)
- SOP: A logical expression expressed as a sum of multiple product terms, each product term being an AND of literals (variables or their complements).

Example:
An SOP form might look like:
F = (x y z') + (x' y z) + (x y' z)

This form is advantageous because each product term directly corresponds to a gate combination, simplifying circuit implementation.

Analyzing Function a: F(x, y, z) = x + y + zb

Step 1: Clarify the Expression

The first step is to interpret the given function accurately. The expression is:

F(x, y, z) = x + y + zb

At first glance, the notation "zb" might seem ambiguous. Typically, in Boolean algebra, variables are represented with lowercase letters, and concatenation indicates AND, while "+" indicates OR. The expression appears to involve variables z and b.

Possible interpretations:

1. The function involves variables x, y, z, and b.
2. The expression is:

F(x, y, z, b) = x + y + (z AND b)

Given the context, it's logical to interpret "zb" as z AND b.

Therefore,

F(x, y, z, b) = x + y + (z b)

Since the variables are x, y, z, and b, the function is four-variable.

Note: The initial problem statement mentions variables x, y, z, but the second function uses only x, y, z. For consistency, we'll assume the first function involves variables x, y, z, and b, making it a four-variable function.

Step 2: Express the Function in Boolean Terms

Expressed explicitly:

F(x, y, z, b) = x + y + (z b)

Our goal: Convert this into its Sum-of-Products form.

Step 3: Derive the SOP Form

Since the function is already expressed as a sum of literals and products, but contains an OR of variables and a product term, we can attempt to expand it into a sum of minterms.

Method 1: Use Boolean algebra laws

- The sum of variables (x + y + z b) can be expanded to cover all minterms where the function evaluates to 1.

Method 2: Use truth table approach

Construct a truth table to identify all minterms where F=1.

Step 4: Constructing the Truth Table

Variables: x, y, z, b

Total minterms: $2^4 = 16$

x	y	z	b	F(x,y,z,b) = x + y + (z b)	Function Output
0	0	0	0	0 + 0 + (00) = 0	0
0	0	0	1	0 + 0 + (01) = 0	0
0	0	1	0	0 + 0 + (10) = 0	0
0	0	1	1	0 + 0 + (11) = 1	1
0	1	0	0	0 + 1 + (00) = 1	1
0	1	0	1	0 + 1 + (01) = 1	1
0	1	1	0	0 + 1 + (10) = 1	1
0	1	1	1	0 + 1 + (11) = 2 (true)	1

Find The Sum Of Products Expansions Of The The Following Boolean Functions A F X Y Z X Y Zb F X Y Z X Z Yc

Find The Sum Of Products Expansions Of The The Following Boolean Functions A F X Y Z X Y Zb F X Y Z X Z Yc

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