Find The Volume Of The Solid Generated By Revolving The Region In The First Quadrant Bounded By The Coordinate

Find The Volume Of The Solid Generated By Revolving The Region In The First Quadrant Bounded By The Coordinate is a fundamental problem in calculus that illustrates the power of integral calculus to determine the volume of three-dimensional objects formed by revolving a region around a specified axis. This type of problem is commonly encountered in calculus courses, engineering, and physical sciences, where understanding the volume of solids of revolution is essential for applications such as manufacturing, physics simulations, and geometric modeling. In this comprehensive guide, we will explore the methods to find the volume of such solids, focusing on regions bounded by curves and axes in the first quadrant, and provide detailed step-by-step procedures to solve these problems efficiently.

Understanding the Concept of Solids of Revolution

What Is a Solid of Revolution?

A solid of revolution is a three-dimensional object created when a two-dimensional region is rotated about an axis. The resulting shape depends on the region's boundary curves and the axis of rotation. These solids are prevalent in calculus, especially in applications involving volume calculations.

Common Axes of Rotation

- x-axis: Rotation around the x-axis often involves integrating with respect to y.
- y-axis: Rotation around the y-axis involves integrating with respect to x.
- Arbitrary lines: Sometimes, rotation is around lines other than the axes, requiring more advanced techniques.

Setting Up the Problem: Region in the First

Quadrant Bounded by the Coordinate

Defining the Region

The region in the first quadrant is typically bounded by:
- The coordinate axes ($x=0$ and $y=0$),
- Curves such as $y=f(x)$, $g(x)$, or other functions.

For example, consider a region bounded by the curves $y=f(x)$ and $y=g(x)$, where these functions are continuous and non-negative in the interval $[a, b]$.

Key Points to Consider

- The region is in the first quadrant, so $x \geq 0$ and $y \geq 0$.
- The boundary curves intersect at points within $[a, b]$.
- The region is finite and well-defined, suitable for integration.

Methods to Find the Volume of the Solid of Revolution

There are primarily two methods to compute the volume of solids of revolution:

1. Disk Method (Solid of Revolution about the x-axis or y-axis)

- Used when the cross-sections perpendicular to the axis of rotation are disks (circles).
- Suitable when the region is described as $y = f(x)$ or $x = g(y)$.

2. Shell Method

- Used when the cross-sections are cylindrical shells.
- Particularly advantageous when rotating around vertical lines ($x = c$) or horizontal lines ($y = c$).

Applying the Disk Method

When to Use the Disk Method

- The region is bounded by $y = f(x)$, and the axis of rotation is the x-axis.
- The region is described as $y = f(x)$, with x in $[a, b]$.

Procedure for the Disk Method

1. Identify the region bounded by the curves.
2. Determine the axis of revolution.
3. Express the radius of the disk as a function of x (or y).
4. Set up the integral for the volume:

$$
V = \pi \int_{a}^{b} [R(x)]^2 dx
$$

where $R(x)$ is the distance from the x-axis to the curve $y = f(x)$.

Example Calculation

Suppose the region is bounded by $y = x^2$ and $y = 4$, from $x=0$ to $x=2$, revolved around the x-axis.

- The outer radius $R(x) = f(x) = y$.
- The volume:

$$
V = \pi \int_{0}^{2} [x^2]^2 dx = \pi \int_{0}^{2} x^4 dx = \pi \left[\frac{x^5}{5} \right]_0^2 = \pi \left(\frac{32}{5} \right) = \frac{32\pi}{5}
$$

Applying the Shell Method

When to Use the Shell Method

- The region is bounded by $y = f(x)$, and the axis of revolution is vertical (e.g., y-axis).
- The region is described in terms of x, but revolving around a vertical line other than the y-axis can make shells preferable.

Procedure for the Shell Method

1. Identify the region and the axis of revolution.
2. The radius of each shell is the distance from the shell to the axis.
3. The height of each shell corresponds to the function value.
4. Set up the integral:

$$
V = 2\pi \int_{a}^{b} \text{(radius)} \times \text{(height)} dx
$$

Example Calculation

Suppose the region between $y = x^2$ and $y = 4$, from $x=0$ to $x=2$, revolved around the y-axis:

- Radius = x
- Height = $4 - x^2$
- Volume:

$$
V = 2\pi \int_{0}^{2} x (4 - x^2) dx
$$

Calculating:

$$
V = 2\pi \int_{0}^{2} (4x - x^3) dx = 2\pi \left[2x^2 - \frac{x^4}{4} \right]_0^2 = 2\pi \left(2 \times 4 - \frac{16}{4} \right) = 2\pi (8 - 4) = 2\pi \times 4 = 8\pi
$$

Step-by-Step Example: Volume of a Solid Revolved Around the x-axis

Problem Statement

Find the volume of the solid generated by revolving the region bounded by the curve $y = \sqrt{x}$ and the x-axis, between $x=0$ and $x=4$, about the x-axis.

Solution Steps

1. Identify the region: Bounded by $y=\sqrt{x}$, $x=0$, $x=4$, and $y=0$.
2. Axis of revolution: x-axis.

3. Method: Disk method, since the cross-sections are disks perpendicular to x.
4. Set up the integral:

$$
V = \pi \int_{0}^{4} [\sqrt{x}]^2 dx = \pi \int_{0}^{4} x dx
$$

5. Calculate the integral:

$$
V = \pi \left[\frac{x^2}{2} \right]_0^4 = \pi \times \frac{16}{2} = 8\pi
$$

Final answer: The volume of the solid is 8π cubic units.

Additional Tips for Solving Volume of Revolution Problems

- Always sketch the region to visualize the problem.
- Determine the correct limits of integration based on the intersection points.
- Choose the most suitable method (disk or shell) based on the axis of rotation and the region's shape.
- Simplify the integral expressions before calculating.
- Be cautious with the functions' domains and intersections to ensure accurate bounds.

Common Challenges and How to Overcome Them

- Complex boundaries: Break down the region into simpler parts.
- Revolving around non-standard axes: Use substitution or shift the coordinate system.
- Multiple regions: Sum the volumes of individual parts.

Conclusion

Calculating the volume of solids generated by revolving regions in the first quadrant bounded by the coordinate axes and curves is a cornerstone of integral calculus. By understanding the principles behind the disk and shell methods, carefully setting up integrals, and visualizing the region, students and professionals can accurately determine these volumes for a variety of applications. Practice with different functions and axes of rotation enhances problem-solving skills and deepens understanding of geometric and calculus concepts. Whether for academic purposes or practical engineering, mastering these techniques provides a powerful toolset for analyzing three-dimensional objects created through revolution.

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Frequently Asked Questions

How do I determine the volume of a solid formed by revolving a region bounded by the coordinate axes in the first quadrant?

To find the volume, set up an integral using the disk or washer method depending on the region's boundaries and revolve around the x-axis or y-axis. Identify the bounds and the function describing the region, then integrate to compute the volume.

What is the general approach to find the volume when revolving a region bounded by the coordinate axes in the first quadrant?

Identify the region's boundary functions, choose the axis of revolution, and apply the appropriate method (disk, washer, or shell). Then, set up and evaluate the definite integral over the region's bounds to find the volume.

Can you explain the disk method for calculating the volume of a solid of revolution in the first quadrant?

Yes. The disk method involves slicing the solid perpendicular to the axis of revolution, forming disks. The volume is found by integrating the cross-sectional area $\pi[r(x)]^2$ or $\pi[r(y)]^2$ over the interval, where $r(x)$ or $r(y)$ is the radius from the axis.

What are common mistakes to avoid when finding the volume of a solid generated by revolving a region in the first quadrant?

Common mistakes include incorrect bounds of integration, using the wrong method (disk vs washer), forgetting to square the radius in the formula, or not correctly identifying the region's boundary functions. Carefully sketching the region helps prevent errors.

How do I set up the integral if the region is bounded by y = f(x), x-axis, and the y-axis?

If revolving around the x-axis, the volume is $V = \pi\int$[a to b] $(f(x))^2$ dx, where a and b are the x-values defining the region. For the y-axis, consider integrating with respect to y, adjusting the bounds accordingly.

What is the difference between using the disk method and the shell method for these problems?

The disk method slices perpendicular to the axis of revolution and is typically easier when revolving around the x-axis or y-axis for functions expressed as y=f(x). The shell method involves cylindrical shells and is useful when integrating with respect to the variable parallel to the axis of revolution, especially for functions expressed as x=f(y).

How do I handle a region bounded by two curves in the first quadrant when calculating the volume of the generated solid?

Identify the intersection points of the curves to determine bounds. Then, choose the method (disk, washer, or shell) suitable for the setup. Subtract the inner volume (if using washers) from the outer to find the total volume, setting up the integral accordingly.

Can symmetry help simplify the calculation of the volume of revolved regions in the first quadrant?

Yes, if the region or the shape is symmetric about an axis, you can compute the volume for one symmetric part and multiply accordingly, reducing the computational effort.

What is the process to verify the correctness of my calculated volume for a solid of revolution?

Check the limits of integration, ensure the formula matches the method used, compare with known volumes for simple shapes, and consider approximating the volume using numerical methods or graphing tools to confirm your result.

Are there any specific tools or software that can assist in calculating volumes of solids generated by revolving regions in the first quadrant?

Yes, tools like Wolfram Alpha, GeoGebra, Desmos, and calculus software such as MATLAB or Mathematica can help set up and evaluate integrals for volumes of revolution, providing visualizations and numerical results.

Additional Resources

Find The Volume Of The Solid Generated By Revolving The Region In The First Quadrant Bounded By The Coordinate Axes And A Curve

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Introduction

Understanding how to find the volume of a solid generated by revolving a region around an axis is a fundamental aspect of integral calculus, particularly in the field of volume computation. This topic involves applying the disk, washer, or shell methods to regions bounded by curves and axes, transforming two-dimensional areas into three-dimensional solids.

In this comprehensive review, we will explore the process of determining the volume of a solid formed by revolving a specific region in the first quadrant, bounded by the coordinate axes and a given curve. We will delve into the foundational concepts, outline the step-by-step methodology, examine various types of bounds and curves, and provide illustrative examples to deepen understanding.

Understanding the Geometric Context

The Region in the First Quadrant

The first quadrant of the Cartesian plane is characterized by positive values of x and y, i.e., $x \geq 0$ and $y \geq 0$. When considering a region within this quadrant bounded by the coordinate axes and a curve, the typical boundary components include:

- The x-axis (where $y = 0$)
- The y-axis (where $x = 0$)
- A curved boundary, described by a function $y = f(x)$ or $x = g(y)$, depending on the problem

The region is often defined over a specific interval, such as $x \in [a, b]$, where $a, b \geq 0$.

The Bounded Region

The bounded region is the area enclosed by:

- The coordinate axes (acting as boundaries)
- The given curve

This region is typically simple in shape, such as a sector, a segment under a curve, or an area between two curves.

The Concept of Solid of Revolution

What Is a Solid of Revolution?

A solid of revolution is a three-dimensional object created when a two-dimensional region in the plane is rotated around a specified axis. This rotation generates a volume, which can be computed using integral calculus.

Common Axes of Rotation

- Revolution around the x-axis: The region is rotated about the x-axis, often leading to a volume calculation using the disk or washer method.
- Revolution around the y-axis: The region is rotated about the y-axis, similarly leading to disk or washer methods, but with different variable considerations.
- Revolution around other lines: Such as lines $x = c$ or $y = c$, which might require shifting axes or using the shell method.

Visualizing the 3D Object

Imagine taking a thin slice of the region—say a vertical strip—and rotating it around the axis. As it rotates, it forms a circular cross-section, and stacking these slices creates the entire solid.

Methods for Calculating the Volume

1. Disk Method

The disk method is applicable when the region is revolved around an axis (usually the x-axis or y-axis), and the cross-sectional slices are disks.

- When revolving around the x-axis:

\[

$$
V = \pi \int_{a}^{b} [f(x)]^2 \, dx
$$

where $y = f(x)$ is the boundary of the region.

- When revolving around the y-axis:

$$
V = \pi \int_{c}^{d} [g(y)]^2 \, dy
$$

2. Washer Method

The washer method extends the disk method to cases where the region is bounded between two curves, leading to hollow cylinders (washers).

- Revolving around the x-axis:

$$
V = \pi \int_{a}^{b} \left([f_{outer}(x)]^2 - [f_{inner}(x)]^2 \right) dx
$$

where $f_{outer}(x)$ and $f_{inner}(x)$ are the outer and inner radii functions.

- Revolving around the y-axis:

$$
V = \pi \int_{c}^{d} \left([g_{outer}(y)]^2 - [g_{inner}(y)]^2 \right) dy
$$

3. Shell Method

The shell method considers cylindrical shells formed when slices are made parallel to the axis of revolution.

- Revolving around the y-axis:

$$
V = 2\pi \int_{a}^{b} x \, f(x) \, dx
$$

for a region between $x=a$ and $x=b$.

- Revolving around the x-axis:

$$
V = 2\pi \int_{c}^{d} y \, g(y) \, dy
$$

Step-by-Step Process to Find the Volume

1. Identify the region:

- Determine the equations of the boundary curves.
- Establish the interval over which the region extends.

2. Decide the axis of revolution:

- Choose the axis around which the region will be revolved.
- This choice influences which method (disk, washer, shell) to use.

3. Set up the integral:

- Express the boundary functions in terms of the variable of integration.
- Decide whether to integrate with respect to x, y, or use shells.

4. Determine the radii or heights:

- For disks/washers, find the radius functions.
- For shells, find the radius and height of the shells.

5. Construct the integral expression:

- Use the appropriate formula based on the method.
- Incorporate the limits of integration.

6. Evaluate the integral:

- Simplify the integrand.
- Use proper integration techniques to find the numerical value.

7. Check the units and logic:

- Confirm the integral bounds match the region.
- Ensure the radius or height functions are correct.

Working Through an Example

Suppose the problem is:

Find the volume of the solid generated by revolving the region in the first quadrant bounded by $y = x^2$, the x-axis, and the vertical line $x = 1$, about the x-axis.

Step 1: Identify the region

- Boundary curves: $y = x^2$, $y = 0$ (x-axis)

- x from 0 to 1

Step 2: Axis of revolution

- About the x-axis

Step 3: Method selection

- Disk method is suitable since the region revolves around the x-axis and the cross-sections are disks.

Step 4: Set up the integral

- The radius of each disk at a point x is $y = x^2$
- Cross-sectional area: $\pi (x^2)^2 = \pi x^4$

Step 5: Integral expression

$$
V = \int_{0}^{1} \pi [x^2]^2 dx = \pi \int_{0}^{1} x^4 dx
$$

Step 6: Evaluation

$$
V = \pi \left[\frac{x^5}{5} \right]_0^1 = \pi \times \frac{1}{5} = \frac{\pi}{5}
$$

Result: The volume of the solid is $\frac{\pi}{5}$ cubic units.

Special Cases and Considerations

- Regions bounded by multiple curves: When more than two curves define the boundary, the integral setup must account for the correct outer and inner radii or shells.
- Revolution about non-primary axes: For axes other than the x- or y-axis, shifts and transformations are necessary.
- Complex functions: For complicated boundary functions, substitution or numerical methods might be required.

Common Mistakes to Avoid

- Mixing up the bounds of integration with the boundaries of the region.
- Confusing the radius and height functions, especially when dealing with washer or shell methods.
- Forgetting to square the radius when calculating the volume (particularly

in disks/washers).
- Not verifying whether the chosen method is the most efficient for the problem.

Advanced Topics and Applications

- Volumes of revolution with parametric equations: When boundary curves are given parametrically.
- Volumes involving multiple regions: Calculated by splitting the region and summing the volumes.
- Physical applications: Calculating the volume of objects in engineering, physics, and manufacturing.
- Surface area of revolution: Although not the focus here, similar integrals are used for surface area calculations.

Summary and Final Thoughts

Calculating the volume of a solid generated by revolving a region in the first quadrant bounded by the axes and a curve is a critical skill in integral calculus. It blends geometric intuition with rigorous mathematical techniques. The key steps involve:

- Carefully identifying the bounded region.
- Choosing an appropriate axis of revolution.
- Selecting the proper method (disk, washer, shell).
- Setting up and evaluating the integral correctly.

Mastery of these concepts enables the solving of a broad class of problems involving three-dimensional objects derived from planar regions. The process reinforces understanding of integration, functions, and geometric reasoning, making it an essential component of advanced calculus studies

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