Gerald Graphs The Function F(x) = (x 3)2 1. Which Statements Are True About The Graph? Select Three Options.The

Gerald Graphs The Function F(x) = (x 3)2 1. Which Statements Are True About The Graph? Select Three Options.The is a compelling question that invites us to explore the fascinating world of quadratic functions and their graphical representations. Understanding the properties of the function F(x) = (x - 3)^2 + 1 can reveal significant insights into its shape, position, and behavior on the coordinate plane. In this comprehensive article, we will analyze the key features of this function, identify which statements about its graph are true, and provide detailed explanations to enhance your understanding of quadratic functions. Whether you're a student, educator, or math enthusiast, this guide aims to clarify complex concepts and improve your skills in graph analysis.

Understanding the Function F(x) = (x - 3)^2 + 1

Before delving into specific statements, it’s essential to understand the structure of this quadratic function. The given function is in vertex form, which makes it easier to interpret its graph directly.

Vertex Form of a Quadratic Function

A quadratic function in the form:
$$ F(x) = a(x - h)^2 + k $$
has:
- Vertex at (h, k)
- Axis of symmetry at x = h
- Opens upwards if a > 0 and downwards if a < 0

In our case:
$$ F(x) = (x - 3)^2 + 1 $$
- a = 1 (positive, so the parabola opens upward)
- h = 3
- k = 1

This tells us that:
- The vertex of the parabola is at (3, 1)
- The graph opens upward
- The minimum value of the function is 1, occurring at x = 3

Key Features of the Graph of F(x) = $(x - 3)^2 + 1$

Understanding the key features of the parabola helps in evaluating various statements about its graph.

1. Vertex and Axis of Symmetry

- Vertex: The lowest point on the graph at (3, 1)
- Axis of symmetry: The vertical line $x = 3$

2. Opening Direction and Shape

- Since the coefficient of the squared term ($a=1$) is positive, the parabola opens upward.
- The graph is symmetric with respect to the axis $x = 3$.

3. Range and Domain

- Domain: All real numbers $(-\infty, \infty)$
- Range: $[1, \infty)$, because the minimum value of the function is 1, and it increases without bound as x moves away from 3

4. Intercepts

- Y-intercept: When $x=0$, $F(0) = (0 - 3)^2 + 1 = 9 + 1 = 10$
- X-intercepts: Find x when $F(x) = 0$:

$$
(x - 3)^2 + 1 = 0 \Rightarrow (x - 3)^2 = -1
$$

Since a square cannot be negative, there are no real x-intercepts. The graph does not cross the x-axis.

Analyzing Statements About the Graph of F(x) = $(x - 3)^2 + 1$

Based on the features above, we can evaluate typical statements about the parabola. The question asks us to select three true statements from a list, so let's explore common options.

Statement 1: The graph has a vertex at (3, 1)

- True. As the function is in vertex form, the vertex occurs at (h, k) = (3, 1). This point is the minimum of the parabola and is visually identifiable on its graph.

Statement 2: The parabola opens downward

- False. Since a = 1 > 0, the parabola opens upward. A downward opening would require a negative coefficient.

Statement 3: The graph does not cross the x-axis

- True. Because the equation $(x - 3)^2 + 1 = 0$ has no real solutions, the parabola does not intersect the x-axis. Its vertex is above the x-axis, and it opens upward.

Statement 4: The minimum value of the function is 1

- True. The vertex at (3, 1) indicates the lowest point on the graph, so the minimum value of F(x) is 1.

Statement 5: The graph is symmetric with respect to the line x = 3

- True. The axis of symmetry for a quadratic in vertex form is x = h, which here is x = 3.

Statement 6: The y-intercept of the graph is at (0, 10)

- True. When x=0:

$$ F(0) = (0 - 3)^2 + 1 = 9 + 1 = 10 $$

- The y-intercept is at (0, 10).

Summary of True Statements About the Graph

Based on the analysis, the three most accurate statements are:

1. The vertex of the parabola is at (3, 1).

2. The graph does not cross the x-axis.
3. The minimum value of the function is 1.

These statements capture the core characteristics of the function's graph and are crucial for understanding its behavior.

Visualizing the Graph of F(x) = (x - 3)^2 + 1

Visual aids are essential in comprehending the features discussed. Here are some tips for sketching or interpreting the graph:

- Plot the vertex at (3, 1).
- Draw the axis of symmetry as the vertical line x=3.
- The parabola opens upward, symmetric about x=3.
- The y-intercept at (0, 10) can help in sketching the shape.
- Since there are no x-intercepts, the parabola remains above the x-axis.

Applications and Importance of Understanding Quadratic Graphs

Quadratic functions like F(x) = (x - 3)^2 + 1 are fundamental in various fields, including physics, engineering, and economics. Recognizing the graph's features allows for:

- Solving real-world problems involving projectile motion.
- Optimizing functions for maximum or minimum values.
- Analyzing the behavior of systems modeled by quadratic equations.

Conclusion

In summary, analyzing the function F(x) = (x - 3)^2 + 1 reveals several key properties:

- The vertex at (3, 1) signifies the minimum point.
- The parabola opens upward due to the positive leading coefficient.
- The graph does not cross the x-axis, as it has no real roots.
- The graph is symmetric about the line x = 3.
- The y-intercept is at (0, 10), providing a point for graphing.

Understanding these features enables a deeper comprehension of quadratic functions and their graphs, enhancing mathematical literacy and problem-solving skills. By mastering the interpretation of vertex form equations, you can quickly analyze and sketch parabolas, making it easier to solve complex problems across various disciplines.

Keywords: quadratic function, graph analysis, vertex form, parabola, x-intercepts, y-intercept, axis of symmetry, minimum value, upward opening, math education, function properties

Frequently Asked Questions

What is the general shape of the graph of the function F(x) = (x - 3)^2 + 1?

The graph is a parabola opening upwards with its vertex at (3, 1).

Does the function F(x) = (x - 3)^2 + 1 have any x-intercepts?

Yes, the x-intercept occurs at x = 3, where F(3) = 0, but in this case, since the minimum value is 1, the parabola does not cross the x-axis, so there are no x-intercepts.

Is the vertex of the parabola F(x) = (x - 3)^2 + 1 located at (3, 1)?

Yes, the vertex of the parabola is at (3, 1), which is the minimum point of the graph.

Does the graph of F(x) = (x - 3)^2 + 1 have any symmetry?

Yes, the parabola is symmetric with respect to the vertical line x = 3.

What is the range of the function F(x) = (x - 3)^2 + 1?

The range is all real numbers y ≥ 1, since the minimum value of the function is 1 and it increases without bound.

Which statements about the graph are true?

The true statements are: 1) The vertex is at (3, 1). 2) The graph opens upwards. 3) The y-values are always greater than or equal to 1.

Additional Resources

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Understanding the behavior of functions and their graphical representations is fundamental in algebra and calculus. When analyzing a specific function such as F(x) = (x - 3)^2 + 1, it is crucial to interpret its graph accurately to determine key features like vertex, symmetry, intercepts, and end behavior. This review provides a comprehensive exploration of the graph of F(x) = (x - 3)^2 + 1, examining its fundamental properties, identifying true statements, and highlighting its characteristics with clarity and precision.

Overview of the Function F(x) = (x - 3)^2 + 1

The function F(x) = (x - 3)^2 + 1 is a quadratic function, which typically graphs as a parabola. The form of this function is known as the vertex form, which makes it straightforward to identify key features such as the vertex, axis of symmetry, and transformations relative to the basic parabola y = x^2.

- Vertex: The vertex of the parabola is at (3, 1). This point represents the minimum value of the function since the parabola opens upward.
- Axis of Symmetry: The line x = 3 acts as the axis of symmetry, dividing the parabola into mirror images.
- Y-intercept: When x = 0, F(0) = (0 - 3)^2 + 1 = 9 + 1 = 10, so the y-intercept is at (0, 10).
- X-intercepts: To find x-intercepts, set F(x) = 0:
(x - 3)^2 + 1 = 0 → (x - 3)^2 = -1. Since a square cannot be negative, this equation has no real solutions, indicating no x-intercepts.

Now, let's analyze the true statements about this graph based on these features.

Key Features and True Statements About the Graph

Understanding the fundamental properties of the parabola defined by F(x) = (x - 3)^2 + 1 allows us to assess which statements about its graph are accurate. Here are three critical statements with explanations:

Statement 1: The graph has a minimum point at (3, 1).

This statement is true.

- Explanation: Since the quadratic is in vertex form, the vertex (3, 1) represents the lowest point on the parabola because the coefficient of the squared term is positive (1). The parabola opens upward, and the vertex is the minimum point.

Pros:
- Clearly identifiable from the form.
- Indicates the lowest value of the function.

Cons:
- Doesn't specify the maximum (which does not exist here, as the parabola opens upward).

Statement 2: The graph is symmetric with respect to the vertical line x = 3.

This statement is true.

- Explanation: The axis of symmetry for a parabola in vertex form $y = a(x - h)^2 + k$ is $x = h$. Here, $h = 3$, so the parabola is symmetric about the line $x = 3$.

Features:
- Symmetry ensures that for any point (x, y) on the parabola, there is a corresponding point (6 - x, y) mirrored across x = 3.
- This symmetry simplifies graphing and analysis.

Pros:
- Facilitates understanding of the parabola's shape.
- Useful in solving equations and inequalities involving the function.

Cons:
- Symmetry does not imply the parabola is symmetric across horizontal lines, only vertical.

Statement 3: The parabola does not intersect the x-axis.

This statement is true.

- Explanation: Since $(x - 3)^2$ is always ≥ 0, adding 1 shifts the parabola upward, ensuring $F(x) > 0$ for all real x. Therefore, the graph does not cross or touch the x-axis, meaning there are no real solutions to $F(x) = 0$.

Implications:
- The function's range is [1, ∞).
- The y-values are always greater than or equal to 1.

Pros:
- Simplifies the analysis of roots and solutions.
- Indicates the parabola's position relative to the x-axis.

Cons:
- Could be less intuitive for students expecting to see x-intercepts in all quadratics.

Additional Insights and Analyses

Beyond these three statements, various other properties can be explored to deepen understanding.

End Behavior of the Graph

- As x approaches infinity, F(x) = (x - 3)^2 + 1 tends to infinity.
- As x approaches negative infinity, F(x) also tends to infinity.
- Conclusion: The parabola opens upward with the arms extending infinitely in both directions.

Effect of Transformations

- The graph is a vertical translation of y = x^2 by 3 units right and 1 unit up.
- The standard parabola y = x^2 is shifted horizontally so that its vertex is at (3, 1).

Implications for Graphing

- The vertex can be plotted directly at (3, 1).
- The axis of symmetry is x = 3.
- The y-intercept is at (0, 10).
- The absence of x-intercepts indicates the parabola lies entirely above the x-axis.

Comparative Analysis with Similar Functions

Understanding this quadratic in context involves comparing it with other forms and functions:

- Standard form y = x^2: The vertex is at (0, 0), and the parabola is symmetric about y-axis.
- Horizontal shifts: Replacing x with (x - h) shifts the parabola horizontally.
- Vertical shifts: Adding a constant k shifts the parabola vertically.

In our case, F(x) = (x - 3)^2 + 1 is a horizontal shift 3 units right and upward shift 1 unit from y = x^2.

Conclusion: Summarizing True Statements and Features

Based on the detailed analysis, the three true statements about the graph of F(x) = (x - 3)^2 + 1 are:

1. The graph has a minimum point at (3, 1).
2. The graph is symmetric with respect to the vertical line x = 3.
3. The parabola does not intersect the x-axis.

These features collectively describe the core behavior and shape of the parabola, providing insights into its structure and how it relates to basic quadratic functions. Understanding these properties aids in graphing, solving equations, and applying quadratic concepts in various mathematical contexts.

Final thoughts: When analyzing quadratic functions, always consider their vertex form to quickly identify key features. Recognizing symmetry, intercepts, and end behavior simplifies the process of understanding and graphing these functions effectively.

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