Given The Domain Of Discourse Z+, Determine The Truth Value (True Or False) Of The Following Sta. (x(x2>x))

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Understanding the Domain of Discourse Z+

Before analyzing the statement, it's essential to clarify what the domain of discourse Z+ refers to. In logic and mathematics, the domain of discourse is the set of all objects over which variables can range. When we specify Z+, we are referring to the set of all positive integers.

Z+ includes:
- 1
- 2
- 3
- 4
- and so forth, extending infinitely.

Characteristics of Z+:
- All elements are greater than zero.
- The set is countably infinite.
- It contains no zero or negative integers.

Understanding this helps in interpreting the statement within the correct context.

Breaking Down the Statement: $x(x^2 > x)$

The statement in question can be interpreted as a logical formula:

- For all x in Z+, the predicate $x(x^2 > x)$ holds.

More explicitly, the statement asserts:

- For every positive integer x, the expression $x^2 > x$ is true.

Restating the statement:

- "For every x in Z+, x multiplied by (x squared greater than x)" — which simplifies to checking whether $x^2 > x$ holds for all positive integers x.

Analyzing the Inequality $x^2 > x$

To determine the truth value, we analyze the inequality:

- $x^2 > x$

Rearranged as:

- $x^2 - x > 0$
- $x(x - 1) > 0$

This form reveals that the inequality depends on the product of x and $(x - 1)$.

Understanding the Sign of $x(x - 1)$

Since x is in $Z+$ (positive integers), $x > 0$.

- For $x = 1$:
- $x(x - 1) = 1(1 - 1) = 1(0) = 0$
- So, $x^2 > x$ translates to $1 > 1$? No, $1 > 1$ is false; it's equal. Therefore, $x^2 = x$ for $x=1$.

- For $x > 1$:
- x □ 2, so $(x - 1)$ □ 1
- Since both x and $(x - 1)$ are positive, their product is positive.
- Thus, $x(x - 1) > 0$, implying $x^2 > x$.

Summary:
- When $x=1$: $x^2 = x$, so the inequality $x^2 > x$ is false.
- When x □ 2: $x^2 > x$ is true.

Evaluating the Truth Value of the Statement

The original statement seems to be a universal quantification:
- "For all x in $Z+$, $x^2 > x$"

Given the analysis:
- For $x=1$: $x^2 = x$, so the inequality does not hold.
- For $x=2, 3, 4, ...$: $x^2 > x$ holds true.

Conclusion:

- The statement "for all x in $Z+$, $x^2 > x$" is False because it fails at $x=1$.

Formal Logic Interpretation

Expressed formally:

- $\forall x \in Z+ \ (x^2 > x)$

Since at $x=1$, $x^2 = x$, the predicate $x^2 > x$ is false, making the entire universal statement false.

Therefore:

- The truth value of the statement is False.

Implications in Mathematical Logic and Number Theory

Understanding the truth value of such statements is crucial in various fields, including number theory, logic, and computer science.

Key points:

- Universal quantification is only true if the predicate holds for every element in the domain.
- A single counterexample (here, $x=1$) invalidates the universal statement.
- When analyzing inequalities over discrete sets like $Z+$, check boundary cases carefully.

Additional Considerations and Variations

While the initial statement considers the entire Z+ domain, modifications could lead to different truth values.

Examples of variations:

1. Existential quantification:
- "There exists an x in Z+ such that $x^2 > x$"
- This is True because, for example, $x=1$ yields $x^2 = x$, so the statement "$x^2 > x$" is false at $x=1$, but at $x=2$, $x^2=4 > 2$, so the existential statement is true.

2. Restricting the domain:
- If the domain were x □ 2, the statement "□x □ 2, $x^2 > x$" would be True.

3. Considering $x=1$ separately:
- Since $x=1$ does not satisfy $x^2 > x$, a more precise statement could be:
- "For all x in Z+ with x □ 2, $x^2 > x$," which is True.

Summary and Final Verdict

- The statement "For all x in Z+, $x^2 > x$" is False due to the counterexample at $x=1$.
- The key insight is understanding the behavior of the inequality at boundary cases within the positive integers.

Conclusion

In summary, when evaluating the truth value of the statement given the domain Z+, the critical step is analyzing the inequality $x^2 > x$ at specific points. Recognizing that at $x=1$, equality holds, not strict inequality, leads us to conclude that the universal statement does not hold for all positive integers.

Final verdict: False

This example highlights the importance of carefully examining boundary cases in mathematical logic and the evaluation of quantified statements within a specified domain.

Additional Resources for Deeper Understanding

- Number Theory Textbooks: Cover fundamental inequalities and properties of integers.
- Logic and Proof Techniques: Learn about quantifiers, counterexamples, and proof strategies.
- Mathematical Induction: Useful for establishing inequalities over infinite sets like Z+.
- Online Resources: Websites like Khan Academy or Brilliant.org offer interactive lessons on inequalities and logical quantification.

Meta Note: Understanding the interplay between inequalities and quantification is fundamental in mathematical logic. Analyzing specific cases within the domain, especially at boundary points, enables precise reasoning and accurate conclusions.

Frequently Asked Questions

What is the domain of discourse Z+ in the given statement?

The domain of discourse Z+ refers to the set of all positive integers (1, 2, 3, ...).

How do you interpret the predicate x(x^2 > x) in the context of Z+?

The predicate x(x^2 > x) states that for a given positive integer x, x squared is greater than x.

Is the statement x(x^2 > x) true for all x in Z+?

No, it is not true for all x in Z+; it is only true for x > 1.

What is the truth value of the statement for x = 1?

For x = 1, the statement becomes 1(1^2 > 1), which simplifies to 1(1 > 1), which is false, so the statement is false for x = 1.

What is the truth value of the statement for x = 2?

For x = 2, the statement is 2(4 > 2), which simplifies to 2(true), so the statement is true for x = 2.

Is the statement true for all x □ 2 in Z+?

Yes, for all x □ 2, x^2 > x holds true, making the statement true for all positive integers greater than 1.

How can we formally express the truth value of the statement in logical form?

The statement can be expressed as □x □ Z+, if x = 1 then false, else true, or more precisely, the statement 'x(x^2 > x)' is true for all x > 1 and false for x = 1.

What is the overall truth value of the statement '$x(x^2 > x)$' over Z+?

The statement is false for $x = 1$ and true for all $x > 1$, so it is not universally true over Z+.

Based on the analysis, what is the final conclusion about the truth value of the statement over Z+?

The statement '$x(x^2 > x)$' is false for $x = 1$ and true for all $x > 1$, so the statement is not universally true; its truth depends on the specific value of x.

Additional Resources

Understanding the Domain of Discourse Z+ and the Logical Statement $\forall x (x^2 > x)$

Introduction

In formal logic, especially within the realms of predicate calculus, the evaluation of the truth value of a statement heavily depends on the domain of discourse—the set of all elements over which variables can range. When the domain is specified as Z+, the set of all positive integers (i.e., natural numbers greater than zero: 1, 2, 3, 4, ...), the interpretation of quantified statements becomes precise and concrete.

The statement under scrutiny is:

$$
\forall x (x^2 > x)
$$

which reads as: "For all x in Z+, x^2 is greater than x."

This piece aims to explore this statement thoroughly, analyzing its truth value within the domain Z+, considering mathematical properties, potential edge cases, and logical implications.

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1. Clarifying the Domain: Z+

Before delving into the statement itself, it is crucial to understand the domain of discourse:

- Z+ (Positive Integers): The set $\{1, 2, 3, 4, \ldots\}$.
- Characteristics:
- Closed under addition, multiplication.
- Contains no zero or negative integers.
- The minimal element is 1.

Understanding that the domain includes all positive integers ensures that the evaluation of the statement is well-defined and avoids ambiguities that might arise if other subsets of integers or real numbers were considered.

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2. Dissecting the Statement

The core of the statement:

$$
\forall x \in Z+,\quad x^2 > x
$$

is a universal quantification; it asserts that every element x in Z+ satisfies the inequality $x^2 > x$.

To verify its truth, we need to analyze the behavior of the function $f(x) = x^2 - x$ across the domain Z+.

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3. Mathematical Analysis of the Inequality $x^2 > x$

3.1. Algebraic Re-arrangement

The inequality:

$$
x^2 > x
$$

can be rewritten as:

$$
x^2 - x > 0
$$

Factoring yields:

$$
x(x - 1) > 0
$$

This quadratic inequality depends on the sign of the product $x(x-1)$.

3.2. Sign Analysis for $x(x-1)$

Since $x \in Z+$, i.e., $x \geq 1$:

- For $x = 1$:

$$
1 \times (1 - 1) = 1 \times 0 = 0
$$

which is not greater than 0; it equals zero.

- For $x > 1$:

$$
x > 1 \implies x \geq 2
$$

Since x is positive:

$$
x > 0 \quad \text{and} \quad x - 1 \geq 1
$$

Thus, for $x \geq 2$:

$$
x(x - 1) \geq 2 \times 1 = 2 > 0
$$

which satisfies the inequality $x^2 > x$.

---

4. Evaluating the Truth Value

Given the above, the key question is whether the inequality holds for all x in Z+.

- At $x=1$:

$$
1^2 = 1,\quad \text{and } 1 \not> 1
$$

meaning:

$$
x^2 > x \quad \text{fails at } x=1
$$

since $1 \not> 1$.

- At $x=2$:

$$
2^2 = 4 > 2 \quad \text{true}
$$

- At $x=3$:

$$
3^2=9 > 3 \quad \text{true}
$$

- At larger x:

$$
x^2 > x \quad \text{obvious as } x^2 \text{ grows faster than } x
$$

Thus, the inequality holds for all $x \geq 2$.

---

5. Final Assessment of the Statement

The original statement:

$$
\forall x \in Z+,\quad x^2 > x
$$

is true if and only if the statement holds for every element in Z+. Given our analysis:

- It fails at $x=1$,
- It holds for all $x \geq 2$.

Therefore, the statement is False over Z+ because the universal quantifier requires the statement to be true for all elements in the domain, and it is not true at $x=1$.

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6. Implications and Nuances

6.1. The Critical Role of the Domain

This example highlights the importance of domain specification in logical statements:

- If the domain were $\{2, 3, 4, 5, \ldots\}$, the statement would be true.
- If the domain included zero or negative integers, the statement's truth value would differ significantly.

6.2. Edge Cases in Quantified Statements

- When dealing with inequalities or properties that are not strictly universal, check boundary points.
- For $x=1$, the inequality reduces to $1^2=1$, which does not satisfy $x^2 > x$.

6.3. Related Inequalities and Theorems

- For all $x \geq 2$, $x^2 > x$,
- The inequality $x^2 > x$ is equivalent to $x(x-1) > 0$,
- The sign of $x(x-1)$ depends on the position of x relative to 1.

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7. Extending the Analysis: Alternative Domains and Statements

7.1. Domain: $\mathbb{N}_0$ (non-negative integers)

- Zero included: $x=0$:

$$
0^2 = 0 \quad \text{and } 0 > 0 \text{?} \quad \text{No}
$$

- The statement would then fail at $x=0$.

7.2. Domain: All integers $\mathbb{Z}$

- For negative integers, say $x = -1$:

$$
(-1)^2 = 1 \quad \text{and } 1 > -1 \quad \text{true}
$$

- But for $x=-2$:

$$
(-2)^2 = 4 > -2 \quad \text{true}
$$

- The inequality holds for all integers x except at $x=0$, where:

$$
0^2=0 \quad \text{and } 0 > 0? \quad \text{No}
$$

- The universal statement over $\mathbb{Z}$ that $x^2 > x$ would be false because of zero.

---

8. Summary and Final Verdict

Aspect	Details
Domain	Z+ (positive integers: 1, 2, 3, ...)
Statement	$\forall x \in Z+,\ x^2 > x$
At $x=1$	$1^2=1$, which does not satisfy $x^2 > x$
At $x \geq 2$	$x^2 > x$ holds true
Conclusion	The statement is False over Z+ because it fails at $x=1$

Hence, the evaluation of the statement $\forall x (x^2 > x)$ within the domain Z+ is:

False

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9. Broader Educational Insights

This example underscores key lessons in formal logic and mathematical reasoning:

- Always carefully specify the domain of discourse.
- Test boundary and critical points to verify universal claims.
- Recognize that properties true for "large enough" elements may not hold at minimal or boundary elements.
- Understand the algebraic manipulation of inequalities to facilitate logical analysis.

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10. Final Remarks

In conclusion, the statement "For all x in Z+, $x^2 > x$" is not universally true within the positive integers. Its falsity hinges on the element $x=1$, which does not satisfy the inequality. This highlights the importance of domain restrictions and boundary checks in logical and mathematical reasoning, especially in predicate calculus.

In essence, the truth value of logical statements is deeply intertwined with the domain of discourse, and meticulous analysis of boundary cases is essential for correct evaluation.

Given The Domain Of Discourse Z Determine The Truth Value

True Or False Of The Following Sta X X2 Gt X

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