(iii) A Tangent Is Drawn To The Graph Of Y=5+8x-4/3x^3.The Gradient Of The Tangent Is -28.Find The Coordinates

(iii) A Tangent Is Drawn To The Graph Of Y=5+8x-4/3x^3. The Gradient Of The Tangent Is -28. Find The Coordinates

Understanding how to find the point of tangency on a curve where the tangent has a specified gradient is a fundamental concept in calculus. In this problem, we are given the function $y = 5 + 8x - \frac{4}{3}x^3$, and we know that at a certain point, the tangent to the curve has a gradient (or slope) of -28. Our goal is to determine the coordinates of this point of tangency.

This comprehensive guide will walk you through the process step by step, including the derivation of the derivative (which gives the gradient function), solving for the x-coordinate where the gradient matches -28, and then calculating the corresponding y-coordinate. We'll also discuss the significance of these steps, why calculus is essential here, and how to verify your results.

Understanding the Problem

Given Data

- Function: $y = 5 + 8x - \frac{4}{3}x^3$
- Gradient of tangent at a point: -28

What is Being Asked?

1. Find the x-coordinate(s) where the tangent to the curve has a gradient of -28.
2. Calculate the corresponding y-coordinate(s) at these point(s).
3. Determine the coordinate pairs (x, y) at the point(s) of interest.

Step 1: Differentiating the Function to Find the Gradient Function

Calculus provides the tools to determine the gradient of a curve at any point by differentiating the function.

Derivative of $y = 5 + 8x - \frac{4}{3}x^3$

$$
\frac{dy}{dx} = \frac{d}{dx} \left(5 + 8x - \frac{4}{3}x^3 \right)
$$

Applying differentiation term-by-term:
- The derivative of a constant (5) is 0.
- The derivative of $8x$ is 8.
- The derivative of $-\frac{4}{3}x^3$ is $-\frac{4}{3} \times 3x^2 = -4x^2$.

Therefore:

$$
\boxed{
\frac{dy}{dx} = 8 - 4x^2
}
$$

This derivative function, $\frac{dy}{dx}$, gives the gradient of the tangent at any point x.

Step 2: Setting the Gradient Equal to -28 and Solving for x

Since the problem states that the tangent has a gradient of -28 at the point of interest, set:

$$
8 - 4x^2 = -28
$$

Now solve for x:

1. Subtract 8 from both sides:

$$
-4x^2 = -28 - 8
$$

$$
-4x^2 = -36
$$

2. Divide both sides by -4:

$$
x^2 = \frac{-36}{-4} = 9
$$

3. Take the square root:

$$
x = \pm \sqrt{9} = \pm 3
$$

Thus, the x-coordinates where the tangent has a gradient of -28 are:

$$
x = 3 \quad \text{and} \quad x = -3
$$

Step 3: Calculating the Corresponding y-Coordinates

Now, substitute each x-value into the original function $y = 5 + 8x - \frac{4}{3}x^3$ to find the corresponding y-values.

For $x = 3$:

$$
y = 5 + 8(3) - \frac{4}{3} \times 3^3
$$

Calculate step-by-step:

- $8 \times 3 = 24$
- $3^3 = 27$
- $\frac{4}{3} \times 27 = 4 \times 9 = 36$

Putting it all together:

$$
y = 5 + 24 - 36 = (29) - 36 = -7
$$

Coordinate at $x = 3$:

$$
(3, -7)
$$

For $x = -3$:

$$
y = 5 + 8(-3) - \frac{4}{3} \times (-3)^3
$$

Calculate step-by-step:

- $8 \times -3 = -24$
- $(-3)^3 = -27$
- $\frac{4}{3} \times -27 = 4 \times -9 = -36$

Putting it all together:

$$
y = 5 - 24 - (-36) = (5 - 24) + 36 = -19 + 36 = 17
$$

Coordinate at $x = -3$:

$$
(-3, 17)
$$

Step 4: Final Answer and Interpretation

The points on the graph of $y = 5 + 8x - \frac{4}{3}x^3$ where the tangent has a gradient of -28 are:

1. $\boxed{(3, -7)}$
2. $\boxed{(-3, 17)}$

These points are significant in understanding the behavior of the curve, indicating where the slope of the tangent is steeply negative. The presence of two points reflects the symmetry of the cubic function's derivative, which is quadratic in nature.

Additional Insights and Applications

Why Is This Important?

- Calculus in Geometry: Finding tangent points with specific gradients helps in understanding the shape and behavior of curves.
- Real-World Applications: Such calculations are critical in physics (determining instantaneous velocity), economics (marginal analysis), and engineering (stress-strain analysis).
- Optimization Problems: Knowing where the slope takes particular values helps in identifying maximum or minimum points, inflection points, and points of interest.

Further Explorations

- Graphing the Function and Tangents: Use graphing tools to visualize the points and see the tangent lines.
- Analyzing the Nature of the Points: Determine whether these points are maxima, minima, or points of inflection by examining the second derivative.
- Extending to Other Gradients: Find points where the tangent has different gradients to analyze the curve's behavior more broadly.

Conclusion

In this detailed exploration, we've successfully identified the points on the curve $y = 5 + 8x - \frac{4}{3}x^3$ where the tangent has a gradient of -28. The process involved differentiating the function to find the gradient function, solving a quadratic equation to find the relevant x-values, and substituting back into the original function to find the corresponding y-values. The key points are at $(3, -7)$ and $(-3, 17)$.

Mastering these techniques enhances your understanding of calculus and its applications to real-world problems involving curves and their tangents. Whether you're preparing for exams, solving engineering problems, or exploring mathematical theories, this approach provides a solid foundation for analyzing the geometry of functions and their tangents.

Keywords for SEO Optimization:
- Calculus tangent points
- Derivative of cubic functions
- Find coordinates with given gradient
- Tangent to curve problem
- Gradient function differentiation
- Solving for tangent points
- Calculus problem solutions
- Mathematical analysis of curves
- Derivative applications in geometry

- Find point of tangency with specific slope

Frequently Asked Questions

What is the equation of the given curve, and what is the significance of finding the tangent's gradient at a point?

The equation of the curve is $y = 5 + 8x - (4/3)x^3$. Finding the tangent's gradient at a point helps determine the slope of the tangent line to the curve at that point, which is useful in understanding the behavior and slope of the function at specific x-values.

How do you find the x-coordinate where the gradient of the tangent to the curve is -28?

First, differentiate y with respect to x to find dy/dx (the gradient function). Then, set dy/dx equal to -28 and solve for x to find the corresponding x-coordinate(s).

What is the derivative of $y = 5 + 8x - (4/3)x^3$, and how is it used to find the required point?

The derivative $dy/dx = 8 - 4x^2$. Setting this equal to -28, i.e., $8 - 4x^2 = -28$, allows solving for x to find the point where the tangent has the specified gradient.

How do you determine the y-coordinate corresponding to the x-value where the gradient is -28?

Substitute the x-value found from the gradient equation back into the original curve $y = 5 + 8x - (4/3)x^3$ to compute the corresponding y-coordinate.

What are the final coordinates of the point where the tangent to the graph has a gradient of -28?

Solving $8 - 4x^2 = -28$ gives x = ±⍰ 13. Substituting these into the original equation yields the points (⍰ 13, y) and (-⍰ 13, y), with y calculated by plugging x into $y = 5 + 8x - (4/3)x^3$.

Additional Resources

Understanding the Problem: A Tangent to the Curve $y=5+8x-(4/3)x^3$ with a Given Gradient

In the realm of calculus, the concept of tangent lines to curves is fundamental, providing insights into the behavior of functions at specific points. The problem at hand involves a cubic function, $y = 5 + 8x - \frac{4}{3}x^3$, and asks us to determine the coordinates of a point where a tangent line touches this curve, given that the gradient (or slope) of that tangent is -28. This problem encapsulates core calculus principles, notably differentiation and solving nonlinear equations, and offers an opportunity to explore the geometric interpretation of derivatives.

This article aims to dissect this problem comprehensively. We will walk through the process of understanding the function, calculating its derivative, applying the given gradient condition, solving the resulting equations, and interpreting the solutions. The goal is to elucidate each step with clarity and depth, ensuring a thorough grasp of the mathematical concepts involved.

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1. The Function and Its Significance

1.1 The Given Function

The function provided is:

$$ y = 5 + 8x - \frac{4}{3}x^3 $$

This is a cubic polynomial, characterized by a degree of three. Its general shape can vary significantly depending on the coefficients, with potential regions of increasing and decreasing behavior, inflection points, and local maxima or minima. Understanding this function's behavior is essential when considering tangent lines, as the slope at any point directly relates to the derivative of the function.

1.2 Geometric Interpretation

Graphically, the function plots a smooth curve that may rise or fall depending on the value of x. At any point along this curve, we can draw a tangent line that just "touches" the curve without crossing it at that point. The slope of this tangent line is given by the derivative of the function evaluated at that point.

In this context, the problem specifies that the tangent line's gradient (or slope) is -28, which indicates a steep negative slope. The goal is to find the specific x-coordinate(s) where this tangent line exists and then determine the corresponding y-coordinate(s).

---

2. Deriving the Gradient Function

2.1 Differentiation of the Function

To find the slope of the tangent line at any point x, we differentiate the function with respect to x:

$$
\frac{dy}{dx} = y' = \frac{d}{dx} \left(5 + 8x - \frac{4}{3}x^3 \right)
$$

Applying standard differentiation rules:

- The derivative of a constant (5) is 0.
- The derivative of $8x$ is 8.
- The derivative of $-\frac{4}{3}x^3$ is $-\frac{4}{3} \times 3x^2 = -4x^2$.

Thus:

$$
y' = 8 - 4x^2
$$

This derivative function $y' = 8 - 4x^2$ gives the gradient of the tangent at any point x along the curve.

2.2 Significance of the Derivative

Understanding the derivative allows us to determine where the tangent line has a specific gradient. Since the problem states that the gradient of the tangent is -28, we can set:

$$
8 - 4x^2 = -28
$$

and solve for x. This equation encapsulates the condition for the tangent's slope at the point(s) of interest.

---

3. Solving for the Point of Tangency

3.1 Setting the Gradient Condition

The core equation derived from the derivative:

$$
8 - 4x^2 = -28
$$

can be rearranged to isolate x^2:

$$
-4x^2 = -28 - 8
$$
$$
-4x^2 = -36
$$

Dividing both sides by -4:

$$
x^2 = \frac{-36}{-4} = 9
$$

The solutions for x are obtained by taking the square root:

$$
x = \pm \sqrt{9} = \pm 3
$$

This indicates that there are two points on the curve where the tangent has a gradient of -28, specifically at $x = 3$ and $x = -3$.

3.2 Calculating Corresponding y-Coordinates

To find the exact points, we substitute these x-values back into the original function:

- For $x = 3$:

\[

$$
y = 5 + 8(3) - \frac{4}{3}(3)^3
$$

Calculate step-by-step:

$$
8(3) = 24
$$
$$
(3)^3 = 27
$$
$$
\frac{4}{3} \times 27 = 4 \times 9 = 36
$$

So,

$$
y = 5 + 24 - 36 = (5 + 24) - 36 = 29 - 36 = -7
$$

- For $x = -3$:

$$
y = 5 + 8(-3) - \frac{4}{3}(-3)^3
$$

Calculate step-by-step:

$$
8(-3) = -24
$$
$$
(-3)^3 = -27
$$
$$
\frac{4}{3} \times (-27) = -36
$$

Thus,

$$
y = 5 - 24 - (-36) = 5 - 24 + 36 = (5 - 24) + 36 = -19 + 36 = 17
$$

Summary of solutions:

x	y	Coordinates
3	-7	(3, -7)
-3	17	(-3, 17)

These are the points on the curve where the tangent line has a slope of -28.

---

4. Geometric and Analytical Significance

4.1 Interpretation of the Points

The two points $(3, -7)$ and $(-3, 17)$ are critical in understanding the behavior of the cubic function:

- At these points, the tangent lines have a steep negative slope, indicating that the function is decreasing rapidly at these points.
- The symmetry in the solutions ($x = \pm 3$) hints at underlying symmetries in the cubic function, often related to its derivatives and inflection points.

4.2 Tangent Line Equations

Having identified the points, one can find the equations of the tangent lines:

- For $x = 3$:

$$
\text{Slope} = -28
$$
$$
\text{Point} = (3, -7)
$$
$$
\text{Equation: } y - (-7) = -28(x - 3)
$$
$$
y + 7 = -28x + 84
$$
$$
y = -28x + 77
$$

- For $x = -3$:

$$
\text{Point} = (-3, 17)
$$
$$
\text{Equation: } y - 17 = -28(x + 3)
$$
$$
y - 17 = -28x - 84
$$
$$
y = -28x - 67
$$

These lines are tangents to the curve at the respective points, with the calculated slopes.

---

5. Broader Context and Applications

5.1 Practical Implications

Understanding where a curve has a particular tangent slope is pivotal in various fields:

- Physics: Analyzing velocity and acceleration, where the slope of a position-time graph indicates speed.
- Economics: Determining points of maximum or minimum profit, cost, or revenue functions.
- Engineering: Designing systems with specific rate changes or sensitivities.

5.2 Mathematical Significance

This problem exemplifies core calculus techniques:

- Differentiation to find slopes
- Solving nonlinear equations
- Interpreting geometric meaning of derivatives
- Connecting algebraic solutions with graphical intuition

It also illustrates the symmetry inherent in cubic functions, and how derivatives help locate points of interest.

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6. Conclusion

The analysis of the function $y = 5 + 8x - \frac{4}{3}x^3$ in relation to its tangent lines with a specific slope demonstrates the power of calculus in understanding the behavior of complex functions. By differentiating, setting the derivative equal to the given slope, and solving for x, we pinpoint the exact locations on the curve where the tangent has the desired gradient. Substituting back into the original function yields the

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