(iv) Suppose Total Costs Are Constrained To Be 2,000 Per Day. Calculate The Economically Efficient Combination

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Understanding how to optimize resource allocation within a fixed budget is a fundamental aspect of economic decision-making. When total costs are constrained to a specific amount—such as 2,000 units per day—it becomes essential to determine the most economically efficient combination of inputs or production activities that maximize output or profit. This article explores the methodology for calculating the optimal combination under such cost constraints, emphasizing the principles of cost minimization and output maximization within budget limits.

Understanding the Concept of Economic Efficiency Under Cost Constraints

What Is Economic Efficiency?

Economic efficiency refers to the allocation of resources in a way that maximizes total output or utility, given the available resources and constraints. In the context of production, it involves selecting input combinations that produce the maximum output at the lowest possible cost or, conversely, producing a specified level of output at the lowest cost.

The Significance of Budget Constraints

When a firm faces a fixed budget—here, 2,000 per day—it must choose among various input combinations or production strategies that fit within this financial limit. The challenge is to identify the combination that yields the highest possible output or profit without exceeding the budget.

Approach to Calculating the Economically Efficient Combination

Step 1: Define the Cost Functions and Inputs

- Identify the types of inputs involved (e.g., labor, capital, raw materials).
- Determine the unit costs of each input.
- Construct the total cost function, which sums the costs of all inputs based on their quantities.

Step 2: Understand the Production Function

- Specify how inputs combine to produce output, typically expressed as a production function $Q = f(L, K, ...)$.
- Determine the marginal products of each input, which indicate how additional units of input affect output.

Step 3: Set the Cost Constraint

The total cost constraint is represented mathematically as:

$C = wL + rK \leq 2000$

where:

- C = total cost (fixed at 2000)
- w = wage rate or cost per unit of labor (L)
- r = rental rate or cost per unit of capital (K)
- L = quantity of labor
- K = quantity of capital

Step 4: Formulate the Optimization Problem

- The goal is to maximize output $Q = f(L, K)$ subject to the cost constraint:

$$\text{Maximize } Q = f(L, K)$$
$$\text{Subject to: } wL + rK = 2000$$

Step 5: Use Lagrangian Optimization Technique

- Set up the Lagrangian function:

$$\mathcal{L} = f(L, K) - \lambda(wL + rK - 2000)$$

- Here, λ is the Lagrange multiplier representing the marginal value of relaxing the cost constraint.
- Find the partial derivatives of $\mathcal{L}$ with respect to L, K, and λ, and set them equal to zero:

$$\partial\mathcal{L}/\partial L = f_L - \lambda w = 0$$
$$\partial\mathcal{L}/\partial K = f_K - \lambda r = 0$$
$$\partial\mathcal{L}/\partial \lambda = wL + rK - 2000 = 0$$

Interpreting the Optimal Solution: The Role of Marginal Products and Input Prices

The Condition for Optimality

- The first-order conditions imply that at the optimum:

$$f_L / w = f_K / r$$

- This is the "equalization of the marginal rate of technical substitution (MRTS)" to the ratio of input prices.
- In words, the firm should allocate inputs so that the marginal product per dollar spent is equal across all inputs.

Implication of the Cost Constraint

- The total expenditure on inputs must exactly equal 2,000 units (assuming full utilization for maximum efficiency).
- The optimal input combination balances the marginal productivity of inputs with their costs, ensuring no resources are wasted.

Practical Calculation: An Example Scenario

Assumptions for the Example

- Suppose:
- The production function is Cobb-Douglas: $Q = A L^\alpha K^\beta$
- Constants:
- $A = 1$ (for simplicity)
- $\alpha = 0.5$
- $\beta = 0.5$
- Input prices:
- w (labor cost) = 50 per unit
- r (capital cost) = 100 per unit

Calculating the Optimal Input Mix

- Total cost constraint:

$$50L + 100K = 2000$$

- The production function:

$$Q = L^{0.5} K^{0.5}$$

- The marginal products:

$$f_L = 0.5 L^{-0.5} K^{0.5}$$
$$f_K = 0.5 L^{0.5} K^{-0.5}$$

Applying the Marginal Product to Price Ratio Condition

$$f_L / w = f_K / r$$

- Substitute the values:

$$(0.5 L^{-0.5} K^{0.5}) / 50 = (0.5 L^{0.5} K^{-0.5}) / 100$$

- Simplify:

$$(L^{-0.5} K^{0.5}) / 100 = (L^{0.5} K^{-0.5}) / 200$$

- Cross-multiplied:

$$200 L^{-0.5} K^{0.5} = 100 L^{0.5} K^{-0.5}$$

- Divide both sides by 100:

$2 \; L^{-0.5} \; K^{0.5} = L^{0.5} \; K^{-0.5}$

- Rewrite:

$2 \; (K / L)^{0.5} = (L / K)^{0.5}$

- Recall that $(K / L)^{0.5} = (L / K)^{-0.5}$
- So:

$2 \; (K / L)^{0.5} = (K / L)^{-0.5}$

- Multiply both sides by $(K / L)^{0.5}$:

$2 \; (K / L)^{1} = 1$

- Therefore:

$2 \; (K / L) = 1$

- Solving for K:

$K = (1/2) \; L$

Using the Cost Constraint to Find Quantities

- Substitute K into the cost constraint:

$50L + 100 \; (1/2) \; L = 2000$

- Simplify:

$50L + 50L = 2000$

- Combine like terms:

$100L = 2000$

- Solve for L:

$L = 20$

- Find K:

$K = (1/2) \; 20 = 10$

Result

- The optimal input combination to produce the maximum output within a 2,000 budget is:

- Labor (L) = 20 units

- Capital (K) = 10 units

- The maximum output can then be calculated:

Q = (20)^{0.5} (10)^{0.5} ≈ 4.472 3.162 ≈ 14.14 units of output

Summary and Key Takeaways

- When total costs are constrained, the key is to allocate resources so that the marginal product per dollar spent is equalized across all inputs.
- The Lagrangian method provides a systematic way to solve the optimization problem subject to budget constraints.
- Understanding the relationship between input prices, marginal products, and the production function is crucial for determining the most efficient input combination.
- Practical examples demonstrate how to apply these principles to real-world scenarios, guiding decision-makers toward cost-effective production strategies.

Conclusion

In conclusion, calculating the economically efficient combination of inputs under a fixed daily cost constraint involves analyzing the

Frequently Asked Questions

What is the significance of constraining total costs to 2,000 per day in determining the economically efficient combination?

Constraining total costs to 2,000 per day sets a budget limit, requiring the calculation of the combination of inputs or outputs that maximizes benefits or minimizes costs within this fixed expenditure, thereby identifying the most economically efficient solution.

How do we approach calculating the economically efficient combination when total costs are limited to 2,000 per day?

We analyze the marginal benefits and costs of different combinations of inputs or outputs, ensuring that total costs do not exceed 2,000, and select the combination where the marginal benefit per dollar spent is maximized, achieving efficiency within the budget constraint.

What role does the concept of marginal analysis play in this cost-constrained optimization problem?

Marginal analysis helps determine the additional benefit gained from increasing inputs or outputs, guiding the selection of the combination where the marginal benefit per unit cost is highest, ensuring the budget constraint is optimally utilized.

Can you explain how to identify the optimal combination of inputs with a total cost cap of 2,000?

Yes, by calculating the marginal cost and marginal benefit of each input, then selecting the combination where the ratio of marginal benefit to marginal cost is highest, without exceeding the total cost of 2,000 per day.

What tools or formulas are useful in calculating the economically efficient combination under a fixed total cost?

Tools like cost-benefit analysis, marginal rate of substitution, and the equimarginal principle are useful, along with formulas that compare marginal benefits to marginal costs across different combinations, ensuring the total cost limit is respected.

How does the concept of opportunity cost influence the calculation of the efficient combination under the cost constraint?

Opportunity cost reflects the value of the next best alternative foregone; when calculating the efficient combination, it helps prioritize options that yield the highest benefits relative to their costs within the 2,000 daily limit.

What are common challenges faced when determining the economically efficient combination with a strict cost constraint?

Challenges include accurately estimating marginal benefits and costs, dealing with nonlinear or complex relationships between variables, and ensuring all constraints are properly incorporated into the optimization process.

How can sensitivity analysis assist in verifying the robustness of the economically efficient combination under the cost constraint?

Sensitivity analysis examines how changes in assumptions or input values affect the optimal solution, helping to verify if the identified combination remains efficient under different scenarios or slight variations in costs or benefits.

Additional Resources

Total Costs Constraints: Unlocking the Path to the Most Economical Production Mix

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Introduction

In the realm of production economics, determining the most efficient combination of inputs or outputs under budget constraints is a fundamental challenge. When a firm faces a total cost constraint—in this case, a maximum allowable expenditure of $2,000 per day—the core objective becomes identifying the economically optimal or cost-effective production combination. This involves balancing different inputs or outputs to maximize efficiency without exceeding the fixed budget.

This article delves into the analytical process necessary to find the economically efficient combination of production when total costs are constrained to be $2,000 per day. We will explore the theoretical foundations, mathematical modeling, and practical steps involved in solving such a problem, providing a comprehensive guide for economists, students, and business decision-makers alike.

Understanding the Context: Cost Constraints and Production Optimization

Before diving into calculations, it’s vital to understand the nature of the problem:

- Total Cost Constraint: The firm cannot spend more than $2,000 daily on inputs or production costs.
- Objective: To identify the combination of inputs or outputs that maximizes production efficiency or profit within this budget.
- Variables:
- Inputs (e.g., labor, capital, raw materials)
- Outputs (goods or services produced)

In many cases, the problem simplifies to choosing quantities of two inputs (say, labor and capital) to maximize output or profit, given the cost constraint.

Step 1: Define the Production Function and Cost Components

Suppose the firm produces output Q using two inputs:

- Labor (L), costing w per unit
- Capital (K), costing r per unit

The production function could take various forms, for example:

$$
Q = f(L, K)
$$

For simplicity and clarity, let's assume a Cobb-Douglas production function:

$$
Q = A \times L^\alpha \times K^{1 - \alpha}
$$

where:

- A is total factor productivity,
- $\alpha \in (0,1)$ is the output elasticity of labor.

The total cost (TC) function is:

$$
TC = wL + rK
$$

Given the cost constraint:

$$
wL + rK \leq 2000
$$

The goal is to maximize Q subject to this cost constraint.

Step 2: Formulate the Optimization Problem

The problem becomes:

$$
\text{Maximize } Q = A \times L^\alpha \times K^{1 - \alpha}
$$
$$
\text{subject to } wL + rK = 2000
$$

This is a classic constrained optimization problem, solvable via methods such as Lagrangian multipliers.

Step 3: Set Up the Lagrangian

Define the Lagrangian function:

$$
\mathcal{L}(L, K, \lambda) = A L^\alpha K^{1 - \alpha} - \lambda (wL + rK - 2000)
$$

where:

- λ is the Lagrange multiplier.

Step 4: Derive First-Order Conditions

Differentiate $\mathcal{L}$ with respect to L, K, and λ:

$$
\frac{\partial \mathcal{L}}{\partial L} = A \alpha L^{\alpha - 1} K^{1 - \alpha} - \lambda w = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial K} = A (1 - \alpha) L^\alpha K^{-\alpha} - \lambda r = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial \lambda} = wL + rK - 2000 = 0
$$

Step 5: Solve the System of Equations

From the first two equations, eliminate λ:

$$
\frac{A \alpha L^{\alpha - 1} K^{1 - \alpha}}{w} = \frac{A (1 - \alpha) L^\alpha K^{-\alpha}}{r}
$$

Simplify:

$$
\frac{\alpha L^{\alpha - 1} K^{1 - \alpha}}{w} = \frac{(1 - \alpha) L^\alpha K^{-\alpha}}{r}
$$

Cross-multiplied:

$$
\alpha L^{\alpha - 1} K^{1 - \alpha} r = (1 - \alpha) L^\alpha K^{-\alpha} w
$$

Divide both sides by $L^{\alpha - 1} K^{-\alpha}$:

$$
\alpha r K^{(1 - \alpha) + \alpha} = (1 - \alpha) w L
$$

Since:

$$
K^{(1 - \alpha) + \alpha} = K^{1}
$$

the equation simplifies to:

$$
\alpha r K = (1 - \alpha) w L
$$

Rearranged:

$$
\frac{K}{L} = \frac{(1 - \alpha) w}{\alpha r}
$$

This ratio indicates the cost-optimal input ratio.

Step 6: Derive the Optimal Input Quantities

Using the cost constraint:

$$
wL + rK = 2000
$$

Replace K with the ratio:

$$
K = \frac{(1 - \alpha) w}{\alpha r} L
$$

Substitute into the cost constraint:

$$
wL + r \times \frac{(1 - \alpha) w}{\alpha r} L = 2000
$$

Simplify:

$$
wL + \frac{(1 - \alpha) w}{\alpha} L = 2000
$$
$$
wL \left(1 + \frac{1 - \alpha}{\alpha}\right) = 2000
$$

Express the sum:

$$
wL \times \frac{\alpha + 1 - \alpha}{\alpha} = 2000
$$
$$
wL \times \frac{1}{\alpha} = 2000
$$

Solve for L:

$$
L = \frac{2000 \alpha}{w}
$$

Similarly, compute K:

$$
K = \frac{(1 - \alpha) w}{\alpha r} \times L = \frac{(1 - \alpha) w}{\alpha r} \times \frac{2000 \alpha}{w} = \frac{(1 - \alpha) \times 2000}{r}
$$

Final Optimal Input Quantities

$$
\boxed{
L^ = \frac{2000 \alpha}{w}
}
$$

$$
\boxed{
K^ = \frac{2000 (1 - \alpha)}{r}
}
$$

These values specify the economically efficient combination of inputs within the $2,000 daily budget.

Step 7: Interpretations and Practical Implications

- Input Ratio: The optimal ratio of capital to labor is dictated by their respective costs and output elasticities:

$$
\frac{K^}{L^} = \frac{(1 - \alpha) w}{\alpha r}
$$

- Budget Allocation: The total cost exactly equals the constraint:

$$
wL^ + rK^ = 2000
$$

- Production Level: The maximum achievable output $Q^$ can be calculated by substituting $L^$ and $K^$ into the production function:

$$
Q^ = A \times (L^)^\alpha \times (K^)^{1 - \alpha}
$$

Practical Example

Suppose:

- $w = \$20$ per labor unit
- $r = \$50$ per capital unit
- $\alpha = 0.6$
- $A = 1$

```
Calculate:

\[
L^ = \frac{2000 \times 0.6}{20} = \frac{1200}{20} = 60
\]
\[
K^ = \frac{2000 \times (1 - 0.6)}{50} = \frac{2000 \times 0.4}{50} =
\frac{800}{50} = 16
\]

Check total cost:

\[
20 \times 60 + 50 \times 16 = 1200 + 800 = 2000
\]

Maximum output:

\[
Q^ = 1 \times 60^{0.6} \times 16^{0.4}
\]

Calculate:

\[
60^{0.6} \approx e^{0.6 \times \ln 60} \approx e^{0.6 \times 4.094} \approx
e^{2.456} \approx 11.65
\]
\[
16^{0.4} \approx e^{0.
```

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