Let The Function F'(x) Be Given By F(x) = 3x^4 8x^3 + 6x + 12 A) Find The Intervals Of Increase And Decreases,

Let The Function F'(x) Be Given By F(x) = 3x^4 8x^3 + 6x + 12 A) Find The Intervals Of Increase And Decreases, this question invites us to explore the fundamental concepts of calculus, specifically the relationship between a function and its derivative. Understanding how to analyze the increasing and decreasing behavior of a function is essential in calculus, as it provides insights into the function's local maxima and minima, concavity, and overall shape. In this comprehensive guide, we will delve into the process of determining the intervals where the function F(x) increases or decreases, based on its derivative F'(x). We will explore key calculus principles, step-by-step solution methods, and practical examples to reinforce your understanding.

Understanding the Function and Its Derivative

Before we proceed with the analysis, it is critical to understand what the function and its derivative represent. The function in question is:

$$F(x) = 3x^4 + 8x^3 + 6x + 12$$

Here, F(x) is a polynomial function, and F'(x), its derivative, provides the slope of the tangent line to the graph at each point x. The derivative essentially tells us whether the function is increasing or decreasing at a particular point:

- If $F'(x) > 0$, then F(x) is increasing at x.
- If $F'(x) < 0$, then F(x) is decreasing at x.
- If $F'(x) = 0$, then F(x) has a potential local maximum, minimum, or point of inflection at x.

Knowing how to find F'(x) allows us to analyze the behavior of F(x) thoroughly.

Calculating the Derivative F'(x)

Given:

$$F(x) = 3x^4 + 8x^3 + 6x + 12$$

The derivative is calculated term-by-term, applying the power rule:

- Derivative of $3x^4$ is $12x^3$.
- Derivative of $8x^3$ is $24x^2$.
- Derivative of $6x$ is 6.
- Derivative of the constant 12 is 0.

Therefore,

$$ F'(x) = 12x^3 + 24x^2 + 6 $$

This derivative function will be the basis for our analysis of increasing/decreasing intervals.

Step-by-Step Approach to Find Intervals of Increase and Decrease

To determine where $F(x)$ is increasing or decreasing, follow these key steps:

1. Find Critical Points

Critical points are values of x where $F'(x) = 0$ or $F'(x)$ is undefined. Since $F'(x)$ is a polynomial, it is defined everywhere, so we only need to solve:

$$ 12x^3 + 24x^2 + 6 = 0 $$

2. Solve the Derivative Equation

Factor the derivative:

$$ 12x^3 + 24x^2 + 6 = 0 $$

Divide through by 6 to simplify:

$$ 2x^3 + 4x^2 + 1 = 0 $$

Now, solving:

$$ 2x^3 + 4x^2 + 1 = 0 $$

This cubic may not factor easily, so we can use rational root theorem or numerical methods to find roots.

Applying Rational Root Theorem:

Possible rational roots are factors of 1 over factors of 2: $ \pm 1, \pm \frac{1}{2} $.

Test $ x = -1 $:

$$ 2(-1)^3 + 4(-1)^2 + 1 = -2 + 4 + 1 = 3 \neq 0 $$

Test $ x = -\frac{1}{2} $:

$$ 2(-\frac{1}{2})^3 + 4(-\frac{1}{2})^2 + 1 $$
$$ = 2(-\frac{1}{8}) + 4(\frac{1}{4}) + 1 $$
$$ = -\frac{1}{4} + 1 + 1 = \frac{3}{4} \neq 0 $$

Test $ x = 1 $:

$$ 2(1)^3 + 4(1)^2 + 1 = 2 + 4 + 1 = 7 \neq 0 $$

Test $ x = \frac{1}{2} $:

$$2(\frac{1}{2})^3 + 4(\frac{1}{2})^2 + 1$$
$$= 2(\frac{1}{8}) + 4(\frac{1}{4}) + 1$$
$$= \frac{1}{4} + 1 + 1 = 2.25 \neq 0$$

Since rational roots are not found, use numerical methods or graphing to approximate roots. Alternatively, we can analyze the sign of the derivative directly.

3. Analyze the Sign of F'(x)

Because solving the cubic explicitly is complex, we can analyze the sign of $F'(x) = 12x^3 + 24x^2 + 6$ by evaluating at key points or using the derivative's behavior.

Notice:

$$F'(x) = 12x^3 + 24x^2 + 6 = 6(2x^3 + 4x^2 + 1)$$

From earlier, the function $2x^3 + 4x^2 + 1$ has no rational roots and is always positive for large positive x (since $2x^3$ dominates).

Let's evaluate at some points:

- At $x = -2$:

$$2(-2)^3 + 4(-2)^2 + 1 = 2(-8) + 4(4) + 1 = -16 + 16 + 1 = 1 > 0$$

- At $x = 0$:

$$0 + 0 + 1 = 1 > 0$$

- At $x = 1$:

$$2(1)^3 + 4(1)^2 + 1 = 2 + 4 + 1 = 7 > 0$$

Since $2x^3 + 4x^2 + 1$ is positive for all tested points and the cubic's behavior suggests it doesn't cross zero, $F'(x) > 0$ for all x.

Conclusion: The derivative is always positive, indicating that $F(x)$ is increasing on $(-\infty, \infty)$.

Implications for the Function's Behavior

Because $F'(x) > 0$ everywhere:

- The function $F(x)$ is strictly increasing over the entire real line.
- There are no critical points where the function switches from increasing to decreasing.
- The function does not have any local maxima or minima.

This simplifies the analysis significantly: the graph of $F(x)$ is monotonically increasing.

Summary of Key Points

- The derivative F'(x) = 12x^3 + 24x^2 + 6 is positive for all real x.
- Since F'(x) > 0 everywhere, F(x) is strictly increasing across all real numbers.
- There are no intervals where the function decreases.
- The function has no critical points leading to local extrema.

Practical Applications and Importance of Analyzing Increasing and Decreasing Intervals

Understanding where a function increases or decreases is fundamental in calculus, with practical applications including:

- Optimizing functions in economics and engineering.
- Determining the shape and behavior of graphs.
- Finding local maxima and minima, essential in design and analysis.
- Analyzing real-world phenomena modeled by functions, such as population growth, profit maximization, or physical systems.

Conclusion

In this detailed analysis, we've demonstrated how to find the intervals of increase and decrease for the function F(x) = 3x^4 + 8x^3 + 6x + 12 by examining its derivative F'(x) = 12x^3 + 24x^2 + 6. Through derivative calculation, solving for critical points, and sign analysis, we've established that the function is strictly increasing over the entire real line. This insight is valuable for understanding the function's overall behavior and can serve as a foundation for further calculus studies, such as analyzing concavity or locating inflection points.

Key Takeaways:

- Always compute the derivative to analyze increasing/decreasing behavior.
- Critical points are found where the derivative equals zero.
- Sign analysis of the derivative reveals the nature of the function's monotonicity.
- For polynomial derivatives that do not cross zero, the function's behavior is uniform across the domain.

Mastering these techniques enhances your calculus skills and allows for more sophisticated analysis of complex functions.

Frequently Asked Questions

What is the function F(x) given in the problem?

The function is F(x) = 3x^4 - 8x^3 + 6x + 12.

How do I find the intervals where $F(x)$ is increasing?

First, find the derivative $F'(x)$, then determine where $F'(x) > 0$; these intervals indicate where $F(x)$ is increasing.

What is the derivative $F'(x)$ of the given function?

$F'(x) = 12x^3 - 24x^2 + 6$.

How do I find critical points for $F(x)$?

Set $F'(x) = 0$ and solve for x to find critical points, which are potential points of increase or decrease.

What steps are involved in determining the intervals of increase and decrease?

Calculate $F'(x)$, find critical points, test intervals around these points to see where $F'(x) > 0$ (increasing) or < 0 (decreasing).

How do I solve the equation $12x^3 - 24x^2 + 6 = 0$?

Factor out common factors: $6(2x^3 - 4x^2 + 1) = 0$, then solve $2x^3 - 4x^2 + 1 = 0$ using methods like substitution or numerical approximation.

What is the significance of the critical points in analyzing the function?

Critical points indicate potential local maxima, minima, or points of inflection, helping to determine where the function increases or decreases.

Can you provide a summary of how to determine the increasing/decreasing intervals for $F(x)$?

Yes, find $F'(x)$, solve for critical points, analyze the sign of $F'(x)$ in each interval between these points, and identify where $F'(x) > 0$ (increasing) and $F'(x) < 0$ (decreasing).

Are there any points where the function $F(x)$ is constant?

Constant points occur where $F'(x) = 0$ over an interval, but for this polynomial, critical points are isolated, so $F(x)$ is only constant at specific x-values where $F'(x)=0$.

What is the overall trend of $F(x)$ based on the first derivative analysis?

Based on the sign of $F'(x)$, you can determine where $F(x)$ is increasing ($F'(x) > 0$) and decreasing ($F'(x) < 0$), revealing the function's overall behavior.

Additional Resources

Let The Function $F'(x)$ Be Given By $F(x) = 3x^4 - 8x^3 + 6x + 12$

Understanding the behavior of a function, particularly its intervals of increase and decrease, is fundamental in calculus. The given function $F(x) = 3x^4 - 8x^3 + 6x + 12$ serves as an excellent example for exploring how derivatives inform us about the function's increasing or decreasing nature. In this review, we will delve into the process of analyzing the function's first derivative, identify critical points, and determine the intervals where the function is increasing or decreasing. We will also highlight important concepts, methods, and best practices for such analysis, providing a comprehensive guide for students and enthusiasts alike.

Understanding the Derivative of the Function

The first step in analyzing the increasing and decreasing intervals of a function is to compute its first derivative, $F'(x)$. The derivative provides the rate of change of the function at any point x. When $F'(x) > 0$, the function is increasing; when $F'(x) < 0$, it is decreasing. Therefore, accurately determining $F'(x)$ is crucial.

Given the function:
$$F(x) = 3x^4 - 8x^3 + 6x + 12$$

Calculating $F'(x)$:
$$F'(x) = \frac{d}{dx} (3x^4) - \frac{d}{dx} (8x^3) + \frac{d}{dx} (6x) + \frac{d}{dx} (12)$$

Applying power rule:
- $\frac{d}{dx} (3x^4) = 12x^3$
- $\frac{d}{dx} (-8x^3) = -24x^2$
- $\frac{d}{dx} (6x) = 6$
- $\frac{d}{dx} (12) = 0$

Thus:
$$F'(x) = 12x^3 - 24x^2 + 6$$

Factoring the Derivative for Critical Point Analysis

To analyze the sign of $F'(x)$, factoring it into simpler components is beneficial. Let's factor $F'(x)$:

$$F'(x) = 12x^3 - 24x^2 + 6$$

First, factor out the greatest common factor (GCF):
$$F'(x) = 6 (2x^3 - 4x^2 + 1)$$

Now, focus on factoring the cubic polynomial inside:

$$ 2x^3 - 4x^2 + 1 $$

Attempting rational root theorem candidates: factors of constant term over factors of leading coefficient:
- Possible roots: $\pm1, \pm \frac{1}{2}$

Test $x=1$:
$$ 2(1)^3 - 4(1)^2 + 1 = 2 - 4 + 1 = -1 \neq 0 $$

Test $x=-1$:
$$ 2(-1)^3 - 4(-1)^2 + 1 = -2 - 4 + 1 = -5 \neq 0 $$

Test $x=\frac{1}{2}$:
$$ 2 \times \left(\frac{1}{2}\right)^3 - 4 \times \left(\frac{1}{2}\right)^2 + 1 = 2 \times \frac{1}{8} - 4 \times \frac{1}{4} + 1 = \frac{1}{4} - 1 + 1 = \frac{1}{4} \neq 0 $$

Test $x=-\frac{1}{2}$:
$$ 2 \times \left(-\frac{1}{2}\right)^3 - 4 \times \left(-\frac{1}{2}\right)^2 + 1 = 2 \times -\frac{1}{8} - 4 \times \frac{1}{4} + 1 = -\frac{1}{4} - 1 + 1= -\frac{1}{4} \neq 0 $$

Since none of these candidates are roots, the cubic does not factor nicely over rationals and may require more advanced methods or numerical approximation to find roots.

Determining Critical Points and Sign Analysis

Even without exact roots, we can analyze the sign of F'(x) by considering the behavior of the cubic $2x^3 - 4x^2 + 1$. The key is to understand where F'(x) changes sign, which occurs at critical points where F'(x) = 0.

Since:
$$ F'(x) = 6(2x^3 - 4x^2 + 1) $$
the critical points are solutions to:
$$ 2x^3 - 4x^2 + 1 = 0 $$

Using numerical methods or graphing tools, we can approximate these roots:

- Approximate root 1: around $x \approx 0.2$
- Approximate root 2: around $x \approx 1.2$

These critical points divide the real line into intervals:
$$
(-\infty, x_1), \quad (x_1, x_2), \quad (x_2, \infty)
$$

Next, pick test points in each interval to determine the sign of F'(x):

- For $x < x_1$, say $x = 0$:
$$ 2(0)^3 - 4(0)^2 + 1 = 1 > 0 $$
So, F'(x) > 0 in $(- \infty, x_1)$.

- For x between x_1 and x_2, say $x=0.5$:
$$ 2(0.5)^3 - 4(0.5)^2 + 1 = 2 \times 0.125 - 4 \times 0.25 + 1 = 0.25 -1 + $$

$$1= 0.25 > 0$$
Still positive, indicating the function is increasing in this interval.

- For $x > x_2$, say $x=2$:
$$2(8) - 4(4) + 1= 16 - 16 + 1= 1 > 0$$
The derivative remains positive, suggesting the function continues increasing beyond the second critical point.

However, since the cubic's behavior indicates potential local maxima or minima, further detailed analysis or plotting can clarify the exact nature.

Intervals of Increase and Decrease

Based on the derivative's sign:

- The function $F(x)$ is increasing where $F'(x) > 0$.
- The function $F(x)$ is decreasing where $F'(x) < 0$.

Given the approximate roots and sign analysis, the function appears to be increasing across all real x, with no intervals where $F'(x)$ is negative. However, more precise numerical analysis suggests potential critical points where the sign might change.

Summary:

- Interval of Increase:
From $-\infty$ to the first critical point, and from the second critical point to ∞.
Likely, the function is increasing over the entire real line.

- Interval of Decrease:
If any, it would occur between critical points where $F'(x) < 0$.
Current analysis suggests no such interval exists.

Conclusion and Practical Implications

Analyzing the intervals of increase and decrease for the function $F(x) = 3x^4 - 8x^3 + 6x + 12$ illustrates the importance of derivative calculations and sign analysis. While the derivative's complexity may pose challenges in exact root finding, approximation methods and graphing tools are invaluable. These methods allow us to visualize critical points and determine the function's behavior effectively.

Pros of this approach:

- Provides a systematic way to analyze function behavior.
- Helps identify local maxima and minima.
- Facilitates understanding of the function's overall shape.

Cons/Challenges:

- Exact roots can be difficult to find analytically for complex derivatives.
- Approximation methods may introduce errors.
- Higher-degree polynomials often require numerical tools or computational software.

Features:

- Derivative calculation using power rule.
- Factoring and sign analysis.
- Use of critical points to determine monotonicity.

In summary, understanding the intervals of increase and decrease through derivative analysis is a cornerstone of calculus. For the given function, while the analysis suggests a predominantly increasing behavior, detailed numerical analysis or graphing would provide definitive insights. This process underscores the importance of combining analytical techniques with technological tools for comprehensive function analysis.

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