Let X Be A Random Variable Which Follows Binomial Distribution With N=400 And P=0.01. Then Find I) P(X<150)

Understanding the Binomial Distribution and Its Application

Let X Be A Random Variable Which Follows Binomial Distribution With N=400 And P=0.01. Then Find I) P(X<150).

In the realm of probability and statistics, the binomial distribution is a fundamental concept used to model the number of successes in a fixed number of independent Bernoulli trials, each with the same probability of success. When working with large sample sizes, calculating binomial probabilities directly can become cumbersome, which is where approximation methods such as the normal approximation prove invaluable.

This article aims to explore how to compute the probability P(X<150) for a binomially distributed random variable X, where the number of trials N=400 and the probability of success p=0.01. We'll walk through the theoretical background, approximation techniques, and step-by-step calculations, providing a comprehensive understanding suitable for students, researchers, and practitioners alike.

Binomial Distribution Fundamentals

Definition and Properties

The binomial distribution describes the probability of observing a specific number of successes in a fixed number of independent Bernoulli trials:

- Number of trials (N): The total number of independent experiments, each with two possible outcomes (success or failure).
- Probability of success (p): The probability that a single trial results in success.
- Number of successes (X): The random variable representing the count of successes in N trials.

The probability mass function (PMF) for a binomial distribution is:

$$ P(X=k) = \binom{N}{k} p^k (1-p)^{N-k} $$

for $k = 0, 1, 2, \ldots, N$.

Parameters in Our Example

Given:
- $N = 400$
- $p = 0.01$

The expected number of successes (mean) and variance are:

- Mean (μ): $np = 400 \times 0.01 = 4$
- Variance (σ^2): $np(1-p) = 400 \times 0.01 \times 0.99 = 3.96$

Since the expected number of successes is small relative to N, and p is small, the binomial distribution in this case is skewed and not symmetric.

Approximating the Binomial Distribution

Calculating $P(X<150)$ directly from the binomial formula is computationally intensive, especially for large N. Instead, approximation methods are employed.

Normal Approximation to the Binomial

The normal approximation is applicable when N is large and p is not extremely close to 0 or 1. The rule of thumb suggests that the approximation is reasonable if:

- $np \geq 5$
- $n(1-p) \geq 5$

In our case:
- $np = 4$ (which is slightly below 5)
- $n(1-p) = 396$ (well above 5)

Given that np is close to 5, the normal approximation can still be used with caution, especially for probability calculations involving larger values like 150.

Applying Continuity Correction

When approximating a discrete distribution with a continuous one, a continuity correction improves accuracy. For $P(X < 150)$, which is equivalent to $P(X \leq 149)$, the continuity correction involves replacing:

$$P(X \leq 149) \approx P(Y \leq 149.5)$$

where $Y \sim N(\mu, \sigma^2)$.

Step-by-Step Calculation of P(X<150)

1. Calculate the Mean and Standard Deviation

- Mean:

$$\mu = np = 400 \times 0.01 = 4$$

- Variance:

$$\sigma^2 = np(1-p) = 3.96$$

- Standard deviation:

$$\sigma = \sqrt{3.96} \approx 1.99$$

2. Set Up the Normal Approximation with Continuity Correction

Since we're interested in $P(X < 150)$, which is $P(X \leq 149)$:

$$P(X < 150) \approx P(Y \leq 149.5)$$

where $Y \sim N(\mu=4, \sigma=1.99)$.

3. Convert to Standard Normal Variable (Z-score)

Calculate the Z-score:

$$Z = \frac{149.5 - \mu}{\sigma} = \frac{149.5 - 4}{1.99} \approx \frac{145.5}{1.99} \approx 73.17$$

This Z-score is extremely large, indicating that $P(X<150)$ is practically 1.

4. Interpret the Result

Because the Z-score is so high (~73), the probability:

$$P(Z \leq 73.17) \approx 1$$

Thus,

$$P(X < 150) \approx 1$$

meaning that the probability of observing fewer than 150 successes when the expected number is only 4 is virtually certain.

Understanding the Implications

The key insight from this calculation is that, given the parameters:

- The probability of having fewer than 150 successes is essentially 100%. This is logical because the expected number of successes is only 4, and 150 is vastly greater than this mean. The probability that the actual number of successes exceeds 150 is astronomically small.

- The normal approximation confirms the intuition: the probability $P(X<150)$ is effectively 1, reflecting the fact that observing such a high number of successes (more than 150) in this scenario is practically impossible.

When to Use Normal Approximation in Binomial Distributions

While the normal approximation is powerful, its accuracy depends on the size of N and the success probability p. Here are guidelines:

- For np and $n(1-p)$ both greater than or equal to 5-10, the

normal approximation is generally reliable.
- When p is very small or very close to 1, or N is small, other methods like the Poisson approximation or exact binomial calculations are preferable.

In our case, since $np = 4$ is slightly below the typical threshold, the normal approximation still provides a close estimate, especially for probabilities involving large values like 150.

Alternative Methods for Calculating Binomial Probabilities

Although the normal approximation is suitable here, other methods include:

1. Exact Binomial Calculation

Computing $P(X<150)$ directly involves summing binomial probabilities:

$$P(X<150) = \sum_{k=0}^{149} \binom{400}{k} p^k (1-p)^{400-k}$$

which is computationally intensive for large N.

2. Poisson Approximation

For small p and large N, the binomial distribution can be approximated by a Poisson distribution with parameter $\lambda = np = 4$.

However, the Poisson approximation is better suited for small p and N such that np is moderate, and it is less accurate here for large deviations like 150.

Summary and Practical Takeaways

- The binomial distribution effectively models the number of successes in N independent Bernoulli trials.
- For large N and small p, the normal approximation simplifies calculations significantly.
- Applying the continuity correction improves approximation accuracy.
- In our specific example, the probability $P(X<150)$ is practically 1

due to the extremely high Z-score resulting from the normal approximation.

Practical implications:

- When dealing with large sample sizes and small success probabilities, expect the count of successes to be very close to the mean.
- The probability of observing a number of successes vastly larger than the mean is negligible.
- Approximation methods are essential tools in statistical analysis, especially when exact calculations are computationally infeasible.

Conclusion

Calculating the probability $P(X<150)$ for a binomial distribution with parameters $N=400$ and $p=0.01$ demonstrates the power of the normal approximation in statistical probability. Despite the large upper limit of 150, the expected number of successes is only 4, making the probability of observing fewer than 150 successes effectively 1. This example underscores the importance of understanding distribution parameters and approximation techniques in statistical analysis, enabling practitioners to make informed decisions in research, data science, quality control, and various fields requiring probabilistic modeling.

Keywords: Binomial Distribution, Normal Approximation, Probability Calculation, Large N, Success Probability, Statistical Analysis, Continuity Correction, Poisson Approximation

Frequently Asked Questions

Given a binomial random variable X with N=400 and P=0.01, how can we approximate P(X < 150)?

Since N is large and p is small, we can use the normal approximation to the binomial distribution. The mean is $\mu = Np = 400\ 0.01 = 4$, and the standard deviation is $\sigma = \text{sqrt}(Np(1 - p)) \approx \text{sqrt}(400\ 0.01\ 0.99) \approx 1.99$. But since $150 >> \mu$, $P(X < 150)$ is approximately 1.

Is it appropriate to use the normal approximation for calculating P(X < 150) when N=400 and p=0.01?

No, because the mean is only 4 and 150 is far in the tail; the normal approximation is not suitable here for $P(X < 150)$. Instead, $P(X < 150)$ is essentially 1.

What is the exact probability $P(X < 150)$ for $X \sim \text{Binomial}(400, 0.01)$?

Since the probability of X being less than 150 is practically 1 (almost certain), the exact probability is very close to 1.

How can the Poisson distribution be used to approximate $P(X < 150)$ in this case?

The Poisson approximation is not suitable here because the mean $\lambda = Np = 4$ is very small compared to 150; thus, it won't accurately estimate $P(X < 150)$.

What is the probability $P(X \geq 150)$ for $X \sim \text{Binomial}(400, 0.01)$?

This probability is effectively zero because the expected value is only 4, so exceeding 150 is extremely unlikely.

If we wanted to find $P(X < 5)$, how would we calculate it exactly?

We would sum the binomial probabilities $P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$.

What is the significance of the mean $\mu=4$ in understanding $P(X<150)$?

Since the mean is 4, and 150 is far greater than 4, the probability $P(X < 150)$ is essentially 1, indicating that X is almost certainly less than 150.

Can the binomial distribution be approximated by a normal distribution for $P(X<150)$?

No, because the normal approximation is only accurate when both np and n(1-p) are large, and for $P(X<150)$ with a mean of 4, it is not appropriate; the probability is nearly 1.

What is the practical interpretation of $P(X < 150)$ for the given binomial distribution?

Since the expected number of successes is only 4, the probability that the number of successes is less than 150 is essentially 100%, meaning it's almost certain.

How would you verify that P(X < 150) is approximately 1 for this distribution?

You can recognize that 150 is many standard deviations above the mean (μ=4), making the probability that X is less than 150 effectively 1 due to the tail behavior of the distribution.

Additional Resources

Binomial Distribution Analysis: Estimating P(X < 150) for N=400 and P=0.01

Understanding the Context: The Binomial Distribution in Focus

Imagine you're a data analyst or a statistician tasked with evaluating the likelihood of a rare event occurring within a large number of trials. This scenario is quintessentially modeled by the binomial distribution—a fundamental concept in probability theory that describes the number of successes in a fixed number of independent Bernoulli trials, each with the same probability of success.

In our case, consider a random variable X that follows a binomial distribution with parameters:

- Number of trials, $N = 400$
- Probability of success per trial, $p = 0.01$

The question at hand is: What is the probability that X is less than 150? In notation:

$$
P(X < 150)
$$

This analysis is not only mathematically intriguing but also practically significant—for instance, understanding rare event probabilities in quality control, epidemiology, or network reliability.

Fundamentals of the Binomial Distribution

Definition and Probability Mass Function

The binomial distribution models the number of successes in N independent trials, each with success probability p. The probability mass function (PMF) is:

$$
P(X = k) = \binom{N}{k} p^k (1 - p)^{N - k}
$$

for $k = 0, 1, 2, \ldots, N$.

In our problem, the parameters are:

- $N = 400$
- $p = 0.01$

which implies that the expected number of successes (mean) is:

$$
\mu = Np = 400 \times 0.01 = 4
$$

and the variance is:

$$
\sigma^2 = Np(1 - p) = 400 \times 0.01 \times 0.99 = 3.96
$$

with standard deviation:

$$
\sigma \approx \sqrt{3.96} \approx 1.99
$$

Approximating $P(X < 150)$: The Challenge

Calculating $P(X < 150)$ directly from the binomial PMF would involve summing over an enormous number of terms:

$$
P(X < 150) = \sum_{k=0}^{149} \binom{400}{k} p^k (1 - p)^{400 - k}
$$

\]

which is computationally infeasible and unnecessary given the scale of the problem. Instead, statistical approximations are used to estimate such probabilities efficiently, especially when N is large.

Applying Normal Approximation to the Binomial Distribution

The Central Limit Theorem and Its Role

For large N, the binomial distribution can be approximated by a normal distribution with the same mean and variance, thanks to the Central Limit Theorem (CLT). This approach simplifies calculations significantly.

Given:

$$
X \sim \text{Binomial}(N=400, p=0.01)
$$

we approximate X by:

$$
Y \sim \mathcal{N}(\mu=4, \sigma^2=3.96)
$$

which means:

$$
Y \sim \mathcal{N}(4, 1.99^2)
$$

Important note: When applying the normal approximation to a discrete distribution, a continuity correction is often employed to improve accuracy.

Continuity Correction and Its Implementation

To estimate $P(X < 150)$, note that:

$$
P(X < 150) = P(X \leq 149)
$$

Using the normal approximation with continuity correction:

$$
P(X \leq 149) \approx P(Y \leq 149 + 0.5) = P(Y \leq 149.5)
$$

This correction accounts for the fact that the binomial distribution is discrete, while the normal distribution is continuous.

Calculating the Approximate Probability

Standardizing the Variable

Transform the problem into a standard normal distribution:

$$
Z = \frac{Y - \mu}{\sigma} \sim \mathcal{N}(0, 1)
$$

Thus,

$$
P(X < 150) \approx P(Y \leq 149.5) = P \left(Z \leq \frac{149.5 - 4}{1.99} \right)
$$

Calculating the standardized value:

$$
z = \frac{149.5 - 4}{1.99} = \frac{145.5}{1.99} \approx 73.13
$$

Interpretation: The z-score of approximately 73.13 is astronomically high.

Understanding the Result

The standard normal distribution's cumulative distribution function (CDF) approaches 1 extremely rapidly as z increases. Specifically:

$$
P(Z \leq 73.13) \approx 1
$$

meaning the probability that a standard normal variable falls below such a high z-score is virtually 1.

Therefore:

$$
P(X < 150) \approx 1
$$

This indicates that, with overwhelming certainty, the number of successes X in 400 trials with success probability 0.01 will be less than 150.

Practical Implications and Insights

Why Is $P(X < 150)$ Near 1?

Given the mean ($\mu=4$) and standard deviation (~2), the typical number of successes is very close to 4, with most outcomes falling within a few successes of this mean. The probability of observing more than a handful of successes is extremely high, but the probability of exceeding even 50 successes is negligible.

Since 150 is vastly larger than the expected value (~4), the probability that X reaches or exceeds 150 is virtually zero. This aligns with properties of the binomial distribution where, for small p, the distribution is heavily skewed towards zero successes, with a negligible tail extending toward larger counts.

Alternative Approaches and Confirmations

Poisson Approximation as a Complementary Method

When p is small and N is large, the binomial distribution can be approximated by a Poisson distribution with parameter $\lambda = Np = 4$.

The Poisson PMF:

$$
P(X = k) = \frac{\lambda^k e^{-\lambda}}{k!}
$$

for $k=0,1,2,\ldots$.

Calculating $P(X < 150)$ directly from the Poisson is trivial because:

$$
P(X < 150) = 1 - P(X \geq 150)
$$

Given the Poisson's tail behavior, $P(X \geq 150)$ is effectively zero, confirming our earlier approximation.

Summary and Final Remarks

- The binomial distribution with parameters $N=400$ and $p=0.01$ has a mean of 4 and standard deviation approximately 2.
- The probability that X is less than 150, $P(X < 150)$, is effectively 1.
- This conclusion is supported by the normal approximation with continuity correction, which yields an astronomically high z-score (~73), implying the probability is indistinguishable from certainty.
- Employing the Poisson approximation further confirms this conclusion, given the rarity of large deviations from the mean in this context.

In essence, for a binomial distribution with such parameters, large deviations are practically impossible, and the probability of X being less than 150 is so close to 1 that for all practical purposes, it can be considered certain.

Key Takeaways for Practitioners

- When dealing with large N, the normal approximation is a powerful tool for estimating binomial probabilities efficiently.
- Continuity correction enhances the accuracy of the normal approximation for discrete distributions.

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- For small \(p\), the Poisson approximation offers an alternative,
especially for tail probability assessments.
- Understanding the scale and parameters helps in interpreting probabilities
and making informed decisions based on statistical models.

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In conclusion, the probability \(P(X < 150)\) in this context is so
overwhelmingly high that it effectively equals 1, demonstrating the power of
approximation techniques in statistical analysis and their practical utility
in complex probability calculations.
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