

Mara Solved An Exponential Equation By Using Logarithms. This Is Her Work.

$7 \cdot 3^x = 18973$

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In the realm of algebra and mathematics, solving exponential equations is a fundamental skill that often requires the application of logarithms. Mara's work exemplifies this process, where she successfully solved the exponential equation $7 \cdot 3^x = 18973$. Her approach demonstrates the power of logarithms in transforming complex exponential problems into manageable algebraic equations. This article provides a comprehensive, SEO-optimized overview of how Mara approached and solved this exponential equation, explaining each step in detail to help students and math enthusiasts understand the process thoroughly.

Understanding the Exponential Equation

Before diving into the solution, it's essential to understand the structure of the given exponential equation:

$$7 \cdot 3^x = 18973$$

This equation involves an exponential term (3^x) , multiplied by 7, equaling a large constant (18973). The goal is to find the value of (x) that satisfies this equation.

Key components:

- Exponential term: (3^x)
- Coefficient: 7
- Constant term: 18973

Understanding the relationship between these components is critical for applying logarithms effectively.

Step-by-Step Solution of the Exponential Equation

Mara's approach involves isolating the exponential component and then using logarithms to solve for x . Here is a detailed step-by-step breakdown:

1. Isolate the exponential term

Start by dividing both sides of the equation by 7 to isolate (3^x) :

$$\begin{aligned} & \left[\right. \\ & 3^x = \frac{18973}{7} \\ & \left. \right] \end{aligned}$$

Calculate the right side:

$$\begin{aligned} & \left[\right. \\ & 3^x = 2710.43 \\ & \left. \right] \end{aligned}$$

This simplifies the problem to solving $(3^x = 2710.43)$.

2. Apply logarithms to both sides

To solve for x , Mara uses logarithms, which allow us to bring the exponent down as a coefficient:

$$\begin{aligned} & \left[\right. \\ & \log(3^x) = \log(2710.43) \\ & \left. \right] \end{aligned}$$

Using the logarithm power rule:

$$\begin{aligned} & \left[\right. \\ & x \cdot \log(3) = \log(2710.43) \\ & \left. \right] \end{aligned}$$

This transforms the exponential equation into a linear form with respect to x .

3. Solve for x

Divide both sides by $\log(3)$:

$$x = \frac{\log(2710.43)}{\log(3)}$$

Calculate the logarithms (using common logarithm base 10 or natural logarithm, the method remains the same):

$$x = \frac{\log(2710.43)}{\log(3)}$$

Approximate values:

$$\begin{aligned} \log(2710.43) &\approx 3.433 \\ \log(3) &\approx 0.477 \end{aligned}$$

Thus,

$$x \approx \frac{3.433}{0.477} \approx 7.20$$

Final answer:

$$x \approx 7.20$$

Why Logarithms Are Essential in Solving Exponential Equations

Mara's solution underscores the importance of logarithms in algebra, especially for equations where the variable appears as an exponent. Here are key reasons:

1. Logarithms Convert Exponential Equations to Linear Equations

By applying logarithms to both sides, the exponent becomes a multiplier, making the equation linear and solvable with basic algebra.

2. Flexibility with Different Bases

Logarithms can be calculated with any base, such as common logarithm ($\log$), base 10) or natural logarithm ($\ln$), base e), providing flexibility depending on the context and available tools.

3. Handling Large or Small Numbers

Logarithms simplify calculations involving very large or small numbers, making it easier to solve for unknown exponents.

Additional Tips for Solving Exponential Equations Using Logarithms

- Always isolate the exponential expression first before applying logarithms.
- Choose the appropriate logarithm base based on the problem or calculator availability.
- Use logarithm properties such as $\log(a \cdot b) = \log a + \log b$ and $\log(a^b) = b \log a$ to simplify calculations.
- Check your solutions by substituting the found value of x back into the original equation.

Common Mistakes to Avoid

- Not isolating the exponential term first before applying logarithms.
- Using incorrect logarithm bases or mixing bases without proper conversion.
- Neglecting to verify the solution by plugging it back into the original equation.
- Ignoring domain restrictions: logarithms are only defined for positive arguments, so ensure the expressions inside the log are positive.

Conclusion: The Power of Logarithms in Solving Exponential Equations

Mara's methodical approach to solving $(7 \cdot 3^x = 18973)$ exemplifies how logarithms serve as essential tools in algebra. By isolating the exponential component, applying logarithms, and then solving for (x) , she efficiently determined the solution. This process not only showcases the practical application of logarithmic properties but also emphasizes their importance in tackling complex exponential equations across various fields such as engineering, physics, economics, and computer science.

Understanding these techniques empowers students and professionals alike to handle a broad spectrum of mathematical challenges with confidence. Whether dealing with exponential growth models, radioactive decay, or compound interest calculations, mastering logarithms is key to unlocking solutions to exponential equations.

Meta Description:

Discover how Mara solved an exponential equation using logarithms, with a detailed step-by-step guide. Learn the importance of logarithms in algebra and how to apply them effectively to solve exponential problems.

Keywords:

Exponential equation, logarithms, solving exponential equations, algebra, Mara's solution, logarithm properties, mathematical solutions, algebra tips, logarithmic equation solving, exponential growth

Frequently Asked Questions

How did Mara use logarithms to solve the exponential equation $7 \cdot 3^x = 18973$?

Mara first divided both sides by 7 to isolate 3^x , resulting in $3^x = 2710.43$. Then, she took the natural logarithm of both sides to bring down the exponent, leading to $x = \ln(2710.43) / \ln(3)$.

What is the importance of logarithms in solving exponential equations

like $7 \cdot 3^x = 18973$?

Logarithms allow us to solve for the exponent when the variable is in the exponent position, by converting the exponential equation into a linear form, making it easier to isolate and compute x .

What steps did Mara follow to find the value of x in the equation $7 \cdot 3^x = 18973$?

She divided both sides by 7, took the natural logarithm of both sides, and then applied the logarithm power rule to solve for x : $x = \ln(18973/7) / \ln(3)$.

How can Mara's approach be generalized to solve other exponential equations of the form $a \cdot b^x = c$?

The general approach is to divide both sides by a to isolate b^x , then take the logarithm of both sides, and finally solve for x using $x = \ln(c/a) / \ln(b)$.

Why is it necessary to use logarithms when solving equations like $7 \cdot 3^x = 18973$?

Because the variable x is in the exponent, logarithms are necessary to 'bring down' the exponent and linearize the equation, enabling us to solve for x using algebraic methods.

What is the approximate value of x that Mara obtained after solving the equation?

Using the logarithmic calculation, Mara found that $x \approx 6.36$, meaning 3^x is approximately equal to 2710.43 in the original equation.

Additional Resources

Mara Solved An Exponential Equation By Using Logarithms — a classic example of how logarithmic techniques can be employed to unravel exponential equations. This process not only demonstrates the power of logarithms but also showcases a step-by-step approach to solving complex equations that might seem daunting at first glance. In this article, we will dissect Mara's work, walk through her methodology, and provide an in-depth explanation of each step to clarify how logarithms serve as a crucial tool in mathematical problem-solving.

Understanding the Problem

At the core of Mara's work is the exponential equation:

$$7 \cdot 3^x = 18973$$

with the goal of solving for x .

This type of problem appears frequently in algebra and higher-level mathematics, especially when dealing with growth and decay models, compound interest, or other phenomena modeled exponentially. The key challenge is to isolate x , which is an exponent, and this is where logarithms come into play.

Why Use Logarithms?

Before diving into Mara's solution, it's important to understand why logarithms are essential in solving exponential equations:

- Exponential functions involve a variable in the exponent, such as 3^x .
- To solve for x , we need to "bring down" the exponent, which is not possible through basic algebraic operations alone.
- Logarithms are the inverse operations of exponentials, allowing us to rewrite the equation in a form that makes solving for x straightforward.

In essence, logarithms help us "undo" the exponential, enabling the algebraic isolation of the variable.

Mara's Step-by-Step Solution

Let's break down Mara's solution into clear, logical steps.

Step 1: Isolate the exponential term

Starting with the original equation:

$$7 \cdot 3^x = 18973$$

Divide both sides by 7 to isolate 3^x :

$$3^x = \frac{18973}{7}$$

Calculating the right side:

$$\lfloor 3^x = 2710.43 \rfloor \text{ (approximate value)}$$

Step 2: Recognize the need for logarithms

Since $\lfloor 3^x \rfloor$ is an exponential expression, and we want to solve for $\lfloor x \rfloor$, we take the logarithm of both sides.

Logarithms allow us to bring the exponent down as a coefficient:

$$\lfloor \log(3^x) = \log(2710.43) \rfloor$$

Step 3: Apply logarithm properties

Recall the logarithmic power rule:

$$\lfloor \log(a^b) = b \cdot \log(a) \rfloor$$

Applying this to the left side:

$$\lfloor x \cdot \log(3) = \log(2710.43) \rfloor$$

Step 4: Solve for $\lfloor x \rfloor$

Divide both sides by $\lfloor \log(3) \rfloor$:

$$\lfloor x = \frac{\log(2710.43)}{\log(3)} \rfloor$$

Now, compute the logarithms:

$$- \lfloor \log(2710.43) \approx 3.433 \rfloor$$

$$- \lfloor \log(3) \approx 0.4771 \rfloor$$

Thus:

$$\lfloor x \approx \frac{3.433}{0.4771} \approx 7.203 \rfloor$$

Final Answer:

$$\lfloor x \approx 7.203 \rfloor$$

Additional Insights and Clarification

Why did Mara choose logarithms?

Mara's choice to use logarithms stems from the nature of the original equation. Since the variable appears as an exponent, logarithms are the natural tool to linearize the exponential component.

The importance of the logarithm base

In the above calculation, we used common logarithms (base 10), but natural logarithms (base e) could also be used, with similar steps:

$$x = \frac{\ln(2710.43)}{\ln(3)}$$

Both methods yield the same result because of the change of base formula:

$$\log_b(a) = \frac{\ln(a)}{\ln(b)}$$

Summary of the method

- Step 1: Isolate the exponential term.
- Step 2: Take logarithms of both sides.
- Step 3: Use the power rule of logarithms to bring down the exponent.
- Step 4: Solve for the variable x .

Practical Applications

Mara's approach is fundamental in various real-world scenarios:

- Population growth models: Solving for time in exponential growth.
- Radioactive decay: Calculating half-lives.
- Finance: Computing compound interest over time.
- Physics: Decay of particles or energy levels.

Understanding how to manipulate exponential equations with logarithms is essential for students and professionals working in scientific and engineering fields.

Additional Tips for Solving Exponential Equations

1. Always check the domain: Ensure the values you take logarithms of are positive.
2. Use calculator logs carefully: Confirm whether you're using the natural log or common log, and stay consistent.
3. Estimate and verify: After computing $\ln(x)$, substitute back into the original equation to verify correctness.
4. Know the properties: Familiarize yourself with logarithmic laws to simplify complex expressions.

Conclusion

Mara's work exemplifies the effective use of logarithms to solve exponential equations, transforming a seemingly complex problem into a manageable algebraic calculation. By understanding the properties of logarithms and the logic behind their application, students and mathematicians alike can confidently approach similar problems. Whether working with growth models, decay processes, or other exponential phenomena, mastering logarithmic techniques is invaluable for accurate and efficient problem-solving.

Remember: The key to solving exponential equations is recognizing when logarithms are needed and applying their properties systematically. Mara's example provides a clear illustration of this powerful mathematical tool in action.

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