Question 2Given That (x+3) Is A Factor Of X3 - 7x = M:Find The Value Of MFactorize X3 - 7x + M CompletelySolve

Question 2Given That (x+3) Is A Factor Of X3 - 7x = M:Find The Value Of MFactorize X3 - 7x + M CompletelySolve

Understanding polynomial factorization and the conditions under which certain factors exist is fundamental in algebra. This article explores a specific problem involving cubic polynomials, their factors, and how to determine unknown constants within them. The problem statement is as follows:

Question 2Given That (x+3) Is A Factor Of X3 - 7x = M:Find The Value Of MFactorize X3 - 7x + M CompletelySolve

This question involves multiple steps—finding the value of M such that (x+3) divides the polynomial evenly, factorizing the polynomial completely, and solving the resulting equations. We'll walk through each stage carefully, providing clarity and detailed explanations to enhance your understanding of polynomial factorization.

Understanding the Problem Statement

Before diving into calculations, it's crucial to understand the core components of the problem:

- The polynomial in question is x^3 - 7x + M.
- The polynomial has a factor (x + 3).
- Our goal is to find the value of M for which (x + 3) is a factor.
- After determining M, we need to factorize x^3 - 7x + M completely.
- Finally, we want to solve the polynomial, if necessary, for roots.

Step 1: Find the Value of M When (x + 3) Is a Factor

The Remainder Theorem plays a key role here. It states that if a polynomial f(x) has a factor (x - a), then f(a) = 0.

In our case, the factor is (x + 3), which can be rewritten as (x - (-3)).

Therefore,

\[

$$
f(-3) = 0
$$

where

$$
f(x) = x^3 - 7x + M
$$

Calculating $f(-3)$:

$$
f(-3) = (-3)^3 - 7(-3) + M = -27 + 21 + M
$$

Simplify:

$$
f(-3) = -6 + M
$$

Since (x + 3) is a factor, f(-3) = 0, so:

$$
-6 + M = 0
$$

$$
M = 6
$$

Result:
The value of M is 6.

Step 2: Rewrite the Polynomial with the Found M

Now that we know M = 6, the polynomial becomes:

$$
x^3 - 7x + 6
$$

Our next step is to factorize this polynomial completely.

Step 3: Factorization of \(x^3 - 7x + 6\) Completely

3.1 Use the Factor Theorem

Since (x + 3) is a factor, we can divide the polynomial by (x + 3) to find the other factors.

3.2 Polynomial Division / Synthetic Division

Let's perform synthetic division of \(x^3 - 7x + 6\) by (x + 3), which corresponds to dividing by x = -3.

Set up synthetic division:

Coefficients of the polynomial:

\[
1 \quad 0 \quad -7 \quad 6
\]

Note: The coefficient of \(x^2\) is 0 because the polynomial is missing an \(x^2\) term.

Perform synthetic division:

\[
\begin{array}{r|rrrr}
-3 & 1 & 0 & -7 & 6 \\
& & -3 & 9 & -6 \\
\hline
& 1 & -3 & 2 & 0 \\
\end{array}
\]

The remainder is 0, confirming (x + 3) is a factor.

The quotient polynomial is:

\[
x^2 - 3x + 2
\]

3.3 Factorize the quadratic \(x^2 - 3x + 2\)

Find roots:

\[
x^2 - 3x + 2 = 0
\]

Factor pairs of 2 that sum to -3:

$$
(x - 1)(x - 2) = 0
$$

Thus, roots are:

$$
x = 1 \quad \text{and} \quad x = 2
$$

Complete factorization:

$$
x^3 - 7x + 6 = (x + 3)(x - 1)(x - 2)
$$

Step 4: Solving the Polynomial Equations

Having fully factorized the polynomial, the solutions to the original polynomial are the roots:

$$
x = -3, \quad x = 1, \quad x = 2
$$

These roots can be interpreted as solutions to the equation:

$$
x^3 - 7x + 6 = 0
$$

Summary of the Solution Process

Step	Description	Result
1	Use the Remainder Theorem to find M	$M = 6$
2	Rewrite polynomial with $M = 6$	$x^3 - 7x + 6$
3	Divide polynomial by $(x + 3)$	Quotient: $x^2 - 3x + 2$
4	Factor quadratic $x^2 - 3x + 2$	$(x - 1)(x - 2)$
5	Complete factorization	$\boxed{(x + 3)(x - 1)(x - 2)}$
6	Roots of the polynomial	$x = -3, 1, 2$

Additional Tips for Polynomial Factorization

- Always look for possible rational roots using the Rational Root Theorem.
- Use synthetic division for efficiency when dividing polynomials by linear factors.
- Remember that a cubic polynomial can be fully factorized into linear factors over real numbers if all roots are real.
- Confirm factorization by expanding the factors to ensure they produce the original polynomial.

Conclusion

This problem demonstrates the importance of the Remainder Theorem in identifying unknown constants within polynomials, as well as the power of synthetic division in factorization. By applying these techniques, we found that $M = 6$ makes $(x + 3)$ a factor of $x^3 - 7x + M$, and we successfully factorized the polynomial into linear factors: $(x + 3)(x - 1)(x - 2)$. The roots, $x = -3, 1, 2$, provide the solutions to the polynomial equation.

Understanding these fundamental algebraic methods enhances your problem-solving skills and prepares you for more complex polynomial challenges. Practice with similar problems will help solidify your grasp of polynomial division, factorization, and root-finding techniques.

Keywords: polynomial factorization, Remainder Theorem, synthetic division, cubic equations, algebra, roots of polynomials, algebraic factorization, solving polynomial equations

Frequently Asked Questions

Given that $(x + 3)$ is a factor of $x^3 - 7x + M$, how can we find the value of M?

Since $(x + 3)$ is a factor, substitute $x = -3$ into the polynomial and set it equal to zero: $(-3)^3 - 7(-3) + M = 0$. Simplify to find M.

What is the step-by-step process to factorize $x^3 - 7x + M$ completely?

First, find M using the factor theorem. Then, perform polynomial division or synthetic division to factor out $(x + 3)$. Finally, factor the remaining quadratic.

How do you apply the factor theorem in this problem?

The factor theorem states that if $(x + 3)$ is a factor, then plugging $x = -3$ into the polynomial gives zero. Use this to find M.

What is the value of M when (x + 3) is a factor of $x^3 - 7x + M$?

Substitute $x = -3$ into $x^3 - 7x + M$: $(-3)^3 - 7(-3) + M = 0 \rightarrow -27 + 21 + M = 0 \rightarrow M = 6$.

How do you completely factorize the polynomial $x^3 - 7x + 6$?

First, verify M = 6. Then, factor out (x + 3) using synthetic division to get a quadratic, and factor the quadratic further.

What is the result of dividing $x^3 - 7x + 6$ by (x + 3)?

Using synthetic division with $x = -3$, the quotient is $x^2 - 3x + 2$, with a remainder of zero indicating exact division.

How can the quadratic $x^2 - 3x + 2$ be factorized?

Factor the quadratic as $(x - 1)(x - 2)$.

What is the complete factorization of $x^3 - 7x + 6$?

The complete factorization is $(x + 3)(x - 1)(x - 2)$.

Why is it important to verify the value of M before factorizing?

Verifying M ensures that (x + 3) is indeed a factor, which is essential for correct factorization and solving the polynomial accurately.

Additional Resources

In-Depth Analysis and Solution of the Polynomial Question: Given That (x + 3) Is a Factor of $x^3 - 7x = M$: Find the Value of M, Factorize $x^3 - 7x + M$ Completely, and Solve

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Introduction

Mathematics, especially algebra, often presents us with polynomial expressions that challenge our understanding of factors, roots, and algebraic identities. The problem at hand involves a cubic polynomial with an unknown constant term, M, and the given condition that (x + 3) is a factor of the polynomial $x^3 - 7x = M$. This question is a classic example of applying polynomial factorization, the Factor Theorem, and algebraic manipulation to determine the unknown constant, factor the polynomial completely, and solve the resulting expressions.

This comprehensive review will guide you through each step with detailed explanations, illustrating fundamental concepts such as polynomial division, the Factor Theorem, synthetic division, and polynomial factorization. The goal is to deepen your understanding of how to approach similar problems, reinforce key algebraic principles, and develop problem-solving strategies.

Understanding the Problem Statement

Let's restate and interpret the problem carefully:

- You are given a cubic polynomial:
$$x^3 - 7x = M$$
- The problem states that $(x + 3)$ is a factor of this polynomial for some value of M.
- Your tasks are:
1. Find the value of M.
2. Factorize $x^3 - 7x + M$ completely.
3. Solve the resulting polynomial equation.

Step 1: Clarifying the Polynomial Expression

The polynomial is expressed as:

$$x^3 - 7x = M$$

which can be rewritten as:

$$x^3 - 7x - M = 0$$

However, from the problem statement, the polynomial of interest is:

$$x^3 - 7x + M$$

The presence of both forms suggests that the polynomial is:

$$P(x) = x^3 - 7x + M$$

and the statement "Given that $(x + 3)$ is a factor" applies to this polynomial $P(x)$.

Important: The key point here is understanding that the polynomial is $P(x) = x^3 - 7x + M$. The initial statement involving $x^3 - 7x = M$ essentially indicates that for the specific value of x, the polynomial equals M. But for the purpose of this problem, the focus is on $P(x) = x^3 - 7x + M$.

Step 2: Applying the Factor Theorem to Find M

The Factor Theorem states that:

> If $(x - a)$ is a factor of a polynomial $P(x)$, then $P(a) = 0$.

In our case, the factor is $(x + 3)$. Note that:

$$
x + 3 = 0 \Rightarrow x = -3
$$

Therefore, if $(x + 3)$ is a factor of $P(x) = x^3 - 7x + M$, then:

$$
P(-3) = 0
$$

Let's evaluate $P(-3)$:

$$
P(-3) = (-3)^3 - 7 \times (-3) + M = -27 + 21 + M
$$

Simplify:

$$
-27 + 21 + M = -6 + M
$$

Set this equal to zero:

$$
-6 + M = 0
$$

Solve for M:

$$
M = 6
$$

Conclusion:
$$
\boxed{
\text{The value of } M \text{ is } 6.
}
$$

Step 3: Complete Factorization of $x^3 - 7x + M$

Now that we've determined $M = 6$, the polynomial becomes:

$$
x^3 - 7x + 6
$$

Our goal is to factorize this completely.

Approach:

1. Use the Rational Root Theorem to identify possible rational roots.
2. Test these roots to find at least one actual root.
3. Use polynomial division (synthetic or long division) to factor out the corresponding linear factor.
4. Factor the resulting quadratic (if possible).

Step 4: Rational Root Theorem and Possible Roots

The Rational Root Theorem suggests that any rational root $\frac{p}{q}$ of the polynomial $a_n x^n + \dots + a_0$ (where coefficients are integers) is such that:

- p divides the constant term (here, 6).
- q divides the leading coefficient (here, 1).

Since the leading coefficient is 1, possible rational roots are divisors of 6:

$$
\pm 1, \pm 2, \pm 3, \pm 6
$$

Step 5: Testing Possible Roots

Evaluate $P(x) = x^3 - 7x + 6$ at these candidates:

- $x = 1$:

$$
1^3 - 7 \times 1 + 6 = 1 - 7 + 6 = 0
$$

- $x = -1$:

$$
(-1)^3 - 7 \times (-1) + 6 = -1 + 7 + 6 = 12 \neq 0

\]

- \(x = 2 \):

\[
8 - 14 + 6 = 0
\]

- \(x = -2 \):

\[
-8 + 14 + 6 = 12 \neq 0
\]

- \(x = 3 \):

\[
27 - 21 + 6 = 12 \neq 0
\]

- \(x = -3 \):

\[
-27 + 21 + 6 = 0
\]

- \(x = 6 \):

\[
216 - 42 + 6 = 180 \neq 0
\]

- \(x = -6 \):

\[
-216 + 42 + 6 = -168 \neq 0
\]

Roots found: \(x = 1, 2, -3 \)

Summary of roots:

\[
x = 1, \quad x = 2, \quad x = -3
\]

Step 6: Fully Factorize the Polynomial

Since the roots are \(1, 2, -3 \), the polynomial factors as:

$$
P(x) = (x - 1)(x - 2)(x + 3)
$$

Verification:

Expand step-by-step:

$$
(x - 1)(x - 2) = x^2 - 3x + 2
$$

Multiply by $(x + 3)$:

$$
(x^2 - 3x + 2)(x + 3)
$$

Using distributive property:

$$
x^2 \times x = x^3
$$
$$
x^2 \times 3 = 3x^2
$$
$$
-3x \times x = -3x^2
$$
$$
-3x \times 3 = -9x
$$
$$
2 \times x = 2x
$$
$$
2 \times 3 = 6
$$

Adding all together:

$$
x^3 + 3x^2 - 3x^2 - 9x + 2x + 6 = x^3 - 7x + 6
$$

which matches our polynomial exactly.

Final Complete Factorization:

$$
\boxed{
x^3 - 7x + 6 = (x - 1)(x - 2)(x + 3)
}
$$

Step 7: Solving the Polynomial Equation

The original question asks to solve the polynomial $x^3 - 7x + 6 = 0$.

From the factorization, the solutions are the roots:

$$
x = 1, \quad x = 2, \quad x = -3
$$

Summary of the Entire Process

- Understanding the problem: Recognized the need to find M given a factor.
- Applying the Factor Theorem: Calculated $M = 6$ by evaluating $P(-3)$.
- Factorization: Used the Rational Root Theorem to identify roots $1, 2, -3$.
- Complete factorization: Expressed as $(x - 1)(x - 2)(x + 3)$.
- Solution: Roots are $x = 1, 2, -3$.

Additional Insights and Tips

- Using the Factor Theorem effectively: Always test potential roots derived from divisors of the constant term to quickly find factors.
- Polynomial division: Synthetic division simplifies the process of checking roots and factoring polynomials.
- Handling multiple roots: When roots repeat, the polynomial can be factored accordingly, but in this case, roots are distinct.
- Verifying solutions: Always expand the factors

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