Question 4 A Consumer's Demands X, Y For Two Different Goods Are Chosen To Maximize The Utility FunctionU=xyThe

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In the realm of microeconomics, understanding consumer behavior is fundamental to analyzing market dynamics and predicting demand patterns. One of the core concepts is how consumers allocate their limited income among different goods to maximize their satisfaction or utility. Utility functions serve as mathematical representations of consumer preferences, allowing economists to analyze optimal consumption choices systematically. Among various utility functions, the multiplicative utility function U=xy stands out due to its unique properties and implications for consumer demand.

This article delves into the problem where a consumer chooses quantities of two goods, X and Y, to maximize their utility function U=xy. We will explore the theoretical foundations, mathematical derivations, and practical implications of this problem. By examining the conditions under which the consumer maximizes utility, we can better understand demand behavior, the concept of marginal utility, and the role of budget constraints. Additionally, we will discuss how to derive demand functions for goods X and Y and analyze the effects of changing prices and income levels on consumer choices.

Whether you are a student of economics, a researcher, or a curious reader interested in consumer theory, this comprehensive guide aims to clarify the principles and calculations involved in utility maximization with the utility function U=xy. Through detailed explanations, step-by-step derivations, and illustrative examples, this article provides a thorough understanding of how consumers make optimal decisions in a two-good scenario.

Understanding the Utility Function U=xy

What Does the Utility Function Represent?

The utility function U=xy represents a specific form of preference where the consumer's satisfaction (utility) depends on the quantities of two goods, X and Y. In this case:

- U is the total utility derived from consuming quantities x of good X and y of good Y.
- The utility is multiplicative, meaning the satisfaction increases as both x and y increase.
- The function implies perfect complementarity to some extent; utility is zero if either x or y is zero.

This utility function belongs to the class of Cobb-Douglas functions, which are widely used in consumer theory due to their properties:

- Constant elasticity of substitution.
- Ease of deriving demand functions.
- Well-behaved preferences that satisfy the assumptions of consumer theory.

Properties of the Utility Function U=xy

Some key properties include:

- Non-Negativity: $U \geq 0$, with the maximum utility achieved for positive quantities of both goods.
- Diminishing Marginal Utility: The marginal utility of each good decreases as the quantity of that good increases, assuming other variables are held constant.
- Convex Preferences: The indifference curves derived from this utility function are convex to the origin, indicating a preference for diversified consumption bundles.

Consumer Optimization Problem

Formulating the Problem

The core problem is to determine the combination of x and y that maximizes utility U=xy subject to the consumer's budget constraint:

$$
p_x x + p_y y = M
$$

where:

- p_x = Price of good X
- p_y = Price of good Y
- M = Consumer's income or budget

The goal is to find the values of x and y that maximize U = xy while satisfying the budget constraint.

Constraints and Assumptions

- The consumer cannot spend more than their income.
- Both goods are divisible.
- The prices are positive and constant.
- The consumer aims to maximize utility given their income and prices.

Mathematical Solution to Utility Maximization

Using the Method of Lagrange Multipliers

To solve the constrained optimization problem, we employ the Lagrangian method:

$$
\mathcal{L} = xy + \lambda (M - p_x x - p_y y)
$$

where:

- λ is the Lagrange multiplier associated with the budget constraint.

Step-by-Step Derivation

Step 1: Calculate the partial derivatives:

$$
\frac{\partial \mathcal{L}}{\partial x} = y - \lambda p_x = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial y} = x - \lambda p_y = 0
$$
$$
\frac{\partial \mathcal{L}}{\partial \lambda} = M - p_x x - p_y y = 0
$$

Step 2: From the first two equations:

$$
y = \lambda p_x
$$
$$
x = \lambda p_y
$$

Dividing the first by the second:

$$
\frac{y}{x} = \frac{\lambda p_x}{\lambda p_y} = \frac{p_x}{p_y}
$$

which yields:

$$
\frac{y}{x} = \frac{p_x}{p_y}
$$

or equivalently:

$$
p_y y = p_x x
$$

Step 3: Use the budget constraint to solve for x and y:

$$
p_x x + p_y y = M
$$

Substitute $y = \frac{p_x}{p_y} x$:

$$
p_x x + p_y \left(\frac{p_x}{p_y} x \right) = M
$$
\[

$$
p_x x + p_x x = M
$$

$$
2 p_x x = M
$$

$$
x^{} = \frac{M}{2 p_x}
$$

Similarly,

$$
y^{} = \frac{p_x}{p_y} x^{} = \frac{p_x}{p_y} \times \frac{M}{2 p_x} = \frac{M}{2 p_y}
$$

Resulting Demand Functions:

$$
\boxed{
x^{} = \frac{M}{2 p_x}
}
$$

$$
\boxed{
y^{} = \frac{M}{2 p_y}
}
$$

Demand functions indicate that the consumer allocates their income equally across goods, weighted by their prices.

Interpreting the Demand Functions

Key Insights:

- The demand for good X depends inversely on its price p_x and directly on income M.
- Similarly for good Y.
- The consumer spends an equal proportion of their income on each good, adjusted by their respective prices.

Implications:

- When p_x increases, demand for X decreases, and vice versa.
- An increase in income M results in proportional increases in demand for both goods.

Marginal Utility and Consumer Behavior

Marginal Utility of Goods X and Y

From $U=xy$, the marginal utilities are:

$$
MU_x = \frac{\partial U}{\partial x} = y
$$

$$
MU_y = \frac{\partial U}{\partial y} = x
$$

These show that:

- The marginal utility of X depends on the quantity of Y.
- The marginal utility of Y depends on the quantity of X.

This interdependence reflects the multiplicative nature of the utility function—consumption of one good enhances the utility derived from the other.

Equilibrium Condition: Equalizing Marginal Utility per Price

Consumers allocate their income so that the last dollar spent on each good yields the same marginal utility:

$$
\frac{MU_x}{p_x} = \frac{MU_y}{p_y}
$$

Substituting the marginal utilities:

$$
\frac{y}{p_x} = \frac{x}{p_y}
$$

which simplifies to the same condition derived earlier:

$$
p_y y = p_x x
$$

This demonstrates that the consumer balances their marginal utility per dollar spent across both goods, leading to the demand functions derived above.

Effects of Price and Income Changes

Impact of Price Changes

- An increase in p_x reduces $x^{\wedge}$, leading to a substitution effect.
- Conversely, a decrease in p_x increases demand for X.
- Similar effects occur with p_y.

Impact of Income Changes

- An increase in M proportionally increases demand for both goods.
- The demand functions show that the income elasticity of demand is 1, indicating unit elasticity.

Graphical Representation

Indifference curves ($U=xy$) are rectangular hyperbolas. Budget lines shift based on income and price changes, illustrating the consumer's optimal choice at the tangency point where the highest indifference curve touches the budget line.

Conclusion and Practical Applications

The analysis of consumer demand under the utility function $U=xy$ reveals several fundamental economic principles:

- Consumers allocate income to maximize utility by equalizing the ratio of marginal utility to price across goods.
- The demand functions are inversely proportional to prices and directly proportional to income.
- The multiplicative utility form implies a preference for diversified consumption; consuming only one good yields zero utility.

Practical applications include:

- Pricing strategies: understanding how price changes influence demand.
- Policy analysis: assessing how income or price subsidies affect consumer behavior.
- Market

Frequently Asked Questions

What does the utility function U = xy represent in consumer choice theory?

The utility function $U = xy$ represents a consumer's level of satisfaction derived from consuming quantities x and y of two different goods, where utility increases as both x and y increase, assuming they are positive.

How does the consumer maximize utility given the utility function U = xy?

The consumer maximizes utility by choosing quantities of x and y such that the marginal utility per dollar spent is equal for both goods, often subject to their budget constraint.

What are the optimal demand functions for goods X and Y derived from the utility function U = xy?

The optimal quantities are determined by setting the ratio of marginal utilities equal to the ratio of prices: $(MU_x/P_x) = (MU_y/P_y)$. For $U = xy$, the demand functions are proportional to the prices and budget constraint, typically leading to $x = (I/2P_x)$ and $y = (I/2P_y)$, where I is income.

If the prices of goods X and Y change, how does that affect the consumer's demand based on U=xy?

An increase in the price of one good will generally decrease the quantity demanded of that good, and the consumer will adjust their consumption to maximize utility, often leading to a new equilibrium where the consumer balances their budget accordingly.

What role does the budget constraint play in maximizing the utility function U = xy?

The budget constraint limits the total expenditure, ensuring that the total spent on goods X and Y does not exceed the consumer's income, which guides the optimal choice of quantities to maximize utility within their budget.

Is the utility function U=xy convex or concave, and what does that imply about consumer preferences?

The utility function U=xy is neither strictly convex nor concave everywhere, but it is strictly quasiconcave, implying that consumer preferences are continuous, monotonic, and exhibit diminishing marginal utility.

How does the concept of marginal utility relate to the utility function U = xy?

Marginal utility is the additional utility gained from consuming an extra unit of a good. For U=xy, the marginal utility of x is y, and of y is x, indicating that each good's marginal utility depends on the amount of the other good consumed.

Can the consumer's demand for goods X and Y be independent in the utility function U=xy?

No, the demand for X and Y are interdependent because the utility function links them multiplicatively; increasing the consumption of one good increases utility only if the other is also consumed, reflecting a complementary relationship.

What happens to the consumer's optimal bundle if the price of good X decreases?

If the price of good X decreases, the consumer can afford to buy more of X while maintaining the same utility level, leading to an increase in the demand for X and potentially for Y due to the utility maximization condition.

How would a change in consumer income affect the demand for goods X and Y under the utility function U=xy?

An increase in income allows the consumer to purchase higher quantities of both goods, shifting the demand upwards proportionally, with the optimal quantities typically increasing in line with income and the prices of the goods.

Additional Resources

Question 4 A Consumer's Demands X, Y For Two Different Goods Are Chosen To Maximize The Utility Function U=xy — this classic problem in consumer theory offers deep insights into how individuals make optimal consumption choices given their preferences and budget constraints. Understanding this problem requires a thorough analysis of utility maximization, marginal analysis, and the principles guiding consumer behavior. In this comprehensive guide, we will explore the intricacies of how a consumer chooses quantities of goods X and Y to maximize their utility function U=xy, including step-by-step methods, economic interpretations, and practical examples. Whether you are a student, economist, or enthusiast, this detailed breakdown will clarify the core concepts behind utility maximization.

Introduction to Utility Maximization

Consumer choice theory is built around the idea that individuals aim to maximize their satisfaction or utility given their limited resources. The utility function, in this case U=xy, represents the consumer's preferences over two goods, X and Y. The goal is to determine the combination of X and Y that provides the highest utility without exceeding the consumer's budget constraint.

Utility Function: U=xy

The utility function U=xy indicates that the consumer's satisfaction depends on the product of the quantities of goods X and Y. This type of utility function suggests perfect complementarity in some contexts but more generally represents a scenario where utility increases as both goods are consumed, and the marginal utility of each good depends on the quantity of the other.

Step 1: Understanding the Budget Constraint

Before maximizing utility, it's essential to understand the budget constraint, which limits consumption choices. The budget constraint can be expressed as:

$P_x X + P_y Y = I$

where:
- P_x = price of good X
- P_y = price of good Y
- I = consumer's income or budget

This linear equation defines all feasible combinations of X and Y the consumer can afford.

Step 2: Setting Up the Optimization Problem

The consumer's goal is:

Maximize U = xy

Subject to:

$P_x X + P_y Y = I$

This involves selecting values of X and Y that satisfy the budget constraint and produce the highest utility.

Step 3: Using the Method of Lagrange Multipliers

The standard approach to constrained optimization problems in economics is the Lagrangian method.

The Lagrangian Function:

$L(X, Y, \lambda) = xy + \lambda (I - P_x X - P_y Y)$

where λ is the Lagrange multiplier representing the marginal utility of income.

Step 4: Deriving First-Order Conditions

Differentiate L with respect to X, Y, and λ:

- $\partial L/\partial X = y - \lambda P_x = 0 \Rightarrow y = \lambda P_x$
- $\partial L/\partial Y = x - \lambda P_y = 0 \Rightarrow x = \lambda P_y$
- $\partial L/\partial \lambda = I - P_x X - P_y Y = 0$

From the first two equations, we get:

$y / P_x = x / P_y$

which simplifies to:

$x / y = P_x / P_y$

This is a key result: the consumer allocates expenditure such that the ratio of marginal utilities equals the ratio of prices.

Step 5: Solving for the Optimal Quantities

Using the ratio:

$x / y = P_x / P_y$

and the budget constraint:

$P_x X + P_y Y = I$

we can substitute Y from the ratio:

$x = (P_x / P_y) y$

then:

$P_x (P_x / P_y) y + P_y y = I$

Simplify:

$(P_x^2 / P_y) y + P_y y = I$

Multiply through by P_y to clear denominators:

$P_x^2 y + P_y^2 y = I P_y$

Factor y:

$y (P_x^2 + P_y^2) = I P_y$

Solve for y:

$y = (I P_y) / (P_x^2 + P_y^2)$

Similarly, x:

$x = (P_x / P_y) y = (P_x / P_y) [(I P_y) / (P_x^2 + P_y^2)] = (I P_x) / (P_x^2 + P_y^2)$

Therefore, the optimal quantities are:

- X\ = $(I P_x) / (P_x^2 + P_y^2)$
- Y\ = $(I P_y) / (P_x^2 + P_y^2)$

Step 6: Interpretation of Results

The solutions indicate that:

- The consumer allocates expenditure proportionally to the prices of goods X and Y.
- The quantities depend on the ratio of prices and total income.
- When prices change, the consumer adjusts consumption to maintain the optimal ratio.

Marginal Utility and Substitution Effect

The ratio of marginal utilities (MU_x / MU_y) equals the price ratio, consistent with the law of equi-marginal utility.

Practical Example

Suppose:

- Income (I) = \$100
- Price of X (P_x) = \$10
- Price of Y (P_y) = \$20

Calculate optimal quantities:

$x = (100\ 10) / (10^2 + 20^2) = 1000 / (100 + 400) = 1000 / 500 = 2$ units

$y = (100\ 20) / 500 = 2000 / 500 = 4$ units

The consumer should purchase 2 units of X and 4 units of Y to maximize utility under these prices and income.

Additional Considerations

Diminishing Marginal Utility

Since the utility function is multiplicative, the marginal utility of each good diminishes as consumption increases, but the combined utility can be increased by balanced consumption.

Effects of Price Changes

- An increase in P_x decreases x, leading to substitution towards Y.
- A decrease in P_y increases Y, encouraging more Y consumption.

Income Effects

Changes in income (I) proportionally affect both x and y, increasing total utility.

Conclusion: The Significance of the Utility Function U=xy

The utility function U=xy demonstrates how consumers balance their consumption of two goods to achieve maximum satisfaction. By applying the principles of constrained optimization, the consumer allocates their budget efficiently—spending in proportion to prices and adjusting consumption to maximize utility.

Understanding this model provides foundational insights into consumer behavior, the effects of price changes, and the principles guiding optimal decision-making in economics. Whether analyzing individual choices or broader market dynamics, the methods and interpretations outlined here serve as essential tools in economic analysis and policy formulation.

Final Thoughts

- The utility maximization problem with U=xy exemplifies the elegant interplay between preferences, prices, and income.
- Mastery of the Lagrangian method and interpretative skills enhances comprehension of consumer theory.
- Such models are pivotal in understanding real-world economic behavior, informing everything from pricing strategies to public policy.

By grasping these concepts, readers can better appreciate how individuals navigate their economic environment to satisfy their needs and desires efficiently.

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