

Select The Correct Answer.A Right Angle Triangle Where ABC Angle Is 90 And Angle ACB Is 45.Which Trigonometric

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Understanding the fundamentals of right-angled triangles is essential for mastering trigonometry, a branch of mathematics focused on the relationships between the angles and sides of triangles. In this article, we will analyze a specific right-angled triangle where angle ABC is 90 degrees, and angle ACB is 45 degrees. Our goal is to determine the correct trigonometric ratios associated with this triangle and explore their applications.

Understanding the Triangle Configuration

Before delving into trigonometric ratios, let's clarify the triangle's structure and the significance of the given angles.

Triangle ABC Description

- Vertices: The triangle is labeled as ABC, with points A, B, and C.
- Angles:
 - Angle ABC: 90 degrees (a right angle at point B).
 - Angle ACB: 45 degrees.
- Sides:
 - The side opposite angle A is BC.
 - The side opposite angle B is AC.
 - The side opposite angle C is AB.

Implications of the Given Angles

- Since angle ABC is 90° , triangle ABC is a right-angled triangle with the right angle at B.
- Angle ACB is 45° , which indicates that the remaining angle, at point A, can be calculated as 45° , making the triangle an isosceles right triangle.

Calculating the third angle:

- Total sum of angles in a triangle = 180°
- Given:
 - $\angle ABC = 90^\circ$
 - $\angle ACB = 45^\circ$

- Therefore,
- $\angle CAB = 180^\circ - 90^\circ - 45^\circ = 45^\circ$

This reveals that triangle ABC is a 45°-45°-90° right triangle, which has specific properties and ratios.

Properties of a 45°-45°-90° Triangle

The 45°-45°-90° triangle holds a special place in trigonometry due to its unique ratios.

Key Characteristics

- Isosceles right triangle: Two angles are equal (both 45°), so the sides opposite these are equal.
- Side ratios:
- The legs (the sides opposite the 45° angles) are equal in length.
- The hypotenuse (the side opposite the right angle) is $\sqrt{2}$ times the length of each leg.

Expressed mathematically:

- If each leg = x ,
- Then hypotenuse = $x\sqrt{2}$.

This ratio simplifies many trigonometric calculations and makes the triangle a perfect model for understanding sine, cosine, and tangent functions at 45° angles.

Determining the Trigonometric Ratios

Given the properties, the key trigonometric functions for the angles in this triangle can be deduced.

Calculating Sine, Cosine, and Tangent for 45°

Since the triangle is isosceles with angles of 45°, the trigonometric ratios are well-known:

1. Sine ($\sin 45^\circ$):

- Definition: $\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$

- For $\theta = 45^\circ$:

- Opposite side = x

- Hypotenuse = $x\sqrt{2}$

- Therefore,

$$\sin 45^\circ = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0.7071$$

2. Cosine ($\cos 45^\circ$):

- Definition: $\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

- Similarly,

$$\cos 45^\circ = \frac{x}{x\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0.7071$$

3. Tangent ($\tan 45^\circ$):

- Definition: $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

- Since both legs are equal,

$$\tan 45^\circ = \frac{x}{x} = 1$$

Summary of key ratios:

Trigonometric Function	Value	Explanation
$\sin 45^\circ$	$\frac{1}{\sqrt{2}}$	Opposite over hypotenuse
$\cos 45^\circ$	$\frac{1}{\sqrt{2}}$	Adjacent over hypotenuse
$\tan 45^\circ$	1	Opposite over adjacent

Application of Trigonometric Ratios in the Triangle

Understanding these ratios allows us to solve various problems involving right-angled triangles.

Calculating Side Lengths

Suppose you know the length of one side, you can determine the others:

- If legs (AB and AC) are of length x :

- Hypotenuse (BC): $x\sqrt{2}$

- Conversely, if the hypotenuse is h :

- Legs: $\left(\frac{h}{\sqrt{2}}\right)$

Example:

- If $(AB = AC = 7)$ units,

- Hypotenuse $(BC = 7 \times \sqrt{2} \approx 9.899)$ units.

Solving for Angles with Trigonometric Ratios

Given side lengths, you can verify the angles:

- For example, if (AB) and (AC) are known:

- $(\sin 45^\circ = \frac{AB}{BC})$

- If $(AB = 5)$, and (BC) is known:

- $(\sin 45^\circ = \frac{5}{BC})$

- Solving for (BC) :

$$BC = \frac{5}{\sin 45^\circ} = \frac{5}{1/\sqrt{2}} = 5 \times \sqrt{2} \approx 7.07$$

This demonstrates the utility of trigonometric ratios in practical calculations.

Real-World Applications of 45°-45°-90° Triangles and Trigonometry

The principles derived from this special triangle are widely applicable across various fields.

Engineering and Architecture

- Designing structures with specific angles.
- Calculating load distributions in frameworks.
- Ensuring precise measurements in construction.

Navigation and Geolocation

- Determining distances and angles using triangulation.

- GPS technology relies on trigonometric calculations involving angles similar to 45° .

Physics and Signal Processing

- Analyzing waveforms and oscillations.
- Calculating components of vectors in physics problems.

Mathematics Education

- Teaching fundamental concepts of angles and ratios.
- Building a foundation for understanding complex trigonometric functions.

Summary and Key Takeaways

- Triangle ABC with angles 90° , 45° , and 45° is an isosceles right triangle.
- The sides obey the ratio: legs are equal, hypotenuse is $(\sqrt{2})$ times the legs.
- The primary trigonometric ratios are:
 - $(\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}})$,
 - $(\tan 45^\circ = 1)$.
- These ratios simplify calculations and have numerous practical applications across various disciplines.

Understanding the properties and ratios of special triangles like the 45° - 45° - 90° triangle is fundamental in mastering trigonometry. Recognizing these patterns enhances problem-solving skills and provides a solid foundation for more advanced mathematical concepts.

In conclusion, when faced with a right-angled triangle where one of the non-right angles is 45° , the trigonometric functions associated with 45° are directly applicable. The symmetry and ratios inherent in the 45° - 45° - 90° triangle make it a powerful tool for various mathematical and real-world applications, emphasizing the importance of mastering these core concepts.

Frequently Asked Questions

In a right-angled triangle ABC with angle A as 90° and angle ACB as 45° , which trigonometric ratio corresponds to side AB relative to side AC?

Sine of angle ACB (45°) is AB divided by AC.

Given a right-angled triangle ABC with angle A = 90° and angle ACB = 45°, what is the value of side AB in terms of side AC?

AB equals AC multiplied by sine 45°, so $AB = AC / \sqrt{2}$.

In triangle ABC, where angle A is 90° and angle ACB is 45°, which trigonometric function can be used to find side BC?

Cosine of 45° can be used to find side BC relative to hypotenuse AC.

What is the measure of the remaining angle ABC in the triangle where angle A = 90° and angle ACB = 45°?

Angle ABC is 45°, since the sum of angles in a triangle is 180°.

In the given triangle, which trigonometric ratio is equal to 1 for angle ACB?

Tangent of 45° is equal to 1.

How can you determine the length of side AC if side AB is known in the triangle with given angles?

Use sine of 45°: $AC = AB / \sin 45^\circ$.

What is the significance of the 45° angle in a right-angled triangle regarding its side ratios?

A 45° angle indicates that the legs adjacent to that angle are equal in length, making the triangle isosceles right-angled.

Additional Resources

Understanding the Trigonometry of a Right-Angled Triangle with a 45-Degree Angle

In the realm of geometry, right-angled triangles hold a special place due to their simplicity and profound applications across various fields such as engineering, architecture, navigation, and physics. When analyzing triangles with specific angles, especially those involving a 45-degree angle, the trigonometric relationships become particularly elegant and insightful. This article delves into a specific scenario: a right-angled triangle where angle ABC is 90 degrees, and angle ACB is 45 degrees. Our goal is to identify the relevant trigonometric functions, understand their significance, and explore the properties of such a triangle in detail.

Fundamental Concepts: Right-Angled Triangles and Trigonometry

Before diving into the specifics of the problem, it is essential to review the foundational concepts of right-angled triangles and the basic trigonometric functions.

Right-Angled Triangle Basics

A right-angled triangle is a triangle that contains a 90-degree angle. The sides of such a triangle are classified as:

- Hypotenuse (c): The side opposite the right angle, the longest side.
- Adjacent side (a): The side adjacent to the angle of interest.
- Opposite side (b): The side opposite the angle of interest.

The angles are typically labeled as A, B, and C, with the right angle often at C, but this can vary depending on the problem context.

Trigonometric Ratios in Right-Angled Triangles

The primary trigonometric functions relate the angles to the ratios of sides:

1. Sine (sin): Opposite / Hypotenuse
2. Cosine (cos): Adjacent / Hypotenuse
3. Tangent (tan): Opposite / Adjacent

These functions are fundamental in solving for unknown sides or angles when some measurements are known.

Analyzing the Triangle: Given Angles and Their Implications

The problem states: "A right-angled triangle where angle ABC is 90 degrees and angle ACB is 45 degrees." To understand this configuration fully, we need to establish the triangle's labeling and interpret the angles correctly.

Clarifying the Triangle's Geometry

Assuming standard notation:

- Vertices: A, B, C
- Angles at vertices: $\angle A$, $\angle B$, $\angle C$
- Sides: Opposite the vertices are sides a, b, c respectively (with side a opposite $\angle A$, etc.)

Given:

- $\angle B = 90^\circ$ (since ABC is a right angle at B)
- $\angle C = 45^\circ$

Since the sum of angles in a triangle is 180° , $\angle A = 180^\circ - 90^\circ - 45^\circ = 45^\circ$.

Thus, the triangle has:

- $\angle A = 45^\circ$
- $\angle B = 90^\circ$
- $\angle C = 45^\circ$

This configuration indicates a right-angled isosceles triangle with the right angle at B and the two acute angles at A and C, each measuring 45° .

Implications of the 45° Angles

Having two angles of 45° and 45° in a right triangle suggests the two sides adjacent to these angles are of equal length. Specifically:

- The sides a (opposite $\angle A$) and c (opposite $\angle C$) are equal.
- The hypotenuse b (opposite right angle at B) relates to these sides through the Pythagorean theorem.

This symmetry simplifies calculations and makes the trigonometric relationships particularly elegant.

Identifying the Relevant Trigonometric Functions

Given the angles, which trigonometric functions are most relevant? Let's analyze each.

Trigonometric Ratios at Specific Angles

For angles of 45° , the basic trigonometric ratios are well-known:

- $\sin(45^\circ) = \cos(45^\circ) = \sqrt{2} / 2 \approx 0.7071$
- $\tan(45^\circ) = 1$

These ratios imply that for angles of 45° , the side lengths relative to the hypotenuse are equal in a right triangle.

Application to Our Triangle

Since:

- $\angle A = 45^\circ$
- $\angle C = 45^\circ$

The trigonometric functions for these angles directly relate to the ratios of sides:

- At $\angle A = 45^\circ$:
 - $\sin(45^\circ) = a / b$
 - $\cos(45^\circ) = c / b$
 - $\tan(45^\circ) = a / c$
- At $\angle C = 45^\circ$:
 - $\sin(45^\circ) = c / b$
 - $\cos(45^\circ) = a / b$
 - $\tan(45^\circ) = c / a$

Because the triangle is isosceles right-angled, $a = c$, and the hypotenuse b relates to either side as:

- $b = a \sqrt{2} = c \sqrt{2}$

Calculating the Sides Using Trigonometry

In this particular configuration, the primary goal is often to determine side lengths or the ratios between sides, which are straightforward given the angles.

Using Basic Trigonometric Ratios

If we assign a variable to the sides:

- Let $a = c = x$ (since sides opposite equal angles are equal)

Then the hypotenuse b can be calculated as:

- $b = x \sqrt{2}$

From the definition of sine or cosine at 45° , for example:

- $\sin(45^\circ) = a / b = x / (x \sqrt{2}) = 1 / \sqrt{2} = \sqrt{2} / 2$

This confirms the known value, illustrating the consistency of these relationships.

Practical Example: Computing Side Lengths

Suppose the length of side a (or c) is known, say $a = 1$ unit. Then:

- $b = 1 \sqrt{2} \approx 1.414$ units

This straightforward calculation demonstrates the utility of trigonometry in solving for sides in such triangles.

Trigonometric Functions' Role in the Triangle's Properties

Understanding the specific trigonometric functions relevant to this triangle allows us to analyze its properties thoroughly.

Key Functions and Their Significance

- Sine (sin): Useful to find the ratio between the side opposite an angle and the hypotenuse. For 45° , $\sin(45^\circ) = \sqrt{2} / 2$, which informs us that the opposite side is approximately 70.7% of the hypotenuse.
- Cosine (cos): Indicates the ratio of the adjacent side to the hypotenuse, also $\sqrt{2} / 2$ at 45° , revealing symmetry in side lengths.
- Tangent (tan): Represents the ratio of the opposite to the adjacent side, which is 1 at 45° , again highlighting the equality of the sides adjacent to these angles.

Implications of Equal Sides and Ratios

Because the angles at A and C are both 45° , the sides a and c are equal, and the ratios of these sides to the hypotenuse involve the same trigonometric functions. This symmetry simplifies many calculations and makes such triangles ideal for illustrating fundamental trigonometric principles.

Applications and Significance of Such Triangles in Real-World Contexts

Understanding the trigonometric relationships in a 45° right triangle is more than an academic exercise; it has practical applications.

Engineering and Architecture

- Structural Design: The isosceles right triangle often appears in truss design, where equal angles and sides optimize weight and strength.
- Pythagorean Triples: The $(1, 1, \sqrt{2})$ ratio forms the basis for numerous Pythagorean triples, fundamental in constructing right triangles with integer sides.

Navigation and Physics

- Vector Resolution: The 45° angles facilitate the decomposition of vectors into equal components, vital in physics and navigation.
- Optics and Wave Mechanics: Symmetrical angles at 45° are used in designing devices like beam splitters and waveguides.

Mathematical Education

- These triangles serve as excellent pedagogical tools to teach the fundamentals of trigonometry, ratios, and geometric reasoning.

Conclusion: The Trigonometric Essence of a 45° Right Triangle

The specific triangle described—right-angled at B with angles of 45° at A and C—exemplifies the elegance of trigonometry. The functions sine, cosine, and tangent at 45° are all equal to $\frac{\sqrt{2}}{2}$ or $\frac{1}{\sqrt{2}}$, reflecting the symmetry and proportionality of the sides. Recognizing these relationships enables precise calculations of side lengths and angles, which are foundational in various scientific disciplines.

This exploration underscores that in a right triangle with two 45° angles, the trigonometric functions simplify to fundamental constants, making it a cornerstone example in understanding

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