Select The Correct Answer. Consider Functions F And G. F(X) = 11x^3 - 3x^2 G(X) = 7x^4 + 9x^3 Which Expression

Select The Correct Answer. Consider Functions F And G. F(X) = 11x^3 - 3x^2 G(X) = 7x^4 + 9x^3 Which Expression is the focus of this comprehensive article. When analyzing functions in algebra, especially polynomial functions like F(x) and G(x), it’s essential to understand how to manipulate and combine them to find expressions with specific properties. Whether you're working on calculus, algebraic simplification, or preparing for exams, mastering the interpretation of such functions is a fundamental skill. This article explores the various operations you can perform with these functions, such as addition, subtraction, multiplication, and composition, and how to determine the resulting expressions. We will also delve into key concepts like derivatives, critical points, and the behavior of these functions, providing clear, step-by-step guidance to help you select the correct expression based on given criteria.

Understanding the Given Functions

Before exploring the possible expressions derived from functions F and G, it’s crucial to understand their structure and characteristics.

Function F(x) = 11x^3 - 3x^2

- This is a cubic polynomial function.
- The highest degree term is 11x^3, which dominates the end behavior.
- The function includes a quadratic term with a negative coefficient, -3x^2.
- It exhibits inflection points and potentially multiple critical points depending on its derivative.

Function G(x) = 7x^4 + 9x^3

- This is a quartic polynomial function.
- The leading term is 7x^4, which determines the end behavior as x approaches ±∞.
- The function has both quartic and cubic terms, influencing its overall shape.
- The positive coefficient of the x^4 term suggests the function opens upwards on both ends.

Exploring Operations Between F and G

When working with two functions, common operations include:

1. Addition: (F + G)(x) = F(x) + G(x)

2. Subtraction: (F - G)(x) = F(x) - G(x)

3. Multiplication: (F G)(x) = F(x) G(x)

4. Composition: (F ∘ G)(x) = F(G(x)) and (G ∘ F)(x) = G(F(x))

Each operation results in a new function with different properties and behaviors. Selecting the correct expression depends on the context—are we asked for the sum, the product, the composition, or perhaps a specific derivative?

Calculating the Sum and Difference of F and G

Sum: (F + G)(x)

- To find the sum, combine like terms:

(F + G)(x) = (11x^3 - 3x^2) + (7x^4 + 9x^3)
= 7x^4 + (11x^3 + 9x^3) - 3x^2
= 7x^4 + 20x^3 - 3x^2

Difference: (F - G)(x)

- Subtract G(x) from F(x):

(F - G)(x) = (11x^3 - 3x^2) - (7x^4 + 9x^3)
= -7x^4 + (11x^3 - 9x^3) - 3x^2
= -7x^4 + 2x^3 - 3x^2

These expressions are straightforward to compute and are useful in analyzing combined behaviors or solving equations.

Product of Functions F and G

Calculating (F G)(x)

- Multiply the functions directly:

$(F\ G)(x) = (11x^3 - 3x^2)\ (7x^4 + 9x^3)$

- Expand using distributive property:

$= 11x^3\ 7x^4 + 11x^3\ 9x^3 - 3x^2\ 7x^4 - 3x^2\ 9x^3$

- Simplify each term:

$= 77x^7 + 99x^6 - 21x^6 - 27x^5$

- Combine like terms:

$= 77x^7 + (99x^6 - 21x^6) - 27x^5$
$= 77x^7 + 78x^6 - 27x^5$

This resulting polynomial provides insight into the combined growth rates and potential roots of the product function.

Function Composition: F(G(x)) and G(F(x))

Function composition involves substituting one function into another, creating a more complex expression.

F(G(x)) = F(G(x))

- Recall: $F(x) = 11x^3 - 3x^2$
- Substitute G(x) into F(x):

$F(G(x)) = 11\ [G(x)]^3 - 3\ [G(x)]^2$

- First, compute $[G(x)]^2$ and $[G(x)]^3$:

$[G(x)]^2 = (7x^4 + 9x^3)^2$
$= (7x^4)^2 + 2\ 7x^4\ 9x^3 + (9x^3)^2$
$= 49x^8 + 126x^7 + 81x^6$

$[G(x)]^3 = [G(x)]\ [G(x)]^2$
$= (7x^4 + 9x^3)\ (49x^8 + 126x^7 + 81x^6)$

- Expand $[G(x)]^3$:

$= 7x^4\ 49x^8 + 7x^4\ 126x^7 + 7x^4\ 81x^6 + 9x^3\ 49x^8 + 9x^3\ 126x^7 + 9x^3\ 81x^6$

- Simplify each term:

$= 343x^{12} + 882x^{11} + 567x^{10} + 441x^{11} + 1134x^{10} + 729x^9$

- Combine like terms:

$= 343x^{12} + (882x^{11} + 441x^{11}) + (567x^{10} + 1134x^{10}) + 729x^9$
$= 343x^{12} + 1323x^{11} + 1701x^{10} + 729x^9$

- Now, F(G(x)) becomes:

$= 11 [G(x)]^3 - 3 [G(x)]^2$
$= 11 (343x^{12} + 1323x^{11} + 1701x^{10} + 729x^9) - 3 (49x^8 + 126x^7 + 81x^6)$

- Final expression:

$= 3773x^{12} + 14553x^{11} + 18711x^{10} + 8001x^9 - 147x^8 - 378x^7 - 243x^6$

This complex composition reveals the intricate behavior of the combined functions and is often used in advanced calculus and algebra.

G(F(x))

- Similarly, substituting F(x) into G(x):

$G(F(x)) = 7 [F(x)]^4 + 9 [F(x)]^3$

- The powers involved become more complex, often requiring binomial expansion or software tools for precise calculation.

Analyzing Derivatives and Critical Points

Understanding the derivatives of functions F and G is vital for analyzing their increasing/decreasing behavior, maxima, minima, and inflection points.

Derivative of F(x)

- $F(x) = 11x^3 - 3x^2$
- F'(x) = derivative of each term:

$F'(x) = 33x^2 - 6x$

- Critical points occur where F'(x) = 0:

$33x^2 - 6x = 0$

$x(33x - 6) = 0$

$x = 0$ or $x = 6/33 = 2/11$

- These points help identify local maxima or minima.

Derivative of G(x)

- $G(x) = 7x^4 + 9x^3$
- $G'(x) = 28x^3 + 27x^2$

- Critical points where $G'(x) = 0$:

$28x^3 + 27x^2 = 0$
$x^2(28x + 27) =$

Frequently Asked Questions

What is the degree of the function F(x) = 11x^3 - 3x^2?

The degree of F(x) is 3.

What is the degree of the function G(x) = 7x^4 + 9x^3?

The degree of G(x) is 4.

Which function has the higher degree, F(x) or G(x)?

G(x) has the higher degree (4) compared to F(x) (3).

What is the leading coefficient of F(x)?

The leading coefficient of F(x) is 11.

What is the leading coefficient of G(x)?

The leading coefficient of G(x) is 7.

If you were to combine F(x) and G(x) into a polynomial, what would be its degree?

The degree would be 4, as G(x) has the highest degree.

Which expression correctly represents the sum of F(x) and

G(x)?

F(x) + G(x) = (11x^3 - 3x^2) + (7x^4 + 9x^3) = 7x^4 + (11x^3 + 9x^3) - 3x^2 = 7x^4 + 20x^3 - 3x^2.

Which expression correctly represents the product of F(x) and G(x)?

F(x) G(x) = (11x^3 - 3x^2) (7x^4 + 9x^3).

What is the degree of the product of F(x) and G(x)?

The degree of the product is 3 + 4 = 7.

Additional Resources

Select The Correct Answer. Consider Functions F And G. F(X) = 11x^3 - 3x^2 G(X) = 7x^4 + 9x^3 Which Expression — this prompt invites a deep exploration into the properties, behaviors, and relationships of two polynomial functions, F(x) and G(x). Understanding such functions is fundamental in algebra and calculus, as they serve as building blocks for more complex mathematical concepts. In this article, we will analyze these functions meticulously, explore various expressions involving them, and evaluate their characteristics to determine the correct answer to the implied question.

Understanding the Given Functions

Before delving into specific expressions or properties, it’s essential to understand the nature of the functions provided.

Function F(x) = 11x^3 - 3x^2

This is a cubic polynomial function characterized by:

- Leading term: 11x^3, which dominates the behavior for large |x| values.
- Degree: 3, indicating the function is cubic and can have up to two inflection points.
- Coefficients: The positive leading coefficient suggests the end behavior as x→∞ is positive, and as x→−∞ is negative.
- Critical points: Found by taking the derivative, which helps identify local maxima and minima.

Function G(x) = 7x^4 + 9x^3

This is a quartic polynomial with:

- Leading term: $7x^4$, which dominates at large $|x|$.
- Degree: 4, indicating the potential for more complex end behavior and multiple turning points.
- Coefficients: Both terms are positive for large x, but the cubic term can influence behavior near the origin.
- Critical points: To analyze maxima and minima, derivatives are necessary.

Analyzing the Behavior of F(x) and G(x)

Understanding the functions' behavior involves examining their derivatives, critical points, and end behaviors.

Derivative of F(x)

$F'(x) = d/dx [11x^3 - 3x^2] = 33x^2 - 6x = 3x(11x - 2)$

- Critical points occur where $F'(x) = 0$, i.e., $x=0$ and $x=2/11$.
- These points help identify local extrema.

Derivative of G(x)

$G'(x) = d/dx [7x^4 + 9x^3] = 28x^3 + 27x^2 = x^2(28x + 27)$

- Critical points at $x=0$ and $x= -27/28$.
- Behavior near these points indicates potential maxima or minima.

End behavior

- As $x\to\infty$, $F(x)\to\infty$ and $G(x)\to\infty$.
- As $x\to-\infty$, $F(x)\to-\infty$ and $G(x)\to\infty$ (since the leading term in G is positive with an even degree).

Exploring Key Expressions Involving F and G

The core of the prompt seems to involve evaluating certain expressions involving these functions. While the original question is incomplete, typical tasks include:

- Finding the sum, difference, or product of F(x) and G(x).
- Comparing F(x) and G(x) at specific points.

- Deriving composite functions such as $F(G(x))$ or $G(F(x))$.
- Computing derivatives or integrals involving these functions.

Let's analyze some common expressions and their implications.

Sum and Difference of F(x) and G(x)

Sum: $(F + G)(x) = (11x^3 - 3x^2) + (7x^4 + 9x^3) = 7x^4 + (11x^3 + 9x^3) - 3x^2 = 7x^4 + 20x^3 - 3x^2$

Difference: $(F - G)(x) = 11x^3 - 3x^2 - (7x^4 + 9x^3) = -7x^4 + (11x^3 - 9x^3) - 3x^2 = -7x^4 + 2x^3 - 3x^2$

These combine to form new polynomials, with the sum being of degree 4 and the difference of degree 4 but with negative leading coefficient.

Composite Functions: F(G(x)) and G(F(x))

$F(G(x)) = 11(G(x))^3 - 3(G(x))^2$

Calculating this explicitly yields a highly complex polynomial involving high powers of x, which can be expanded for detailed analysis.

Similarly,

$G(F(x)) = 7(F(x))^4 + 9(F(x))^3$

Again, expanding these would produce very high-degree polynomials, useful in understanding the composition behavior.

Critical Points and Extrema

Analyzing the critical points of these functions helps understand their maximum and minimum values.

Critical Points of F(x)

- $x=0$: $F(0)=0$
- $x=2/11$: $F(2/11) = 11(2/11)^3 - 3(2/11)^2 = 11(8/1331) - 3(4/121) \approx$ negligible, but calculations reveal the nature of the extremum.

Use second derivative test:

$F''(x) = d/dx [33x^2 - 6x] = 66x - 6$

- At x=0: $F''(0) = -6 < 0$ ⇒ local maximum.
- At x=2/11: $F''(2/11) = 66(2/11) - 6 = 12 - 6 = 6 > 0$ ⇒ local minimum.

Critical Points of G(x)

- x=0: G(0)=0
- x= -27/28: G(-27/28) can be computed for extrema.

Second derivatives:

$G''(x) = d/dx [28x^3 + 27x^2] = 84x^2 + 54x$

- At x=0: $G''(0)=0$, indicating an inflection point or requiring further analysis.
- At x= -27/28: substitution gives insight into the concavity.

Graphical Insights and Visualization

Plotting the functions over various intervals provides visual clues about their behavior:

- F(x) exhibits cubic growth with a local maximum at x=0 and a local minimum near x=2/11.
- G(x), being quartic, shows a "W"-shaped curve with minima and maxima influenced by the cubic term.

Visual analysis confirms the algebraic findings and aids in understanding the intersection points, asymptotic behavior, and relative magnitudes.

Applications and Relevance of These Functions

Understanding such polynomial functions and their combinations has practical applications:

- Physics: Modeling motion where acceleration is polynomial.
- Economics: Cost functions with polynomial growth.
- Engineering: Signal processing involving polynomial approximations.

The analysis of critical points, derivatives, and compositions enhances problem-solving skills in calculus and helps in optimizing functions or predicting their behavior.

Pros and Cons of Polynomial Function Analysis

Pros:

- Clear analytical framework for understanding function behavior.
- Polynomial functions are differentiable everywhere, making calculus tools applicable.
- Symmetry and end behavior are straightforward to analyze.

Cons:

- High-degree polynomials can become complex to expand and analyze explicitly.
- Numerical approximations may be necessary for precise critical point evaluations.
- Graphs can sometimes be deceptive without detailed calculations.

Conclusion: Determining the Correct Expression

While the original prompt lacks specific options, the analysis above provides a comprehensive foundation to evaluate various expressions involving F(x) and G(x). The key steps involve:

- Computing derivatives to find critical points.
- Analyzing end behavior for limits.
- Combining functions via sum, difference, or composition.
- Using graphical insights for confirmation.

Understanding these functions' properties enables us to select the correct expression based on criteria such as maxima, minima, roots, or asymptotic behavior.

In summary, mastering the analysis of polynomial functions like F and G is vital for advanced mathematics and real-world applications. Whether the task is to find the maximum value, intersection points, or evaluate complex compositions, a systematic approach rooted in calculus and algebra ensures accurate and insightful outcomes.

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