

Suppose That 10 Percent Of People Are Left Handed. If 6 People Are Selected At Random, What Is The Probability

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Understanding probability is essential in various fields, from statistics and psychology to everyday decision-making. In this article, we explore a specific probability problem involving left-handedness, providing a comprehensive explanation suitable for learners and enthusiasts alike. We will analyze the problem, explain relevant concepts, and guide you through the calculations step by step.

Understanding the Scenario

Before diving into the calculations, let's carefully interpret the problem statement:

- Given: 10% of the population is left-handed.
- Question: If 6 people are randomly selected, what is the probability that a certain event related to their handedness occurs?

Typically, such problems ask for the probability that a specific number of the selected individuals are left-handed or right-handed. For clarity, we will consider common scenarios:

1. The probability that exactly k out of 6 are left-handed.
2. The probability that at least a certain number of them are left-handed.
3. The probability that none are left-handed, or all are left-handed, etc.

For this article, we will focus on calculating the probability that exactly k of the 6 people are left-handed, for any value of k from 0 to 6.

Modeling the Problem: Binomial Distribution

The problem can be modeled using the binomial distribution, which describes the number of successes (in this case, being left-handed) in a fixed number of independent trials (each person selected), each with the same probability of success.

Key parameters:

- Number of trials, $n = 6$.
- Probability of success in each trial, $p = 0.10$ (since 10% are left-handed).
- Number of successes, k , varies from 0 to 6.

Binomial probability formula:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

Where:

- $\binom{n}{k}$ is the binomial coefficient, representing the number of ways to choose k successes out of n trials.
- p^k is the probability of success in k trials.
- $(1 - p)^{n - k}$ is the probability of failure in the remaining trials.

Calculating Specific Probabilities

Let's compute the probability for specific values of k, such as:

- Exactly 0 left-handed people.
- Exactly 1 left-handed person.
- Exactly 3 left-handed people.
- Exactly 6 left-handed people.

Probability that None are Left-Handed (k=0)

$$P(X=0) = \binom{6}{0} (0.10)^0 (0.90)^6 = 1 \times 1 \times 0.531441 = 0.531441$$

Interpretation: There is approximately a 53.14% chance that none of the 6 people are left-handed.

Probability that Exactly 1 Person is Left-Handed (k=1)

$$P(X=1) = \binom{6}{1} (0.10)^1 (0.90)^5 = 6 \times 0.10 \times 0.59049 = 6 \times 0.10 \times 0.59049 \approx 0.354294$$

Interpretation: About a 35.43% chance that exactly 1 person out of 6 is left-handed.

Probability that Exactly 3 People are Left-Handed (k=3)

$$P(X=3) = \binom{6}{3} (0.10)^3 (0.90)^3 = 20 \times 0.001 \times 0.729 = 20 \times 0.001 \times 0.729 = 0.01458$$

Interpretation: Approximately a 1.46% chance that exactly 3 individuals are left-handed.

Probability that All 6 are Left-Handed (k=6)

$$P(X=6) = \binom{6}{6} (0.10)^6 (0.90)^0 = 1 \times 0.000001 \times 1 = 0.000001$$

$$P(X=6) = \binom{6}{6} (0.10)^6 (0.90)^0 = 1 \times 0.000001 \times 1 = 0.000001$$

Interpretation: An extremely low probability (~0.0001%) that all 6 are left-handed.

Calculating Probabilities for Range of Outcomes

Often, questions involve probabilities that at least a certain number of people are left-handed, or up to a certain number. For example, what is the probability that at least 2 are left-handed?

Probability that at least 2 are left-handed

$$P(X \geq 2) = 1 - P(X=0) - P(X=1)$$

Using previous calculations:

$$P(X=0) \approx 0.531441$$

$$P(X=1) \approx 0.354294$$

So,

$$P(X \geq 2) = 1 - 0.531441 - 0.354294 = 1 - 0.885735 = 0.114265$$

Interpretation: There's approximately an 11.43% chance that at least 2 of the 6 people are left-handed.

Probability that no more than 1 are left-handed

$$P(X \leq 1) = P(X=0) + P(X=1) \approx 0.885735$$

This signifies an 88.57% chance that at most 1 person among the 6 is left-handed.

Using Binomial Distribution Tables and Calculators

While manual calculations are insightful, for larger n or multiple probabilities, it's efficient to use binomial distribution tables, statistical software (like R, Python, or calculators), or online binomial

probability calculators.

Example: Calculating $P(X \leq 2)$

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$$

where

$$P(X=2) = \binom{6}{2} (0.10)^2 (0.90)^4 = 15 \times 0.01 \times 0.6561 = 15 \times 0.01 \times 0.6561 = 0.098415$$

Adding all:

$$P(X \leq 2) = 0.531441 + 0.354294 + 0.098415 \approx 0.98415$$

Meaning there is about a 98.42% chance that among 6 people, at most 2 are left-handed.

Real-World Applications and Significance

Understanding these probabilities has practical implications:

- Market Research: Companies might analyze preferences or behaviors that follow similar binomial patterns.
- Healthcare Studies: Researchers studying the prevalence of traits or conditions in small samples.
- Educational Assessments: Estimating the likelihood of certain student characteristics in a class.
- Psychology and Sociology: Investigating the distribution of handedness in populations.

Knowing the probability distribution helps in designing studies, interpreting data, and making informed decisions.

Summary and Key Takeaways

- The problem of determining the probability that a certain number of people are left-handed among a sample can be modeled using the binomial distribution.
- The probability of exactly k successes (left-handed individuals) in n trials is given by:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}$$

- For a sample of 6 people with 10% left-handedness:
- The probability that none are left-handed (~54%).
- The probability that exactly one is left-handed (~35%).
- The probability that all are left-handed (~0.0001%).
- The probability that at least 2 are left-handed (~11%).
- These calculations can be extended to compute probabilities for any range of successes, aiding in statistical analysis and decision-making.

Conclusion

Probability calculations involving binomial distributions provide valuable insights into scenarios like the prevalence of left-handedness among a small sample. By understanding and applying the binomial formula, one can assess the likelihood of various outcomes, which is essential in research, analysis, and everyday reasoning. Whether predicting rare events or common ones, mastering these concepts enhances statistical literacy and decision-making skills.

Remember: Always verify your calculations with reliable tools or software, especially as the number of trials increases or the probabilities become more complex.

Frequently Asked Questions

What is the probability that exactly 2 out of 6 randomly selected people are left-handed if 10% of the population is left-handed?

Using the binomial probability formula, $P = C(6,2) (0.1)^2 (0.9)^4 \approx 0.214$.

What is the probability that none of the 6 selected people are left-handed?

The probability is $(0.9)^6 \approx 0.5314$, since all 6 are right-handed.

What is the probability that at least one person out of 6 is left-handed?

It's 1 minus the probability that none are left-handed: $1 - (0.9)^6 \approx 1 - 0.5314 = 0.4686$.

What is the probability that exactly 3 out of 6 selected people are left-handed?

Using the binomial formula, $P = C(6,3) (0.1)^3 (0.9)^3 \approx 0.0081$.

If 6 people are selected, what is the expected number of left-handed individuals?

The expected value is $n p = 6 \cdot 0.1 = 0.6$.

What is the probability that more than 2 out of 6 are left-handed?

It's the sum of probabilities for 3, 4, 5, and 6 left-handed people, or $1 - \text{sum of probabilities for 0, 1, and 2}$.

How does increasing the sample size from 6 to 12 affect the probability of selecting exactly 1 left-handed person?

The probability increases slightly, calculated as $P = C(12,1) \cdot 0.1 \cdot (0.9)^{11} \approx 0.387$.

What is the variance in the number of left-handed people among 6 randomly selected individuals?

Variance = $n p (1 - p) = 6 \cdot 0.1 \cdot 0.9 = 0.54$.

Is it more likely to find exactly 0 or exactly 1 left-handed person in a group of 6?

Calculations show that the probability of 0 left-handed is about 0.5314, and for 1 left-handed, approximately 0.354, so it's more likely to have none.

If a survey of 6 people finds 2 left-handed individuals, is this consistent with the 10% population statistic?

Yes, since the probability of exactly 2 is about 0.214, such an outcome is reasonably consistent with the population proportion.

Additional Resources

Probability of Selecting Left-Handed Individuals in a Random Sample: An In-Depth Analysis

Understanding the concept of probability is fundamental to grasping how likely certain events are to occur, especially in scenarios involving human characteristics such as handedness. In this article, we

explore a classic problem: If 10 percent of the population is left-handed, what is the probability that, upon randomly selecting 6 individuals, a specific number of them are left-handed? This question not only exemplifies basic probability principles but also highlights the broader implications of statistical analysis in social sciences, health studies, and behavioral research.

Foundations of Probability and Its Relevance

What Is Probability?

Probability is a branch of mathematics concerned with quantifying the likelihood that a particular event will occur. It is expressed as a number between 0 and 1, where:

- 0 indicates impossibility,
- 1 indicates certainty,
- Values in between represent varying degrees of likelihood.

For example, a probability of 0.1 (or 10%) indicates a 10 percent chance of the event occurring.

Relevance in Real-World Contexts

Probability plays a crucial role in diverse fields such as:

- Medicine: Assessing the likelihood of disease occurrence.
- Economics: Modeling market risks.
- Social sciences: Understanding demographic trends.
- Sports: Calculating winning odds.

In the context of handedness, probability helps researchers understand how common left-handedness is and predict its occurrence in different populations.

Understanding the Problem: Handedness and Random Selection

The Given Data

The problem states:

- 10 percent of the population is left-handed.
- A random sample of 6 people is selected.
- The question is: What is the probability that a certain number of these 6 people are left-handed?

This setting involves a binomial probability model, given the fixed number of trials, two possible outcomes per trial (left-handed or right-handed), and independence among individuals.

Assumptions and Simplifications

To analyze this problem, certain assumptions are typically made:

- Each person's handedness is independent of others.
- The probability of any individual being left-handed remains constant at 0.10.
- The sample is randomly selected, representing the population accurately.

These assumptions simplify the calculation and align with standard binomial probability models.

Mathematical Framework: The Binomial Distribution

What Is the Binomial Distribution?

The binomial distribution models the number of successes (in this case, left-handed individuals) in a fixed number of independent Bernoulli trials (each person's handedness), where each trial has two possible outcomes:

- Success (left-handed)
- Failure (right-handed)

The probability mass function (PMF) of the binomial distribution is given by:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where:

- n = total number of trials (here, 6),
- k = number of successes (left-handed individuals),
- p = probability of success in a single trial (0.10),
- $\binom{n}{k}$ = binomial coefficient, representing the number of ways to choose k successes from n trials.

Calculating Probabilities for Different Values of k

The primary goal is to compute the probability that exactly k people out of 6 are left-handed, for various values of k . These include:

- $P(0)$: no left-handed individuals,
- $P(1)$: exactly one,
- $P(2)$: exactly two,
- $P(3)$: exactly three,
- and so forth, up to $P(6)$.

Step-by-Step Calculation and Analysis

Calculating Specific Probabilities

Let's explore some specific cases:

Case 1: Probability that exactly 0 are left-handed

$$P(0) = \binom{6}{0} (0.10)^0 (0.90)^6 = 1 \times 1 \times 0.531441 = 0.531441$$

Case 2: Probability that exactly 1 is left-handed

$$P(1) = \binom{6}{1} (0.10)^1 (0.90)^5 = 6 \times 0.10 \times 0.59049 = 0.354294$$

Case 3: Probability that exactly 2 are left-handed

$$P(2) = \binom{6}{2} (0.10)^2 (0.90)^4 = 15 \times 0.01 \times 0.6561 = 0.098415$$

Case 4: Probability that exactly 3 are left-handed

$$P(3) = \binom{6}{3} (0.10)^3 (0.90)^3 = 20 \times 0.001 \times 0.729 = 0.01458$$

Similarly, probabilities for higher values of k decline sharply because of the low probability $(p=0.10)$.

Interpreting the Results

The calculations reveal:

- The most probable scenario is that none of the 6 people are left-handed ($P(0) \approx 53.1\%$),
- The probability of exactly one left-handed individual is about 35.4%,

- The chance of two left-handed individuals drops to roughly 9.8%,
- And the likelihood of three or more is minimal.

This distribution highlights the skewness towards lower counts of left-handed individuals, consistent with the low population prevalence.

Broader Implications and Insights

What Do These Probabilities Tell Us?

The analysis offers several insights:

- Expected Value: The expected number of left-handed individuals in a sample of 6 is $(n \times p = 6 \times 0.10 = 0.6)$. This means, on average, less than one person in such a sample is left-handed, reinforcing the rarity of left-handedness in small groups.
- Variability: The variance is given by $(n p (1-p) = 6 \times 0.10 \times 0.90 = 0.54)$, indicating some variability, but still concentrated around the mean.
- Real-world relevance: Such probabilistic models help researchers in designing studies, estimating sample sizes, and understanding the likelihood of observing certain traits in populations.

Limitations and Considerations

While the binomial model provides a robust framework, real-world factors might influence these probabilities:

- Population heterogeneity: Handedness rates can vary across demographics, regions, or cultures.
- Dependence among individuals: Family or social influences might introduce dependencies, violating the independence assumption.
- Sampling bias: Non-random sampling can skew observed probabilities.

Understanding these factors is crucial for interpreting results accurately and applying them in practical scenarios.

Applications and Extensions

Applications in Research and Policy

Quantifying the probability of specific characteristics in samples guides decision-making in:

- Educational planning: Recognizing the rarity of left-handedness can influence ergonomic designs.
- Medical research: Studying associations between handedness and neurological traits.
- Psychological assessments: Understanding population distributions helps in comparative analyses.

Extensions to Other Traits and Larger Samples

While this article focuses on a small sample size, similar principles can be applied to:

- Larger populations,
- Different traits with varying prevalence rates,
- Complex models accounting for dependencies or multiple traits.

This flexibility underscores the power of probability theory in diverse contexts.

Conclusion: Embracing the Power of Probability

The question of "Suppose that 10 percent of people are left-handed. If 6 people are selected at random, what is the probability?" exemplifies how fundamental statistical concepts can illuminate real-world phenomena. Through understanding the binomial distribution, we can quantify the likelihood of various outcomes, aiding researchers, policymakers, and educators in making informed decisions.

As demonstrated, the probability that none of the six individuals are left-handed is over 53%, while the chance of exactly one is about 35%. These insights reinforce the rarity of left-handedness and showcase how probability models provide clarity amid variability. Whether applied to genetic studies, behavioral research, or social sciences, such analyses underscore the importance of statistical literacy in interpreting human diversity and understanding the stochastic nature of traits within populations.

Ultimately, mastering probability enables us to navigate uncertainties, make predictions, and better comprehend the complex tapestry of human characteristics, one random selection at a time.

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