Suppose That The Functions G And F Are Defined As Follows. G(x)=(-5+x)(-4+x) F(x)=-7+8x (a) Find ((g)/(f))(1).

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Understanding the behavior of functions and their ratios is fundamental in algebra and calculus. In this article, we will explore how to evaluate the ratio of two functions, $\frac{g(x)}{f(x)}$, at a specific point, namely $x=1$, given the functions:

- $G(x) = (-5 + x)(-4 + x)$
- $F(x) = -7 + 8x$

This problem involves multiple steps, including substituting the given x-value into each function, simplifying the expressions, and then calculating the ratio. By the end of this article, you will have a comprehensive understanding of how to approach similar problems involving function ratios, as well as insights into algebraic manipulation and evaluation.

Understanding the Given Functions

Before diving into calculations, it's essential to understand the structure of each function.

Function G: $G(x) = (-5 + x)(-4 + x)$

This function is defined as the product of two binomials:

- $-5 + x$
- $-4 + x$

Multiplying these binomials will result in a quadratic expression, which can be expanded using the distributive property (FOIL method).

Function F: $F(x) = -7 + 8x$

This is a linear function, straightforward to evaluate at any point.

Step-by-Step Solution to Find $\left(\frac{g}{f} \right)(1)$

The goal is to compute:

$$
\left(\frac{g}{f} \right)(1) = \frac{G(1)}{F(1)}
$$

Let's proceed with each component.

Step 1: Calculate $G(1)$

Given:

$$
G(x) = (-5 + x)(-4 + x)
$$

Substitute $x=1$:

$$
G(1) = (-5 + 1)(-4 + 1)
$$

Compute each:

$$
-5 + 1 = -4
$$
$$
-4 + 1 = -3
$$

Therefore:

$$
G(1) = (-4)(-3) = 12
$$

Step 2: Calculate $F(1)$

Given:

$$
F(x) = -7 + 8x

$$

Substitute $x=1$:

$$
F(1) = -7 + 8 \times 1 = -7 + 8 = 1
$$

Step 3: Compute the ratio $\frac{G(1)}{F(1)}$

Now, substituting the calculated values:

$$
\frac{G(1)}{F(1)} = \frac{12}{1} = 12
$$

Answer: $\boxed{12}$

Additional Insights: Simplifying the Functions

While the calculation above is straightforward, understanding how to manipulate the functions algebraically can be beneficial, especially if similar problems involve more complex expressions or require general formulas.

Expanding $G(x)$

Instead of leaving $G(x)$ in factored form, you can expand it:

$$
G(x) = (-5 + x)(-4 + x)
$$

Applying the distributive property:

$$
G(x) = (-5)(-4) + (-5)(x) + x(-4) + x \times x
$$

Simplify each term:

$$
G(x) = 20 - 5x - 4x + x^2
$$

Combine like terms:

$$
G(x) = x^2 - 9x + 20
$$

This quadratic form can be useful for further analysis or for evaluating at different points.

Verifying $G(1)$ with the expanded form

Substitute $x=1$:

$$
G(1) = (1)^2 - 9 \times 1 + 20 = 1 - 9 + 20 = 12
$$

which matches our previous calculation, confirming consistency.

Understanding Function Ratios in Algebra

Evaluating ratios like $\frac{g(x)}{f(x)}$ at specific points is common in various mathematical contexts, including limits, derivatives, and real-world applications such as physics and economics.

When is the ratio undefined?

The ratio $\frac{g(x)}{f(x)}$ is undefined when $f(x) = 0$. For the given $F(x) = -7 + 8x$, set it equal to zero:

$$
-7 + 8x = 0 \Rightarrow 8x = 7 \Rightarrow x = \frac{7}{8}
$$

At $x = \frac{7}{8}$, the ratio $\frac{g(x)}{f(x)}$ is undefined.

Applications of function ratios

- Rate comparisons: Comparing speeds, growth rates, or other ratios.
- Limits: Analyzing the behavior of ratios as x approaches specific values.
- Optimization: Finding maximum or minimum ratios in applied problems.

Common Mistakes to Avoid

- Forgetting to substitute x before simplifying: Always substitute before simplifying or evaluating.
- Dividing by zero: Confirm that the denominator does not equal zero at the point of evaluation.
- Misapplying algebraic operations: Be cautious with signs, especially when expanding binomials.

Summary and Final Remarks

In this comprehensive guide, we've demonstrated how to evaluate the ratio of two functions at a specific point, using the functions:

- $G(x) = (-5 + x)(-4 + x)$
- $F(x) = -7 + 8x$

By substituting $x=1$, expanding where necessary, and simplifying, we found:

$$
\boxed{ \left(\frac{g}{f} \right)(1) = 12 }
$$

Understanding these procedures enhances your algebraic skills and prepares you for more advanced topics involving function analysis, calculus, and real-world problem-solving.

Practice Problems for Further Mastery

1. Given $H(x) = (x+3)(x-2)$ and $K(x) = 4 - x$, find $\left(\frac{H}{K} \right)(2)$.

2. If $M(x) = 2x^2 - 5x + 3$ and $N(x) = x - 1$, evaluate $\left(\frac{M}{N} \right)(3)$.

3. For $P(x) = (x-1)^2$ and $Q(x) = 3x + 2$, determine $\left(\frac{P}{Q} \right)(-2)$.

By practicing similar problems, you'll strengthen your understanding of function ratios, algebraic manipulation, and evaluation techniques.

Frequently Asked Questions

Given the functions G(x)=(-5 + x)(-4 + x) and F(x)=-7 + 8x, how do you compute (G/F)(1)?

First, find G(1): G(1) = (-5 + 1)(-4 + 1) = (-4)(-3) = 12. Then, find F(1): F(1) = -7 + 8(1) = -7 + 8 = 1. Finally, (G/F)(1) = G(1) / F(1) = 12 / 1 = 12.

What is the value of G(1) when G(x) = (-5 + x)(-4 + x)?

G(1) = (-5 + 1)(-4 + 1) = (-4)(-3) = 12.

How do you evaluate F(1) for F(x) = -7 + 8x?

F(1) = -7 + 8(1) = -7 + 8 = 1.

What is the value of the quotient (G/F)(1) given G(1)=12 and F(1)=1?

The quotient (G/F)(1) = 12 / 1 = 12.

How can you generalize the process of finding (G/F)(x) for any x?

Calculate G(x) by plugging x into G(x)=(-5 + x)(-4 + x), then calculate F(x)= -7 + 8x, and finally divide G(x) by F(x).

Are there any restrictions on x for the function (G/F)(x)?

Yes, since division by zero is undefined, F(x) must not be zero. For F(x) = -7 + 8x, F(x)=0 when x=7/8.

What is the importance of verifying the domain when computing (G/F)(x)?

Verifying the domain ensures that the denominator F(x) is not zero, preventing undefined expressions and ensuring valid calculations.

How do you interpret the result (G/F)(1)=12 in the

context of these functions?

It means that when x=1, the ratio of G(x) to F(x) is 12, indicating how G and F relate at that specific point.

Additional Resources

Suppose That The Functions G And F Are Defined As Follows. G(x)=(-5+x)(-4+x) F(x)=-7+8x (a) Find ((g)/(f))(1).

Introduction

In the realm of algebra and functions, understanding how to manipulate and analyze functions is fundamental. Whether you're tackling a math problem for school or applying these concepts in more advanced fields like engineering or data science, clarity about how functions work and how to evaluate their combinations is essential. Today, we'll explore a specific problem involving two functions, G and F, and guide you through the process of calculating the value of their quotient at a particular point. This problem not only tests your algebraic skills but also deepens your understanding of function operations, including composition and evaluation at specific points.

Understanding the Problem

The problem provides two functions:

- $G(x) = (-5 + x)(-4 + x)$
- $F(x) = -7 + 8x$

The goal is to find the value of $\frac{G(x)}{F(x)}$ at $x=1$, i.e., compute $\left(\frac{G}{F}\right)(1)$.

To approach this problem systematically, we need to:

1. Understand the definitions and structure of the functions.
2. Simplify $G(x)$ to a more manageable form.
3. Evaluate both $G(1)$ and $F(1)$.
4. Compute the quotient $\frac{G(1)}{F(1)}$.

Let's delve into each of these steps in detail.

Section 1: Analyzing and Simplifying the Functions

The Function $G(x)$

Given:

$$ G(x) = (-5 + x)(-4 + x) $$

This is a product of two linear binomials. To simplify, we can expand this expression using the distributive property (FOIL method):

$$
G(x) = (-5 + x)(-4 + x)
$$

Applying FOIL:

- First: $(-5)(-4) = 20$
- Outer: $(-5)(x) = -5x$
- Inner: $x \times (-4) = -4x$
- Last: $x \times x = x^2$

Adding all these together:

$$
G(x) = 20 - 5x - 4x + x^2
$$

Combine like terms:

$$
G(x) = x^2 - 9x + 20
$$

Now, $G(x)$ is expressed as a quadratic function:

$$
G(x) = x^2 - 9x + 20
$$

The Function $F(x)$

Given:

$$
F(x) = -7 + 8x
$$

This is already in simplified linear form.

Section 2: Evaluating the Functions at $x=1$

With simplified forms, we proceed to evaluate each function at $x=1$.

Calculating $G(1)$

$$
G(1) = (1)^2 - 9 \times 1 + 20 = 1 - 9 + 20
$$

Perform the calculations:

$$
1 - 9 = -8
$$

$$
-8 + 20 = 12
$$

So,

$$
G(1) = 12
$$

Calculating $F(1)$

$$
F(1) = -7 + 8 \times 1 = -7 + 8 = 1
$$

Section 3: Computing the Quotient $\frac{G(1)}{F(1)}$

Now that we have values for both functions at $x=1$:

$$
G(1) = 12
$$

$$
F(1) = 1
$$

The quotient is:

$$
\left(\frac{G}{F}\right)(1) = \frac{G(1)}{F(1)} = \frac{12}{1} = 12
$$

Section 4: Interpreting the Result and Its Significance

The calculation yields a straightforward result:

$$
\boxed{12}
$$

This value indicates that at $x=1$, the ratio of the functions G and F is 12. Such problems are common in algebra and serve as foundational exercises in understanding how functions interact and how their values can be combined through operations like division.

Why is this important?

- Function Evaluation: It reinforces the process of substituting specific values into functions.
- Algebraic Manipulation: It demonstrates the importance of simplifying functions before substitution to make calculations manageable.
- Understanding Ratios: It illustrates how the ratio of two functions can vary depending on the input, which is essential in real-world applications like physics, economics, and engineering.

Section 5: Broader Applications and Related Concepts

While this problem is computational, it opens doors to several broader mathematical concepts:

1. Function Composition and Ratios
Understanding how to evaluate $\frac{G(x)}{F(x)}$ at specific points is foundational in analyzing the behavior of functions relative to each other, such as in rates and proportional relationships.

2. Domain Considerations
Before division, it’s critical to ensure $F(x) \neq 0$ to avoid undefined expressions. In this case:

$$
F(1) = 1 \neq 0
$$

which validates the calculation.

3. Graphical Interpretation
Plotting $G(x)$ and $F(x)$ can offer visual insights into their behaviors, intersections, and the points where their ratio reaches specific values.

4. Extension to More Complex Functions
This approach can be extended to more complex functions, including rational functions, polynomial functions, and functions involving radicals or exponential components.

Conclusion

Evaluating the quotient of functions at specific points is a key skill in algebra that combines function analysis, algebraic manipulation, and substitution. In this particular problem, by simplifying the function \(G(x) \) to a quadratic form and evaluating both \(G(1) \) and \(F(1) \), we found that \(\left(\frac{G}{F}\right)(1) = 12 \). This process exemplifies the importance of understanding function structure and the systematic approach needed for accurate calculation. Mastery of these concepts not only enhances mathematical proficiency but also provides essential tools applicable across various scientific and engineering disciplines.

Summary of Steps:

1. Simplify \(G(x) \) to a standard polynomial form.
2. Evaluate \(G(1) \) and \(F(1) \).
3. Calculate the quotient \(\frac{G(1)}{F(1)} \).
4. Interpret the result within the context of the problem.

By following these steps, students and professionals alike can confidently analyze functions and their ratios, paving the way for more advanced mathematical exploration and practical application.

Suppose That The Functions G And F Are Defined As Follows G X 5 X 4 X F X 7 8x A Find G F 1

Suppose That The Functions G And F Are Defined As Follows G X 5 X 4 X F X 7 8x A Find G F 1

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