The System Function Of A Causal LTI System Is Given As H1(s)=2s + 5/s^2+5s+6 A) (2) Write Down The Differential

The System Function Of A Causal LTI System Is Given As H1(s)=2s + 5/s^2+5s+6 A) (2) Write Down The Differential

Understanding the system function of a causal Linear Time-Invariant (LTI) system is fundamental in control systems engineering, signal processing, and various fields of electrical engineering. The given system function, H1(s)=2s + 5 / (s^2 + 5s + 6), encapsulates the behavior of the system in the Laplace domain. This article delves into the detailed analysis of this system function, exploring how to derive the corresponding differential equation, its significance, and practical applications.

Introduction to System Functions in Causal LTI Systems

In the realm of control systems and signal processing, the system function, also known as the transfer function, plays a pivotal role. It characterizes the input-output relationship of a system in the Laplace domain, providing insights into system stability, transient response, and frequency characteristics.

What is a Causal LTI System?

- Causality: The system's output depends only on present and past inputs, not future inputs.
- Linearity: The principle of superposition applies; responses to individual inputs can be summed.
- Time-Invariance: System properties do not change over time.

Significance of the System Function H(s)

- Facilitates the analysis of system behavior without solving differential equations directly.
- Enables the use of algebraic methods to analyze system stability and response.
- Provides a pathway to design controllers and filters.

Analyzing the Given System Function H1(s)

The provided function is:

$$ H_1(s) = \frac{2s + 5}{s^2 + 5s + 6} $$

This rational function can be decomposed into simpler components to facilitate the derivation of the differential equation.

Step 1: Factor the Denominator

The quadratic denominator factors as:

$$s^2 + 5s + 6 = (s + 2)(s + 3)$$

This factorization indicates the system's poles are at $s = -2$ and $s = -3$, which influence the system's stability and transient response.

Step 2: Express H(s) in a more manageable form

The system function becomes:

$$H_1(s) = \frac{2s + 5}{(s + 2)(s + 3)}$$

Deriving the Differential Equation from the System Function

Converting a transfer function into its corresponding differential equation involves the following process:

Step 1: Express the transfer function as a ratio of polynomials

$$H_1(s) = \frac{Y(s)}{X(s)}$$

where:

- $Y(s)$: Laplace transform of the output signal $y(t)$
- $X(s)$: Laplace transform of the input signal $x(t)$

Assuming zero initial conditions, the relation between input and output in the Laplace domain is:

$$Y(s) = H_1(s) \times X(s)$$

Step 2: Cross-multiplied form

$$(s + 2)(s + 3) Y(s) = (2s + 5) X(s)$$

Expanding:

$$(s^2 + 5s + 6) Y(s) = (2s + 5) X(s)$$

Step 3: Convert to time domain

Recall that:

- $s Y(s) \leftrightarrow \frac{dy(t)}{dt}$
- $s^2 Y(s) \leftrightarrow \frac{d^2 y(t)}{dt^2}$
- $s X(s) \leftrightarrow \frac{dx(t)}{dt}$
- $X(s) \leftrightarrow x(t)$

Applying inverse Laplace transforms:

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6 y(t) = 2 \frac{dx(t)}{dt} + 5 x(t)$$

This is the differential equation governing the system.

Final Differential Equation of the System

The derived differential equation is:

$$
\boxed{
\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6 y(t) = 2 \frac{dx(t)}{dt} + 5 x(t)
}
$$

Key points about this differential equation:

- It is a second-order linear differential equation.
- The coefficients are constants, characteristic of an LTI system.
- The right side involves derivatives and the input, indicating the system's response depends on both the current and rate of change of the input.

Understanding the Differential Equation and System Behavior

The differential equation provides a comprehensive picture of how the system responds to various inputs.

Homogeneous Equation (System's Natural Response)

$$\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6 y(t) = 0$$

- Characteristic equation: $s^2 + 5s + 6 = 0$
- Roots: $s = -2, -3$

The natural response decays exponentially with time constants determined by these roots.

Particular Solution (Response to Input)

The non-homogeneous part involves the input $x(t)$ and its derivative, implying that the system's output depends directly on both the input and its rate of change.

Stability Analysis

- Since both poles are at $s = -2$ and $s = -3$, which are in the left-half of the s-plane, the system is stable.
- The transient response will decay exponentially without oscillations, given the real roots.

Practical Applications of the Derived Differential Equation

This differential equation is instrumental in various practical applications:

- Control System Design: Designing controllers to achieve desired transient and steady-state responses.
- Signal Processing: Filtering signals where the system acts as a second-order filter.
- Electrical Circuit Analysis: Modeling RLC circuits with specific input sources.
- Mechanical Systems: Analyzing mass-spring-damper systems with external forces.

Summary and Key Takeaways

In this comprehensive analysis, we:

- Started with the given system function $H_1(s) = \frac{2s + 5}{s^2 + 5s + 6}$
- Factorized the denominator to identify system poles
- Derived the corresponding differential equation governing the system's behavior
- Analyzed the stability and transient response characteristics
- Highlighted practical applications across engineering domains

Key points to remember:

- The system function directly leads to the differential equation describing the system
- The poles of the transfer function determine system stability and response
- The differential equation provides insights into how inputs influence outputs over time

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Conclusion

Understanding the system function of a causal LTI system and converting it into a differential equation is essential for analyzing and designing control systems and filters. The process involves algebraic factorization, Laplace domain transformations, and inverse Laplace transforms to arrive at a time-domain differential equation. This equation encapsulates the system's dynamic behavior, stability, and response to inputs, serving as a foundation for further analysis and engineering applications.

If you wish to explore more about transfer functions, system stability, or differential equations in control systems, numerous online resources and textbooks provide in-depth explanations and practical examples to enhance your understanding.

Frequently Asked Questions

What is the transfer function H1(s) of the given causal LTI system?

The transfer function is $H1(s) = (2s + 5) / (s^2 + 5s + 6)$.

How can we factor the denominator $s^2 + 5s + 6$ in the transfer function?

The quadratic $s^2 + 5s + 6$ factors as $(s + 2)(s + 3)$.

What is the differential equation corresponding to the transfer function $H1(s) = (2s + 5) / (s^2 + 5s + 6)$?

The differential equation is: $(s^2 + 5s + 6)Y(t) = (2s + 5)X(t)$, which in the time domain translates to $d^2y/dt^2 + 5dy/dt + 6y(t) = 2\ dx/dt + 5\ x(t)$.

How do you derive the differential equation from the given transfer function?

By taking the inverse Laplace transform of the transfer function multiplied by the input, we convert the ratio into a differential equation relating input and output. Specifically, replacing s with derivatives: $(d^2/dt^2 + 5\ d/dt + 6)\ y(t) = (2\ d/dt + 5)\ x(t)$.

What is the significance of the pole-zero plot for this transfer function?

The pole-zero plot reveals the system's stability and dynamic behavior; for this transfer function, the poles at s = -2 and s = -3 indicate a stable system with exponential decay responses.

Can you simplify the transfer function H1(s) for easier analysis?

Yes, factor the numerator and denominator: H1(s) = (2s + 5) / [(s + 2)(s + 3)]. This helps in partial fraction decomposition and inverse Laplace transforms.

What are the steps to implement this LTI system in a real-world application?

First, derive the differential equation from H1(s), then design an appropriate filter circuit or digital implementation based on the differential equations, ensuring stability and desired response.

Additional Resources

Understanding the System Function of a Causal LTI System: $H_1(s) = (2s + 5) / (s^2 + 5s + 6)$

Introduction to Causal LTI Systems and Their System Functions

In the realm of signals and systems, Linear Time-Invariant (LTI) systems are fundamental due to their predictable behavior and analytical tractability. These systems are characterized by two key properties:

- Linearity: The response to a linear combination of inputs is the same linear combination of the responses to each input.
- Time-Invariance: The system's characteristics do not change over time.

Understanding the system function—also known as the transfer function—is crucial because it encapsulates the entire behavior of the system in the complex frequency domain. The transfer function provides insight into stability, frequency response, and system poles and zeros.

The Given System Function $H_1(s)$

The problem presents us with a specific system function:

$$
H_1(s) = \frac{2s + 5}{s^2 + 5s + 6}
$$

This rational function relates the Laplace transforms of the output and input signals:

$$
Y(s) = H_1(s) \times X(s)
$$

\]

where:

- \(Y(s)\) is the Laplace transform of the output signal.
- \(X(s)\) is the Laplace transform of the input signal.

Significance of the System Function in Causal LTI Systems

The system function provides essential information, including:

- Poles and Zeros: Roots of the denominator and numerator respectively, which determine system stability and frequency response.
- Frequency Response: Evaluated on the imaginary axis, i.e., \(s = j\omega\).
- Time-Domain Behavior: Through inverse Laplace transforms, the system's impulse, step, and other responses can be derived.

Step 1: Expressing the System Function Clearly

The system function is given as a rational function:

\[
H_1(s) = \frac{Numerator}{Denominator}
\]

with:

- Numerator: \(2s + 5\)
- Denominator: \(s^2 + 5s + 6\)

Step 2: Factorizing the Denominator

Factorization simplifies the analysis of poles:

\[
s^2 + 5s + 6 = (s + 2)(s + 3)
\]

Thus, the system function becomes:

\[
H_1(s) = \frac{2s + 5}{(s + 2)(s + 3)}
\]

Step 3: Analyzing Poles and Zeros

Poles:

- Located at $s = -2$ and $s = -3$.

Zero:

- Located at the root of numerator $2s + 5 = 0 \Rightarrow s = -\frac{5}{2} = -2.5$.

Step 4: Causality and Stability Considerations

Causality implies that the system's output at any time depends only on current and past inputs. For causal systems, the inverse Laplace transform exists and is well-defined if the system function is proper or strictly proper (degree of numerator less than or equal to degree of denominator).

In this case:

- Numerator degree: 1
- Denominator degree: 2

Since numerator degree < denominator degree, $H_1(s)$ is strictly proper, satisfying causality conditions.

Stability in continuous-time LTI systems requires all poles to be in the left half-plane (LHP):

- Both poles at $s = -2$ and $s = -3$ are in LHP.

Therefore, the system is causal and stable.

Step 5: Writing the Differential Equation Corresponding to the System

General Approach:

Transform the transfer function back into a differential equation by cross-multiplied Laplace domain relations, then applying inverse Laplace transforms.

Step 6: Deriving the Differential Equation

Starting with:

$$
H_1(s) = \frac{Y(s)}{X(s)} = \frac{2s + 5}{s^2 + 5s + 6}
$$

This implies:

$$
Y(s) = H_1(s) \times X(s)
$$

Rearranged:

$$
(s^2 + 5s + 6)Y(s) = (2s + 5)X(s)
$$

Step 7: Applying the Inverse Laplace Transform

Recall the correspondence between Laplace domain and time domain:

- $s \leftrightarrow \frac{d}{dt}$
- Multiplication by s corresponds to differentiation
- Division by s corresponds to integration

Expressed as a differential equation:

$$
\left(\frac{d^2}{dt^2} + 5 \frac{d}{dt} + 6 \right) y(t) = 2 \frac{d}{dt} x(t) + 5 x(t)
$$

which can be written explicitly as:

$$
\boxed{
\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6 y(t) = 2 \frac{dx(t)}{dt} + 5 x(t)
}
$$

Step 8: Differential Equation Summary

The differential equation governing the system is:

$$
\boxed{
\frac{d^2 y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6 y(t) = 2 \frac{dx(t)}{dt} + 5 x(t)
}
$$

This equation relates the input $x(t)$ and output $y(t)$, encapsulating the system's behavior.

Step 9: Impulse and Step Response Analysis

Understanding the system's response to fundamental inputs like an impulse $\delta(t)$ or a step $u(t)$ is crucial.

Impulse Response $h(t)$:

- Found by inverse Laplace transform of $H_1(s)$.
- Since $H_1(s)$ is proper, partial fractions are used.

Step 10: Partial Fraction Expansion of $H_1(s)$

Express:

$$
H_1(s) = \frac{2s + 5}{(s + 2)(s + 3)}
$$

Set:

$$
\frac{2s + 5}{(s + 2)(s + 3)} = \frac{A}{s + 2} + \frac{B}{s + 3}
$$

Multiply through:

$$
2s + 5 = A(s + 3) + B(s + 2)
$$

which leads to:

$$
2s + 5 = A s + 3A + B s + 2B
$$
$$
(2s + 5) = (A + B)s + (3A + 2B)
$$

Matching coefficients:

- For s:

$$
2 = A + B
$$
- For constants:

\[

$$
5 = 3A + 2B
$$

Solve the system:

$$
A + B = 2
$$

$$
3A + 2B = 5
$$

Express $B = 2 - A$. Substitute into second:

$$
3A + 2(2 - A) = 5
$$

$$
3A + 4 - 2A = 5
$$

$$
A + 4 = 5
$$

$$
A = 1
$$

Then:

$$
B = 2 - A = 2 - 1 = 1
$$

Step 11: Final Partial Fraction Decomposition

$$
H_1(s) = \frac{1}{s + 2} + \frac{1}{s + 3}
$$

Step 12: Inverse Laplace Transform for Impulse Response

Recall:

$$
\mathcal{L}^{-1}\left\{\frac{1}{s + a}\right\} = e^{-a t} u(t)
$$

where $u(t)$ is the unit step function.

Thus,

$$
h(t) = e^{-2t} u(t) + e^{-3t} u(t)
$$

This is the impulse response of the system.

Step 13: Step Response

The step response can be obtained by multiplying the transfer function by $1/s$ (Laplace of step input):

$$
Y(s) = H_1(s) \times \frac{1}{s}
$$

and then taking the inverse Laplace transform.

Summary and Key Takeaways

- The system function $H_1(s)$ encapsulates the dynamic behavior of the causal LTI system.
- The system is causal because the numerator degree is less than the denominator degree.
- The poles at -2 and -3 ensure system stability.
- The zero at -2.5 influences the frequency response.
- The corresponding differential equation:

\[
\frac{d^2

The System Function Of A Causal Lti System Is Given As H1 S 2s 5 S 2 5s 6 A 2 Write Down The Differential

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