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Introduction

Understanding electric forces and fields generated by multiple charges is fundamental in physics, especially in electrostatics. In this article, we analyze a specific configuration involving four point charges arranged at the corners of a square. The setup includes three identical positive charges (+q) and one negative charge (-q), all placed at the vertices of a square with side length 'a'. Our goal is to determine the magnitude of the net electric force experienced by one of the charges, typically focusing on the negative charge or a particular positive charge, depending on the context. This problem exemplifies the principles of Coulomb's law, superposition, and vector addition, which are essential in understanding electrostatic interactions in multi-charge systems.

Setup of the Problem

Charge Configuration

Imagine a square with four vertices labeled as A, B, C, and D:

- Charges at vertices:
- A: +q
- B: +q
- C: +q
- D: -q

Assuming the square ABCD has side length 'a', the arrangement is as follows:

```
A (+q) ---- B (+q)
||
||
D (-q) ---- C (+q)
```

However, for simplicity, and based on typical problem setups, assume the negative charge is at a specific vertex, for example, at D, and the three positive charges occupy the remaining three vertices.

Objective

Calculate the magnitude of the net electric force acting on a particular charge—most commonly, the negative charge at D—due to the other three charges in the square.

Fundamental Concepts and Principles

Coulomb's Law

The foundation for electrostatic force calculations is Coulomb's law, which states:

$$
F = \frac{k |q_1 q_2|}{r^2}
$$

where:

- F is the magnitude of the force between two point charges,
- k is Coulomb's constant ($8.99 \times 10^9 \, \mathrm{Nm^2/C^2}$),
- q_1, q_2 are the magnitudes of the charges,
- r is the distance between the charges.

The direction of the force is along the line connecting the charges, attractive if charges are opposite, repulsive if like charges.

Superposition Principle

The net electric force on a charge is the vector sum of the forces exerted by all other charges in the system. This involves:

- Calculating individual forces,
- Resolving forces into components,
- Summing components to find the resultant force.

Calculating the Forces in the Square Configuration

Distances Between Charges

In a square of side 'a':

- Adjacent vertices are separated by distance 'a'.
- Diagonally opposite vertices are separated by $a\sqrt{2}$.

Assuming the negative charge is at D, the other charges are at A, B, and C:

- Distance from D to A: a (horizontal or vertical side),
- Distance from D to B: $a \sqrt{2}$ (diagonal),
- Distance from D to C: a (adjacent vertex).

Force Between Charges

Using Coulomb's law, the forces are:

- Force F_{DA} between D (-q) and A (+q):

$$
F_{DA} = \frac{k q^2}{a^2}
$$

- Force F_{DB} between D (-q) and B (+q):

$$
F_{DB} = \frac{k q^2}{(a \sqrt{2})^2} = \frac{k q^2}{2 a^2}
$$

- Force F_{DC} between D (-q) and C (+q):

$$
F_{DC} = \frac{k q^2}{a^2}
$$

All these forces are attractive because the charges are opposite in sign, thus pulling D towards A, B, and C.

Determining the Net Force on the Negative Charge D

Step 1: Establish Force Directions

- F_{DA} acts along the side AD, directed toward A.
- F_{DC} acts along the side DC, directed toward C.
- F_{DB} acts along the diagonal DB, directed toward B.

Since D is negative, the attraction pulls it toward the positive charges.

Step 2: Resolve Forces into Components

To find the net force, resolve each force into components along the axes (say, x and y axes):

- For F_{DA}:

- Acts along the side of the square; assume A is at the origin and D at the coordinate (0, a).
- Force points toward A, which is at (0,0).

- For F_{DC}:

- Similar reasoning, C is at (a,0), D at (0,a).

- For F_{DB}:

- B at (a,a), D at (0,a).

Calculating components:

- F_{DA}:

- Magnitude: $\frac{k q^2}{a^2}$
- Direction: toward A at (0,0). From D at (0,a), the force points downward (negative y-direction).

Components:

$$
F_{DA_x} = 0
$$
$$
F_{DA_y} = - \frac{k q^2}{a^2}
$$

- F_{DC}:

- Magnitude: $\frac{k q^2}{a^2}$
- From D at (0,a) toward C at (a,0). The line connecting D to C makes a diagonal.

- The vector from D to C:

$$
\vec{r}_{DC} = (a - 0, 0 - a) = (a, -a)
$$

- The magnitude:

$$
r_{DC} = a \sqrt{2}
$$

- Force magnitude:

$$
F_{DC} = \frac{k q^2}{2 a^2}
$$

- Components:

$$
F_{DC_x} = F_{DC} \times \frac{a}{a \sqrt{2}} = \frac{k q^2}{2 a^2} \times \frac{a}{a \sqrt{2}} = \frac{k q^2}{2 a^2} \times \frac{1}{\sqrt{2}} = \frac{k q^2}{2 a^2 \sqrt{2}}
$$

$$
F_{DC_y} = F_{DC} \times \frac{-a}{a \sqrt{2}} = - \frac{k q^2}{2 a^2 \sqrt{2}}
$$

- F_{DB}:

- Magnitude: $\frac{k q^2}{2 a^2}$

- From D (0,a) to B (a,a):

- Vector:

$$
\vec{r}_{DB} = (a - 0, a - a) = (a, 0)
$$

- Components:

$$
F_{DB_x} = \frac{k q^2}{2 a^2} \times 1 = \frac{k q^2}{2 a^2}
$$

$$
F_{DB_y} = 0
$$

Step 3: Summing Components to Find Net Force

Sum the x and y components separately:

- X-components:

$$
F_{net, x} = F_{DC_x} + F_{DB_x} = \frac{k q^2}{2 a^2 \sqrt{2}} + \frac{k q^2}{2 a^2}
$$

- Y-components:

$$
F_{net, y} = F_{DA_y} + F_{DC_y} + 0 = - \frac{k q^2}{a^2} - \frac{k q^2}{2 a^2 \sqrt{2}}
$$

Calculating the Magnitude of the Resultant Force

The overall net force magnitude:

$$
F_{net} = \sqrt{(F_{net, x})^2 + (F_{net, y})^2}
$$

Substituting the expressions:

$$
F_{net} = \sqrt{\left(\frac{k q^2}{2 a^2 \sqrt{2}} + \frac{k q^2}{2 a^2} \right)^2 + \left(- \frac{k q^2}{a^2} - \frac{k q^2}{2 a^2 \sqrt{2}} \
$$

Frequently Asked Questions

What is the configuration of the charges in the given problem?

The problem describes three identical positive charges (+q) placed at three corners of a square and a fourth negative charge (-q) at the remaining corner, forming a square of side length A.

How do you calculate the electric field at the position of the negative charge due to the three positive charges?

You calculate the electric field contributions from each of the three positive charges at the location of the negative charge, using Coulomb's law, and then vectorially add these fields to find the net electric field.

What is the magnitude of the electric field at the position of the negative charge due to a single positive charge?

The magnitude of the electric field due to a single positive charge at a distance A (the side length of the square) is $E = k|q|/A^2$, where k is Coulomb's constant.

How do symmetry considerations simplify the calculation of the net electric field?

Symmetry allows us to note that two of the positive charges produce electric fields that can cancel or add in specific directions, reducing the vector addition complexity when summing the fields at the negative charge's position.

What is the final expression for the magnitude of the electric field at the negative charge's location?

The magnitude of the net electric field at the negative charge's position is $E = (2kq)/A^2$, directed along the diagonal connecting the negative charge to the centroid of the positive charges.

Why do the electric fields due to the positive charges at the negative charge's location combine in a specific manner?

Because of the geometric symmetry of the square, the electric fields from the three positive charges have components that either reinforce or cancel each other, resulting in a net field aligned along the diagonal of the square.

What is the significance of the side length A in calculating the electric field in this configuration?

The side length A determines the distance between charges, directly influencing the magnitude of each Coulombic electric field component, and thus is essential in calculating the overall electric field at the negative charge's position.

Additional Resources

Three Identical Charges +q And A Fourth Charge -q Form A Square Of Side A. (a) Find The Magnitude Of The Electric Field At The Center.

Understanding the electric field generated by multiple point charges is a fundamental problem in electrostatics, offering insights into how charge configurations influence electric potential and field distributions. In this detailed guide, we explore the scenario where three identical charges +q and a fourth charge -q form a square of side a, and we aim to find the magnitude of the electric field at the center of this square. This problem combines principles of Coulomb's law, vector addition, and symmetry considerations, making it a valuable exercise for students and enthusiasts alike.

Introduction to the Problem

Imagine a square with four points located at its corners. Three of these corners are occupied by identical positive charges (+q), while the fourth corner holds a negative charge (-q). The side length of this square is denoted by a.

Our goal: Determine the magnitude of the electric field at the geometric center of the square due to these four charges.

Key points:
- Charges: Three are +q, one is -q.
- Geometry: Square of side a.
- Location: The point of interest is the center of the square.

Physical Principles and Concepts

Before delving into calculations, let's review the fundamental principles involved:

- Coulomb's Law: The electric field $\vec{E}$ due to a point charge q at a distance r is given by:

$$
\vec{E} = \frac{k |q|}{r^2} \hat{r}
$$

where:
- $k = \frac{1}{4\pi \varepsilon_0}$,
- $\hat{r}$ is the unit vector pointing from the charge to the point where the field is measured.

- Superposition Principle: The net electric field at a point is the vector sum of the electric fields due to each individual charge.

- Symmetry: The geometric arrangement allows us to analyze the vector contributions systematically, considering symmetry for simplification.

Step-by-Step Solution Approach

To find the electric field at the center, we follow these steps:

1. Identify the positions of charges and the center.

2. Calculate the distance from each charge to the center.

3. Determine the electric field vector contributed by each charge at the center.

4. Use symmetry to simplify calculations where possible.

5. Sum the vectors to find the net electric field.

---

1. Coordinates and Geometry Setup

Let's define a coordinate system for clarity:

- Place the square in the xy-plane.
- Let the center of the square be at the origin, $O(0,0)$.
- Assign the corners as follows:

Corner	Coordinates	Charge	Label
A	$\left(-\frac{A}{2}, \frac{A}{2} \right)$	+q	Q_A
B	$\left(\frac{A}{2}, \frac{A}{2} \right)$	+q	Q_B
C	$\left(\frac{A}{2}, -\frac{A}{2} \right)$	-q	Q_C
D	$\left(-\frac{A}{2}, -\frac{A}{2} \right)$	+q	Q_D

Suppose the negative charge -q is at corner C, and the positive charges +q are at corners A, B, and D.

---

2. Distances from Charges to the Center

Since the square is symmetric:

$$
r = \text{distance from each corner to center} = \frac{A}{\sqrt{2}}
$$

This is because the distance from a corner (x,y) to the center $(0,0)$:

$$
r = \sqrt{\left(\pm \frac{A}{2} \right)^2 + \left(\pm \frac{A}{2} \right)^2} = \sqrt{\frac{A^2}{4} + \frac{A^2}{4}} = \sqrt{\frac{A^2}{2}} = \frac{A}{\sqrt{2}}
$$

---

3. Electric Field Due to Each Charge

The magnitude of the electric field due to each charge at the center:

$$
E = \frac{k |q|}{r^2} = \frac{k |q|}{(A/\sqrt{2})^2} = \frac{k |q|}{A^2/2} = \frac{2k |q|}{A^2}
$$

- For positive charges (+q), the field vectors point away from each charge toward the center.
- For the negative charge (-q), the field vector points toward the charge from the center (since the field points toward negative charges).

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4. Direction of Electric Field Vectors

Let's analyze the directions:

- Charges at A, B, D are positive: their fields point from the charges to the center.

- The charge at C is negative: its field points from the center toward the charge.

Expressing these directions:

- At corners A, B, D: The vectors point inward, towards the center, along the lines from each corner to the origin.
- At corner C: The vector points outward, from the center toward the negative charge.

---

5. Calculating the Components

Let's compute the electric field contributions from each charge in vector form:

- Charge at A $\left(-\frac{a}{2}, \frac{a}{2} \right)$:

- Vector from A to center: $\vec{r}_A = (0 - (-a/2), 0 - a/2) = \left(\frac{a}{2}, -\frac{a}{2} \right)$

- Magnitude: $r = \frac{a}{\sqrt{2}}$

- Unit vector: $\hat{r}_A = \frac{1}{r} \left(\frac{a}{2}, -\frac{a}{2} \right) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)$

- Electric field due to Q_A:

$$
\vec{E}_A = \frac{k q}{r^2} \hat{r}_A = \frac{2k q}{a^2} \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right) = \frac{2k q}{a^2} \times \frac{1}{\sqrt{2}} (1, -1) = \frac{\sqrt{2} k q}{a^2} (1, -1)
$$

- Charge at B $\left(\frac{a}{2}, \frac{a}{2} \right)$:

- Vector from B to center: $\vec{r}_B = (-a/2, -a/2)$

- Unit vector:

$$
\hat{r}_B = \left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \right)
$$

- Electric field:

$$
\vec{E}_B = \frac{\sqrt{2} k q}{a^2} (-1, -1)
$$

- Charge at D $\left(-\frac{a}{2}, -\frac{a}{2} \right)$:

- Vector from D to center: $(a/2, a/2)$

- Unit vector:

$$
\hat{r}_D = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
$$

- Electric field:

$$

\vec{E}_D = \frac{\sqrt{2} k q}{A^2} (1, 1)
\]

- Charge at C \(\left(\frac{A}{2}, -\frac{A}{2} \right)\):

- Vector from C to center: \((-A/2, A/2) \)

- For the negative charge, the electric field points toward the charge, so the vector points from the center to the charge (opposite direction of Coulomb field):

- The Coulomb field points from charge to the center:

\[
\vec{E}_C = - \frac{\sqrt{2} k q}{A^2} \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = - \frac{\sqrt{2} k q}{A^2} (-1, 1) = \frac{\sqrt{2} k q}{A^2} (

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