Through What Potential Difference Must An Electron Be Accel- Erated From Rest To Have A De Broglie Wavelength

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Understanding the relationship between the potential difference applied to an electron and its resulting de Broglie wavelength is fundamental in quantum mechanics and electron microscopy. This article aims to elucidate the principles governing this relationship, derive the necessary formulas, and provide practical calculations to determine the potential difference required for electrons to exhibit specific wavelengths. Whether you're a student, researcher, or enthusiast, grasping this concept is essential for exploring the wave-particle duality of electrons and their applications in modern technology.

Introduction to De Broglie Wavelength

Wave-Particle Duality

The wave-particle duality is a cornerstone of quantum physics, asserting that particles such as electrons exhibit both particle-like and wave-like properties. Louis de Broglie proposed that particles have an associated wavelength, now known as the de Broglie wavelength, expressed as:

- $\lambda = \frac{h}{p}$

where:
- λ is the de Broglie wavelength,
- h is Planck's constant ($6.626 \times 10^{-34}\, \text{Js}$),
- p is the momentum of the particle.

Significance of the De Broglie Wavelength

The de Broglie wavelength determines how wave-like a particle behaves. For electrons accelerated to high energies, their wavelengths become comparable to the size of atoms, enabling applications such as electron microscopes, which surpass optical microscopes in resolution.

Relation Between Electron Energy and Wavelength

Electron Kinetic Energy from Potential Difference

When an electron is accelerated through a potential difference V, it gains kinetic energy equal to the work done by the electric field:

$$
\text{Kinetic Energy (KE)} = eV
$$

where:
- $e = 1.602 \times 10^{-19}\, \text{C}$ is the elementary charge,
- V is the potential difference in volts.

Since the electron starts from rest, its kinetic energy is entirely due to this acceleration:

$$
KE = \frac{1}{2}mv^2
$$

with:
- $m = 9.109 \times 10^{-31}\, \text{kg}$ (mass of the electron),
- v is the velocity of the electron after acceleration.

Equating the two expressions:

$$
eV = \frac{1}{2}mv^2
$$

which allows us to find the velocity:

$$
v = \sqrt{\frac{2eV}{m}}
$$

Electron Momentum and De Broglie Wavelength

The momentum p of the electron is:

$$
p = mv
$$

Substituting the expression for v:

$$
p = m \sqrt{\frac{2eV}{m}} = \sqrt{2meV}
$$

Therefore, the de Broglie wavelength becomes:

$$
\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}
$$

This fundamental equation relates the potential difference V to the wavelength λ.

Deriving the Formula for Potential Difference Based on Desired Wavelength

By rearranging the previous equation:

$$
\lambda = \frac{h}{\sqrt{2meV}}
$$

we get:

$$
\sqrt{2meV} = \frac{h}{\lambda}
$$

Squaring both sides:

$$
2meV = \frac{h^2}{\lambda^2}
$$

Solving for V:

$$
V = \frac{h^2}{2me \lambda^2}
$$

This formula allows us to calculate the potential difference needed to accelerate an electron from rest to have a specific de Broglie wavelength.

Practical Calculations and Examples

Calculating the Potential Difference for a Given Wavelength

Suppose we want an electron to have a de Broglie wavelength of $\lambda = 0.1\, \text{nm} = 1 \times 10^{-10}\, \text{m}$. Using the formula:

$$
V = \frac{h^2}{2me \lambda^2}
$$

Plugging in the known values:

$$
h = 6.626 \times 10^{-34}\, \text{Js}
$$
$$
m = 9.109 \times 10^{-31}\, \text{kg}
$$
$$
e = 1.602 \times 10^{-19}\, \text{C}
$$
$$
\lambda = 1 \times 10^{-10}\, \text{m}
$$

Calculating numerator:

$$
h^2 = (6.626 \times 10^{-34})^2 = 4.39 \times 10^{-67}\, \text{Js}^2
$$

Calculating denominator:

$$
2me \lambda^2 = 2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times (1 \times 10^{-10})^2
$$
$$
= 2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times 1 \times 10^{-20}
$$
$$
= 2 \times 9.109 \times 1.602 \times 10^{-31 -19 -20}
$$
$$
= 2 \times 14.588 \times 10^{-70} = 29.176 \times 10^{-70} = 2.9176 \times 10^{-69}
$$

Now, compute V:

$$
V = \frac{4.39 \times 10^{-67}}{2.9176 \times 10^{-69}} \approx 15.05\, \text{V}
$$

\]

Thus, an electron must be accelerated through approximately 15 volts to have a de Broglie wavelength of 0.1 nm.

Implication for Electron Microscopy

In electron microscopes, electrons are accelerated through high voltages to achieve very short wavelengths, enabling imaging at atomic resolutions. For example, to attain a wavelength of approximately 0.005 nm (which is comparable to interatomic distances), the required potential difference is:

$$
V = \frac{h^2}{2me \lambda^2}
$$

Plugging in $\lambda = 0.005\, \text{nm} = 5 \times 10^{-12}\, \text{m}$:

$$
V \approx \frac{4.39 \times 10^{-67}}{2.9176 \times 10^{-69} \times (5 \times 10^{-12}/1 \times 10^{-10})^2}
$$

Note that:

$$
\frac{5 \times 10^{-12}}{1 \times 10^{-10}} = 0.05
$$

and

$$
(0.05)^2 = 0.0025
$$

Thus,

$$
V \approx \frac{4.39 \times 10^{-67}}{2.9176 \times 10^{-69} \times 0.0025} \approx \frac{4.39 \times 10^{-67}}{7.294 \times 10^{-72}} \approx 601.7\, \text{V}
$$

So, an acceleration voltage of roughly 600 volts is needed for electrons to have a wavelength suitable for atomic-resolution imaging.

Additional Considerations and Limitations

Relativistic Effects at High Energies

The derivation above assumes non-relativistic mechanics, which is valid for relatively low voltages (up to a few tens of kilovolts). However, at higher energies (above approximately 100 keV), relativistic effects become significant, affecting the electron's mass and velocity. The relativistic momentum p is given by:

$$
p = \frac{mv}{\sqrt{1 - v^2/c^2}}
$$

where c is the speed of light. In such cases, the de Broglie wavelength formula must be modified to include relativistic corrections:

$$
\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV(1 + \frac{eV}{2mc^2})}}
$$

which complicates calculations but increases accuracy at high energies.

Practical Limitations

While increasing the accelerating potential reduces the de Broglie wavelength, practical limits exist:

- Equipment Constraints: High-voltage accelerators are expensive and complex.
- Material Limitations: Components must withstand high voltages without breakdown.
- Relativistic Corrections: As energies increase, relativistic effects must be considered for precise calculations.
- Beam Coherence: Shorter wavelengths require high-quality electron sources with minimal energy spread.

Summary and Conclusions

The

Frequently Asked Questions

What is the formula to determine the potential difference required to accelerate an electron to a specific de Broglie wavelength?

The potential difference V is given by $V = (h^2) / (8\ m\ e\ \lambda^2)$, where h is Planck's constant, m is the mass of the electron, e is the elementary charge, and λ is the de Broglie wavelength.

How does the de Broglie wavelength relate to the kinetic energy of an electron accelerated through a potential difference?

The de Broglie wavelength λ is related to the electron's kinetic energy KE by $\lambda = h / \sqrt{(2meV)}$, where V is the potential difference, indicating that higher V results in a shorter wavelength.

Why is it important to know the potential difference when studying electron wave behavior in experiments?

Knowing the potential difference allows precise control over the electron's wavelength, enabling the study of wave properties, interference, and diffraction phenomena in experiments.

What physical constants are needed to calculate the potential difference for a given de Broglie wavelength?

The constants required are Planck's constant (h), the mass of the electron (m), and the elementary charge (e).

If an electron is accelerated from rest through a potential difference of 100 V, what is its de Broglie wavelength?

Using $\lambda = h / \sqrt{(2meV)}$, the wavelength is approximately 3.7 nanometers, calculated with known constants.

How does decreasing the potential difference affect the de Broglie wavelength of an electron?

Decreasing the potential difference reduces the electron's kinetic energy, resulting in a longer de Broglie wavelength.

Can the potential difference required to achieve a specific de Broglie wavelength be practically achieved in laboratory settings?

Yes, potential differences in the range of a few hundred volts to several kilovolts are commonly used in laboratories to produce electrons with desired wavelengths for experiments.

Additional Resources

Through What Potential Difference Must An Electron Be Accelerated From Rest To Have A De Broglie Wavelength

In the realm of quantum mechanics, particles such as electrons exhibit properties that blur the line between classical and quantum physics. One of the most fascinating phenomena is wave-particle duality, where particles can display wave-like behavior under certain conditions. A fundamental aspect of this behavior is encapsulated in the concept of the De Broglie wavelength, which relates a particle's momentum to a characteristic wavelength. This principle is not only pivotal in understanding microscopic systems but also underpins technologies like electron microscopes and quantum computing. A key question that often arises is: Through what potential difference must an electron be accelerated from rest to acquire a specific De Broglie wavelength? This article delves into the physics behind this question, providing a clear, comprehensive explanation suitable for students, educators, and enthusiasts alike.

Understanding the De Broglie Wavelength

The Wave-Particle Duality Concept

The wave-particle duality is a cornerstone of quantum mechanics, first proposed by Louis de Broglie in 1924. It asserts that particles such as electrons, protons, and even larger molecules exhibit both particle-like and wave-like properties. While classical physics treats particles and waves as distinct entities, quantum mechanics reveals that particles can have wave characteristics, especially at microscopic scales.

De Broglie hypothesized that the wavelength (λ) associated with a particle is inversely proportional to its momentum (p):

$$
\lambda = \frac{h}{p}
$$

where:
- h is Planck's constant ($6.626 \times 10^{-34}\,\mathrm{Js}$),
- p is the momentum of the particle.

This simple yet profound relation bridges the gap between classical momentum and quantum wave properties, enabling us to predict when wave-like behavior becomes significant.

Significance in Physics and Technology

The De Broglie wavelength becomes particularly relevant when the particle's wavelength is comparable to the dimensions of the system under study. For electrons, this wavelength determines phenomena such as diffraction and interference, which are observable in experiments like electron diffraction through crystals. Furthermore, understanding and controlling the wavelength via the particle's velocity or energy is crucial in designing electron microscopes, where shorter wavelengths allow imaging at higher resolutions.

Relating Electron Energy to Wavelength

Classical and Quantum Perspectives on Electron Energy

An electron initially at rest, when accelerated through a potential difference V, gains kinetic energy. In classical terms, the kinetic energy ($K.E.$) acquired by the electron is given by:

$$
K.E. = eV
$$
where e is the elementary charge ($1.602 \times 10^{-19}\, \mathrm{C}$).

In quantum mechanics, this kinetic energy translates into momentum, which in turn sets the De Broglie wavelength. To connect these concepts, we need to express the electron's momentum in terms of its energy.

The Non-Relativistic Approximation

For many practical scenarios where the electron’s velocity is much less than the speed of light ($c \approx 3 \times 10^8\, \mathrm{m/s}$), a non-relativistic approximation suffices. Under this approximation:

$$
K.E. = \frac{1}{2}mv^2
$$
and the momentum:

$$
p = mv
$$
where m is the rest mass of the electron ($9.109 \times 10^{-31}\, \mathrm{kg}$).

Since $K.E. = eV$, we can write:

$$
\frac{1}{2} m v^2 = eV
$$

Rearranged to find the momentum:

$$
p = mv = \sqrt{2 m e V}
$$

Substituting into De Broglie's relation:

$$
\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m e V}}
$$

This formula directly links the accelerating potential V to the resulting De Broglie wavelength λ.

Deriving the Potential Difference for a Given Wavelength

Mathematical Relationship

The core question is: Given a desired De Broglie wavelength λ, what potential difference V must an electron be accelerated through?

Starting from the relation:

$$
\lambda = \frac{h}{\sqrt{2 m e V}}
$$

Rearranged to solve for V:

$$
V = \frac{h^2}{2 m e \lambda^2}
$$

This formula allows us to compute the necessary accelerating potential once the target wavelength is specified.

Step-by-Step Calculation

Suppose we want an electron to have a De Broglie wavelength of $\lambda = 0.1\, \mathrm{nm} = 1 \times 10^{-10}\, \mathrm{m}$. Plugging into the formula:

\[

$$
V = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times (1 \times 10^{-10})^2}
$$

Calculating numerator:

$$
(6.626 \times 10^{-34})^2 = 4.39 \times 10^{-67}
$$

Calculating denominator:

$$
2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times 1 \times 10^{-20} = 2 \times 9.109 \times 1.602 \times 10^{-70} \approx 29.2 \times 10^{-70} = 2.92 \times 10^{-69}
$$

Finally:

$$
V = \frac{4.39 \times 10^{-67}}{2.92 \times 10^{-69}} \approx 150.3\, \mathrm{V}
$$

Thus, an electron must be accelerated through approximately 150 volts to have a De Broglie wavelength of 0.1 nm.

Relativistic Considerations

When and Why Relativity Matters

The non-relativistic approximation holds well for lower energies, typically up to a few tens of volts. However, as the potential difference increases, electron velocities approach a significant fraction of the speed of light, necessitating relativistic corrections.

For high energies, the total energy E of the electron becomes:

$$
E = \sqrt{(pc)^2 + (m c^2)^2}
$$

where $m c^2 \approx 0.511\, \mathrm{MeV}$ is the rest energy of the electron.

The relativistic expression for momentum:

$$

p = \frac{\sqrt{E^2 - (m c^2)^2}}{c}
\]

and the kinetic energy:

\[
K.E. = E - m c^2
\]

Relating \(V\) and \(\lambda\) in the relativistic case involves more elaborate formulas, but for most practical purposes involving potential differences under a few hundred volts, the non-relativistic approximation remains sufficiently accurate.

Practical Applications and Implications

Electron Microscopy and Material Science

The ability to control an electron’s wavelength by adjusting the accelerating voltage is fundamental in electron microscopy. Shorter wavelengths (achieved with higher voltages) allow for higher resolution imaging, enabling scientists to observe structures at the atomic level. For example, electrons accelerated through 100 keV (kiloelectron volts) have wavelengths around 0.0037 nm, orders of magnitude smaller than visible light, which explains the extraordinary resolving power of electron microscopes.

Particle Accelerators and Quantum Experiments

In particle physics, understanding the relationship between potential difference and wavelength allows researchers to tune particle beams for specific experiments, such as diffraction studies or probing the structure of materials. Precise control over electron energy and wavelength enables the observation of phenomena like quantum interference and wavefunction behavior.

Designing Experiments with Targeted Wavelengths

Scientists wishing to observe diffraction patterns from crystals or other periodic structures can determine the required acceleration voltage for electrons to produce the desired wavelength. For example, to observe diffraction from a lattice with a spacing of 0.2 nm, electrons must be accelerated through a potential difference that grants them a wavelength comparable to or smaller than 0.2 nm, calculated via the formulas outlined above.

Summary and Key Takeaways

- The De Broglie wavelength links a particle's momentum to its wave properties, given by $\lambda = h / p$.
- Accelerating an electron from rest through a potential difference V imparts kinetic energy eV, which relates to its momentum.
- Under non-relativistic assumptions, the potential difference needed for a specific wavelength λ is:

$$
V = \frac{h^2}{2 m e \lambda^2}
$$

- For practical calculations, this formula helps determine the acceleration voltage necessary to produce electrons

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