Two Point Charges Q And -q Are Located On The Z Axis At Z +a And Z--a, Respectively. (a) Find The Electrostatic

Two Point Charges Q And -q Are Located On The Z Axis At Z +a And Z--a, Respectively. (a) Find The Electrostatic is a classic problem in electrostatics that helps us understand how electric charges interact and influence the electric potential and field in space. When dealing with point charges placed along the z-axis, the goal often involves calculating the resulting electric potential and electric field at any point in space, especially along the axis itself or at specific points of interest.

In this article, we will explore how to find the electrostatic potential and electric field created by two point charges positioned symmetrically along the z-axis. We will delve into the fundamental principles of Coulomb's law, electric potential, and field calculations, providing a comprehensive guide for students, educators, and enthusiasts interested in the physics of electric charges.

Understanding the Setup of the Problem

Coordinates and Positions of Charges

The problem involves two point charges:

- Charge Q located at position z = +a on the z-axis.
- Charge -q located at position z = -a on the z-axis.

These positions are symmetric about the origin, which simplifies many calculations due to symmetry properties. The charges are fixed in space, and the goal is to analyze the electric potential and electric field they produce at a general point in space, often denoted as (x, y, z).

Key Concepts in Electrostatics

Before diving into calculations, it’s important to review some fundamental concepts:

- **Coulomb's Law:** Describes the force between two point charges:

$$F = \frac{1}{4\pi\epsilon_0} \frac{|Q_1 Q_2|}{r^2}$$

- **Electric Potential (V):** Scalar quantity representing potential energy per unit charge at a point in space:

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{q_i}{r_i}$$

- **Electric Field (E):** Vector field indicating the force per unit positive charge at a point:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \sum q_i \frac{\hat{r}_i}{r_i^2}$$

Where:
- ϵ_0 is the permittivity of free space.
- r_i is the distance from the charge q_i to the point of interest.
- $\hat{r}_i$ is the unit vector pointing from the charge to the point.

Calculating the Electric Potential at a Point

General Expression for Electric Potential

Since electric potential is a scalar quantity, it can be directly summed from each point charge's contribution:

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r_Q} - \frac{q}{r_{-q}} \right)$$

Where:
- r_Q is the distance from the charge Q at $\left(0, 0, +a\right)$ to the point $\vec{r} = (x, y, z)$.
- r_{-q} is the distance from the charge $-q$ at $\left(0, 0, -a\right)$ to the point $\vec{r}$.

Calculating the Distances

The distances are given by:

$$ r_Q = \sqrt{(x)^2 + (y)^2 + (z - a)^2} $$
$$ r_{-q} = \sqrt{(x)^2 + (y)^2 + (z + a)^2} $$

At points along the z-axis ($x = 0, y = 0$), these simplify to:

$$ r_Q = |z - a| $$
$$ r_{-q} = |z + a| $$

Therefore, the potential at any point along the z-axis becomes:

$$ V(z) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{|z - a|} - \frac{q}{|z + a|} \right) $$

This expression reveals how the potential varies along the axis, providing insight into regions of high and low potential, and the influence of charge magnitudes and positions.

Calculating the Electric Field

Vector Nature of Electric Field

Unlike potential, the electric field is a vector quantity. To find the electric field at a point, we sum the contributions from each charge, considering both magnitude and direction:

$$ \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{r_Q^3} \vec{r}_Q - \frac{q}{r_{-q}^3} \vec{r}_{-q} \right) $$

Where:
- $\vec{r}_Q = \vec{r} - \vec{r}_Q'$,
- $\vec{r}_{-q} = \vec{r} - \vec{r}_{-q}'$.

Electric Field Along the Z-Axis

Along the z-axis, the symmetry simplifies calculations:

$$ E_z(z) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q (z - a)}{|z - a|^3} - \frac{q (z + a)}{|z + a|^3} \right) $$

This expression indicates the net electric field at any point along the axis, pointing in the positive or negative z-direction depending on the relative magnitudes and positions of the charges.

Special Cases and Physical Interpretations

Potential and Field at Specific Points

- At the origin ($z=0$):

$$V(0) = \frac{1}{4\pi\epsilon_0} \left(\frac{Q}{a} - \frac{q}{a} \right) = \frac{1}{4\pi\epsilon_0} \frac{Q - q}{a}$$

- Far away from the charges ($z \to \pm \infty$):

$$V(z) \to 0$$

- At the positions of the charges:

The potential theoretically goes to infinity at the exact location of each point charge, reflecting the idealized point charge model.

Physical Significance of Results

The calculations reveal how the electric potential and field are influenced by the relative magnitudes of Q and q, and their positions. For example:

- If $Q = q$, the potential along the axis simplifies, and the net field at large distances tends to zero, indicating a neutral overall configuration.
- The superposition principle allows the addition of potentials and fields from multiple charges, which is fundamental in electrostatics.

Applications and Further Considerations

- **Electrostatic Shielding:** Understanding how charges influence fields helps in designing shields.
- **Capacitor Design:** Arrangements similar to this setup are foundational in capacitor theory.
- **Modeling Atomic and Molecular Structures:** The principles extend to understanding interactions at

microscopic scales.

In advanced studies, additional factors such as dielectric materials, boundary conditions, and dynamic effects can modify these basic calculations, but the principles remain the same.

Conclusion

Understanding how point charges influence the electric potential and field in space is central to electrostatics. When charges are positioned symmetrically along the z-axis, the calculations become more manageable, providing clear insights into the behavior of electric fields and potentials. By applying Coulomb's law, the principle of superposition, and symmetry considerations, we can accurately analyze complex charge distributions. Whether designing electronic components, studying atomic interactions, or exploring fundamental physics, mastering these calculations is essential for any aspiring physicist or engineer.

This comprehensive guide to finding the electrostatic potential and electric field for two point charges on the z-axis offers foundational knowledge applicable across many fields in science and technology.

Frequently Asked Questions

What is the expression for the electrostatic potential due to two point charges Q and -q located on the z-axis at positions +a and -a?

The electrostatic potential at any point along the z-axis is given by $V(z) = (1/4\pi\varepsilon_0)\ [Q/|z - a| - q/|z + a|]$.

How do you find the electric field on the z-axis due to two point charges Q and -q positioned at z = +a and z = -a?

The electric field components along the z-axis are $E_z = (1/4\pi\varepsilon_0)\ [Q/(z - a)^2 - q/(z + a)^2]$, pointing along the z-direction, considering the sign of each charge.

What is the significance of the positions +a and -a for charges Q and -q in the electrostatics problem?

These positions specify that the charges are symmetrically placed along the z-axis at distances a from the origin, which simplifies analysis due to symmetry.

How can the potential at the origin (z=0) be calculated for charges Q and -q located at z = +a and -a?

At z=0, the potential is $V(0) = (1/4\pi\varepsilon_0)\ [Q/|a| - q/|a|] = (1/4\pi\varepsilon_0)(Q - q)/a$, assuming both charges are at a distance a from the origin.

What is the condition for the net electric field to be zero along the z-axis between the charges?

The net electric field is zero at a point where the magnitudes of the fields due to each charge are equal and opposite, which occurs at z where $Q/(z - a)^2 = q/(z + a)^2$.

How does the electrostatic potential behave far away from the two charges on the z-axis?

At large distances ($z \gg a$), the potential behaves like that of a combined charge, approximately $V(z) \approx (1/4\pi\varepsilon_0)\ (Q - q)/z$, diminishing as $1/z$.

How does the symmetry of the charge placement influence the electric field and potential calculations?

The symmetrical placement at $\pm a$ simplifies the problem, allowing for straightforward superposition and analysis of the fields and potentials along the z-axis.

What are the physical implications if Q = q in this configuration?

If Q = q, the net charge at the positions cancels out, resulting in a zero net charge, and the potential and field would reflect a dipole configuration with specific characteristics, such as zero potential at the midpoint.

Additional Resources

Electrostatics and the Interaction of Two Point Charges on the Z-Axis: An In-Depth Analysis

In the realm of classical electromagnetism, understanding the behavior of point charges and their resulting electric fields forms the foundation of many scientific and engineering applications. When two point charges are positioned along a common axis—particularly the Z-axis—analysts can leverage symmetry and mathematical principles to derive meaningful insights into their electrostatic interactions. Specifically, the configuration involving charges \(Q \) and \(-q \), located at positions \(Z = +a \) and \(Z = -a \) respectively, serves as an instructive example for exploring electric potential, field distributions, and the resulting forces.

This article offers a comprehensive, detailed examination of this system, beginning with fundamental principles, progressing through mathematical derivations, and concluding with physical interpretations and implications. Our approach emphasizes clarity, analytical rigor, and contextual understanding, suitable for students, educators, and professionals seeking an in-depth review of this classical electrostatic problem.

Understanding the Setup: The Two Charges on the Z-Axis

Positioning of the Charges

The system features two point charges aligned along the Z-axis in a three-dimensional coordinate space:

- Charge Q located at $Z = +a$
- Charge $-q$ located at $Z = -a$

Here, a is a positive real number representing the distance from the origin to each charge. The charges are fixed in space, and the entire analysis hinges on the electrostatic principles governing their interactions.

This configuration is symmetric about the origin due to the equal magnitude of distances but opposite signs of the charges. Such symmetry simplifies many calculations and provides insights into the nature of the electric field and potential distributions.

Significance and Applications

Understanding this particular arrangement is fundamental in multiple contexts:

- Dipole Formation: The pair of charges essentially form an electric dipole, especially as Q and q are of comparable magnitudes.
- Modeling Molecular Structures: Many molecules can be approximated as dipoles with charges separated by a fixed distance.
- Electrostatic Shielding and Capacitance: Insights from such configurations underpin the design of capacitors and shielding devices.
- Theoretical Foundations: It serves as a textbook example for applying Coulomb's law, superposition principle, and potential theory.

Mathematical Foundations of Electrostatics

Coulomb's Law and Electric Potential

The cornerstone of electrostatics is Coulomb's law, which states that the electric force $\mathbf{F}$ between two point charges q_1 and q_2, separated by a distance r, is given by:

$$
\mathbf{F} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}
$$

where:

- ε_0 is the vacuum permittivity,
- $\hat{\mathbf{r}}$ is the unit vector from the source charge to the field point.

The electric potential V at a point P due to a point charge q located at $\mathbf{r}_q$ is:

$$
V(\mathbf{r}) = \frac{1}{4\pi \varepsilon_0} \frac{q}{|\mathbf{r} - \mathbf{r}_q|}
$$

The total potential from multiple point charges is the algebraic sum of individual potentials owing to superposition.

Calculating the Electric Potential at an Arbitrary Point

Coordinate System and Notation

To analyze the system, adopt a coordinate system where the charges are fixed on the Z-axis:

- Charge Q at $Z = +a$ (coordinate $(0, 0, +a)$)
- Charge $-q$ at $Z = -a$ (coordinate $(0, 0, -a)$)
- Observation point P at (x, y, z)

The distance from P to each charge is:

$$
r_{Q} = |\mathbf{r} - \mathbf{r}_Q| = \sqrt{x^2 + y^2 + (z - a)^2}
$$

$$
r_{-q} = |\mathbf{r} - \mathbf{r}_{-q}| = \sqrt{x^2 + y^2 + (z + a)^2}
$$

Expressing these in cylindrical coordinates (ρ, ϕ, z):

$$
\rho = \sqrt{x^2 + y^2}
$$

The symmetry about the Z-axis suggests that the potential depends only on ρ and z, simplifying the analysis.

Expression for the Total Electric Potential

Applying superposition, the potential at P due to both charges is:

$$
V(\rho, z) = \frac{1}{4\pi \varepsilon_0} \left(\frac{Q}{r_Q} - \frac{q}{r_{-q}} \right)
$$

where

$$
r_Q = \sqrt{\rho^2 + (z - a)^2}
$$

$$
r_{-q} = \sqrt{\rho^2 + (z + a)^2}
$$

This expression encapsulates the combined electrostatic influence of the two charges at any point in space.

Electric Field Derived from the Potential

Gradient of the Potential and Field Components

The electric field $\mathbf{E}$ is related to the potential V via:

$$
\mathbf{E} = - \nabla V
$$

In cylindrical coordinates, the components are:

$$
E_\rho = - \frac{\partial V}{\partial \rho}
$$

$$
E_z = - \frac{\partial V}{\partial z}
$$

Given the potential expression, these derivatives involve applying the chain rule:

$$
\frac{\partial V}{\partial \rho} = \frac{1}{4\pi \varepsilon_0} \left(-Q \frac{\partial}{\partial \rho} \frac{1}{r_Q} + q \frac{\partial}{\partial \rho} \frac{1}{r_{-q}} \right)
$$

$$
\frac{\partial V}{\partial z} = \frac{1}{4\pi \varepsilon_0} \left(-Q \frac{\partial}{\partial z} \frac{1}{r_Q} + q \frac{\partial}{\partial z} \frac{1}{r_{-q}} \right)
$$

Calculating these derivatives yields explicit expressions for the electric field components at any point.

Physical Implications of the Field Distribution

The resulting field pattern exhibits features characteristic of dipole fields, especially at points far from the charges, where the potential resembles that of a dipole:

$$
V \sim \frac{1}{4\pi \varepsilon_0} \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{r^2}
$$

where $\mathbf{p} = Q \mathbf{d}$ is the dipole moment, with $\mathbf{d}$ being the displacement vector from $-q$ to Q.

Electrostatic Force and Energy Considerations

Force on Each Charge

The force on a charge due to the other can be derived using Coulomb's law:

$$
\mathbf{F} = \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} \hat{\mathbf{r}}
$$

- The force on Q due to $-q$:

$$
\mathbf{F}_Q = - \frac{1}{4\pi \varepsilon_0} \frac{Q q}{(2a)^2} \hat{\mathbf{z}}
$$

- The force on $-q$ due to Q:

$$
\mathbf{F}_{-q} = \frac{1}{4\pi \varepsilon_0} \frac{Q q}{(2a)^2} \hat{\mathbf{z}}
$$

The forces are equal in magnitude and opposite in direction, consistent with Newton's third law.

Note: These forces are attractive if Q and $-q$ have opposite signs, which is the case here, leading to a mutual attraction along the Z-axis.

Potential Energy of the System

The electrostatic potential energy U stored in the system is:

$$
U = \frac{1}{4\pi \varepsilon_0} \frac{Q \times (-q)}{2a} = - \frac{1}{4\pi \varepsilon_0} \frac{Q q}{2a}
$$

The negative sign indicates that the system is in a bound state with lower potential energy when the charges

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