Two Positive Point Charges Q Are Placed On The X-axis, One At X = A And One At X = -a.a. Derive An Expression

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Introduction

The study of electrostatics involves understanding the behavior of electric charges at rest and the fields they produce. When multiple point charges are positioned in space, analyzing the resultant electric field and potential becomes essential, especially in solving problems related to force interactions and potential energy. One common configuration involves placing two identical positive point charges along a line, such as the x-axis, and determining the electric potential or field at any arbitrary point due to these charges.

This article aims to derive a comprehensive expression for the electric potential at a point in the vicinity of two positive point charges placed on the x-axis at known positions \(x = A \) and \(x = -a \). We will systematically develop the expression, explore its physical implications, and discuss its applications.

Understanding the Physical Setup

Coordinates and Positions of Charges

- Charge 1 (Q): Located at position $x = A$ on the x-axis.
- Charge 2 (Q): Located at position $x = -a$ on the x-axis.

Both charges are positive and identical in magnitude for simplicity, but the derivation can be extended to different magnitudes as necessary.

Objective

- To derive an expression for the electric potential V at a general point P with coordinate x on the x-axis due to these two charges.

Assumptions

- The charges are point charges, so their potentials are singular at their positions.
- The space is free of other charges or influences.
- The electric potential is scalar, additive, and obeys superposition principle.

Basic Principles of Electric Potential

Electric Potential of a Point Charge

The potential V at a point P due to a single point charge Q located at x_0 is given by:

$$
V = \frac{kQ}{r}
$$

where:

- \(k = \frac{1}{4\pi \varepsilon_0} \) (Coulomb’s constant),

- \(r \) is the distance between the charge and the point \(P \).

Superposition Principle

The total potential at \(P \) due to multiple charges is the algebraic sum of the potentials due to each charge:

\[
V_{total} = V_1 + V_2 + \dots
\]

Derivation of the Electric Potential Expression

Step 1: Coordinates of the Point \(P \)

Let point \(P \) be located at coordinate \(x \) on the x-axis.

Step 2: Distances from \(P \) to Each Charge

- Distance from \(P \) to charge at \(x = A \):

\[
r_1 = |x - A|
\]

- Distance from \(P \) to charge at \(x = -a \):

\[

r_2 = |x + a|

\]

Step 3: Write the Potential at \(P \)

Since both charges are positive and equal to \(Q \), the potential at \(P \) is:

\[

V(x) = \frac{kQ}{|x - A|} + \frac{kQ}{|x + a|}

\]

This is the general expression valid for any position \(x \) on the x-axis.

Step 4: Simplify the Expression

In many cases, the absolute value can be addressed based on the relative positions of \(x \), \(A \), and \(a \). For the purpose of a general expression, the potential is:

\[

\boxed{

V(x) = kQ \left(\frac{1}{|x - A|} + \frac{1}{|x + a|} \right)

}

\]

Analyzing Special Cases and Applications

Case 1: Point \(P \) Located Between the Charges

If \(-a < x < A \), then:

- $x - A < 0 \Rightarrow |x - A| = A - x$
- $x + a > 0 \Rightarrow |x + a| = x + a$

Thus, the potential becomes:

$$
V(x) = kQ \left(\frac{1}{A - x} + \frac{1}{x + a} \right)
$$

Case 2: Point P Located Outside the Charges

- To the right of both charges ($x > A$):

$$
V(x) = kQ \left(\frac{1}{x - A} + \frac{1}{x + a} \right)
$$

- To the left of both charges ($x < -a$):

$$
V(x) = kQ \left(\frac{1}{A - x} + \frac{1}{-a - x} \right)
$$

Applications in Physics and Engineering

- Electrostatic Force Calculations: Understanding potential distribution helps in calculating forces on test charges.
- Capacitance and Energy Storage: Potential expressions are foundational for capacitor design involving multiple charges.
- Electrostatic Shielding: Knowledge of potential fields guides shielding strategies in electronic devices.
- Field Visualization: Deriving potential functions aids in plotting equipotential surfaces and electric field

lines.

Extending the Derivation: Potential at a Point Off the X-Axis

While the above derivation assumes a point along the x-axis, the potential at a point P located at (x, y, z) involves the 3D distance:

$$
r_i = \sqrt{(x - x_i)^2 + y^2 + z^2}
$$

and the potential becomes:

$$
V(\mathbf{r}) = \sum_{i} \frac{k Q_i}{r_i}
$$

This generalizes the problem to three dimensions, essential for real-world applications involving spatial charge distributions.

Summary and Key Takeaways

- The potential at any point along the x-axis due to two positive point charges located at $x = A$ and $x = -a$ is:

$$
\boxed{

$$V(x) = kQ \left(\frac{1}{|x - A|} + \frac{1}{|x + a|} \right)$$

}

\]

- The expression simplifies based on the position of P relative to the charges.
- Superposition principle applies, allowing the addition of potentials from multiple charges.
- Understanding this potential distribution aids in solving complex electrostatics problems involving multiple charges.

Final Remarks

The analysis of two positive point charges on the x-axis provides foundational insights into electrostatic interactions and potential theory. The derived expression is critical for solving problems involving electric potential, field calculations, and energy considerations in various scientific and engineering contexts. Mastery of these derivations enhances problem-solving skills and deepens understanding of electrostatic phenomena.

References

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Frequently Asked Questions

What is the setup for deriving the electric potential due to two positive point charges placed on the x-axis at positions x = a and x = -a?

The setup involves two positive point charges, each of magnitude Q, positioned along the x-axis at points $x = a$ and $x = -a$. To derive the electric potential at a point P on the x-axis, we consider the distances from P to each charge and sum their individual potentials, since potential is scalar.

How do you express the electric potential at a point P on the x-axis due to two charges Q at x = a and x = -a?

The electric potential V at point P (located at x) is given by $V = (kQ / |x - a|) + (kQ / |x + a|)$, where k is Coulomb’s constant. Each term represents the potential due to one charge at the point P.

What assumptions are made while deriving the potential expression for two positive point charges on the x-axis?

The assumptions include that the charges are point charges, the medium is vacuum or air (permittivity ε_0), electrostatic conditions hold (no changing magnetic fields), and the superposition principle applies, allowing the potentials to be summed algebraically.

Can you derive the general expression for the potential at any point on the x-axis due to two charges at ±a?

Yes. The potential at a point x on the x-axis is $V(x) = (kQ / |x - a|) + (kQ / |x + a|)$. This expression accounts for the contributions from both charges at their respective distances from x.

How does the potential vary along the x-axis for two positive charges placed symmetrically at ±a?

Along the x-axis, the potential is symmetric with respect to the origin. It has maximum values near the charges and decreases with distance, approaching zero at points far away from both charges. The potential is positive everywhere due to both charges being positive.

What is the significance of deriving the expression for potential due to two charges on the x-axis in electrostatics?

Deriving this expression helps understand the electric field distribution, potential energy, and behavior of test charges in the vicinity of multiple charges. It's fundamental for analyzing electrostatic configurations and potential interactions in systems with multiple point charges.

How would the potential expression change if the charges were of opposite signs?

If the charges are of opposite signs, the potential at a point x becomes $V(x) = (kQ / |x - a|) - (kQ / |x + a|)$, with the second term negative, leading to potential cancellation and different field behaviors, including potential points where the total potential is zero.

What is the importance of considering the superposition principle in deriving the potential for multiple charges?

The superposition principle allows the total potential at a point to be calculated as the algebraic sum of potentials due to individual charges. This simplifies the analysis of complex charge configurations and is fundamental in electrostatics for accurately determining the resultant potential.

Additional Resources

Two Positive Point Charges Q Are Placed On The X-axis, One At X = A And One At X = -a. Derive An Expression

When analyzing two positive point charges Q are placed on the x-axis, one at X = A and the other at X = -a, a fundamental problem in electrostatics emerges: determining the electric potential and electric field at any arbitrary point in space due to these two charges. This scenario not only encapsulates core principles of Coulomb's law but also offers insight into superposition, symmetry, and the behavior of electric fields generated by discrete charge distributions. In this guide, we will explore the derivation of the general expression for the potential and electric field at any point in the xy-plane caused by these two charges positioned along the x-axis.

Understanding the Physical Setup

Coordinates and Placement of Charges

- Charge 1 (Q): Located at $X = A$ on the x-axis.
- Charge 2 (Q): Located at $X = -a$ on the x-axis.

Both charges are positive and identical in magnitude. Their positions are symmetric with respect to the origin, which simplifies the analysis due to symmetry.

Why This Configuration Matters

- The symmetry about the origin provides an elegant way to analyze the electric potential and field.
- It serves as a foundational model for understanding more complex charge distributions.
- The derivation can be extended to other configurations, such as different magnitudes or positions.

Fundamental Principles

Coulomb's Law

The electric potential V at a point $P(x,y)$ due to a point charge Q located at (x_0, y_0) is:

$$
V = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{r}
$$

where r is the distance between the charge and the point P:

$$
r = \sqrt{(x - x_0)^2 + (y - y_0)^2}
$$

Superposition Principle

The total potential at a point P due to multiple point charges is the algebraic sum of potentials due to each charge:

$$
V_{total} = V_1 + V_2 + \dots
$$

Similarly, the electric field $\vec{E}$ is a vector sum of fields due to individual charges.

Deriving the Electric Potential at a Point $P(x,y)$

Step 1: Express the distances from each charge to point P

- Distance from Q at $(A, 0)$:

$$
r_1 = \sqrt{(x - A)^2 + y^2}
$$

- Distance from Q at $(-a, 0)$:

$$
r_2 = \sqrt{(x + a)^2 + y^2}
$$

Step 2: Write the potential contributions

- Potential due to charge at $(A, 0)$:

$$
V_1 = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{r_1}
$$

- Potential due to charge at $(-a, 0)$:

$$
V_2 = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{r_2}
$$

Step 3: Total potential at $P(x,y)$

\[

$$
V(x,y) = V_1 + V_2 = \frac{Q}{4\pi \varepsilon_0} \left(\frac{1}{r_1} + \frac{1}{r_2} \right)
$$

This expression provides the general potential at any point (x,y).

Deriving the Electric Field at $P(x,y)$

The electric field is a vector quantity, obtained by superposing the fields due to each charge.

Step 1: Electric field due to each charge

- Electric field at P due to charge at $(A, 0)$:

$$
\vec{E}_1 = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{r_1^3} \cdot \vec{r}_1
$$

where $\vec{r}_1 = (x - A) \hat{i} + y \hat{j}$.

- Electric field at P due to charge at $(-a, 0)$:

$$
\vec{E}_2 = \frac{1}{4\pi \varepsilon_0} \cdot \frac{Q}{r_2^3} \cdot \vec{r}_2
$$

where $\vec{r}_2 = (x + a) \hat{i} + y \hat{j}$.

Step 2: Total electric field

$$
\vec{E} = \vec{E}_1 + \vec{E}_2
$$

which can be explicitly written as:

$$
\vec{E}(x, y) = \frac{Q}{4 \pi \varepsilon_0} \left(\frac{\vec{r}_1}{r_1^3} + \frac{\vec{r}_2}{r_2^3} \right)
$$

Expressed in components:

$$
E_x = \frac{Q}{4 \pi \varepsilon_0} \left(\frac{x - A}{r_1^3} + \frac{x + a}{r_2^3} \right)
$$

$$
E_y = \frac{Q}{4 \pi \varepsilon_0} \left(\frac{y}{r_1^3} + \frac{y}{r_2^3} \right)
$$

Special Cases and Symmetry

1. On the x-axis ($y=0$)

- Potential simplifies to:

$$
V(x, 0) = \frac{Q}{4\pi \varepsilon_0} \left(\frac{1}{|x - A|} + \frac{1}{|x + a|} \right)
$$

- Electric field components:

$$
E_x = \frac{Q}{4\pi \varepsilon_0} \left(\frac{x - A}{|x - A|^3} + \frac{x + a}{|x + a|^3} \right)
$$

$$
E_y = 0
$$

2. At the midpoint between charges ($x=0$, $y=0$)

- Symmetry causes the x-components of electric field to cancel if $A = a$, simplifying the analysis.

Practical Applications and Implications

- Electrostatic potential maps: Visualizing the potential distribution around two charges.
- Electric field lines: Understanding how the field emanates and interacts between charges.
- Force calculations: Using the derived fields to compute forces on other charges placed within the field.
- Capacitance and energy storage: Analyzing configurations of multiple charges.

Extending the Model

While this analysis considers identical positive charges, the framework can be generalized to:

- Charges of different magnitudes, Q_1 and Q_2.

- Different positions, not necessarily symmetric.
- Charges placed in three-dimensional space.

The core principles—superposition, Coulomb's law, and coordinate geometry—remain the same, with necessary modifications to the expressions.

Summary

- The electric potential at any point (x, y) due to two positive point charges placed at $(A, 0)$ and $(-a, 0)$ is:

$$
V(x, y) = \frac{Q}{4\pi \varepsilon_0} \left(\frac{1}{\sqrt{(x - A)^2 + y^2}} + \frac{1}{\sqrt{(x + a)^2 + y^2}} \right)
$$

- The electric field components are:

$$
E_x = \frac{Q}{4 \pi \varepsilon_0} \left(\frac{x - A}{r_1^3} + \frac{x + a}{r_2^3} \right)
$$

$$
E_y = \frac{Q}{4 \pi \varepsilon_0} \left(\frac{y}{r_1^3} + \frac{y}{r_2^3} \right)
$$

- These expressions encapsulate the superposition principle and symmetry considerations, providing a comprehensive mathematical description of the electrostatic environment generated by the two charges.

Final Remarks

Understanding the behavior of electric potentials and fields due to multiple point charges on the x-axis is crucial in electrostatics. It bridges fundamental concepts with real-world applications, such as designing capacitors, understanding charge distributions in molecules, and analyzing electric field patterns. The derivations outlined here serve as a foundation for deeper explorations into electrostatics, enabling students and professionals alike to model complex systems with confidence and precision.

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