Verify That The Following Function Is A Probability Mass Function, And Determine The Requested Probabilities.

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When working with discrete random variables in probability theory, one of the fundamental concepts is the probability mass function (PMF). A PMF assigns probabilities to each possible value that a discrete random variable can take. Ensuring that a given function qualifies as a PMF is crucial for the validity of probabilistic models and for calculating meaningful probabilities. This article provides a comprehensive guide to verifying whether a function is a PMF and demonstrates how to determine specific probabilities based on that function.

Understanding the Probability Mass Function (PMF)

Definition of a Probability Mass Function

A probability mass function, denoted as $p(x)$, is a function that assigns probabilities to each possible value x of a discrete random variable X. For $p(x)$ to be a valid PMF, it must satisfy two key properties:

- **Non-negativity:** $p(x) \geq 0$ for all x.
- **Normalization:** The sum of probabilities over all possible values must equal 1:

$$
\sum_{x} p(x) = 1
$$

Importance of Verifying a PMF

Validating whether a function is a PMF is essential because:

- It ensures the probabilities are meaningful and consistent.
- It allows accurate calculation of the probabilities of events.
- It forms the basis for further statistical analysis, such as calculating expectations, variances, and probabilities of compound events.

Steps to Verify That a Function Is a Probability Mass Function

Step 1: Identify the Domain of the Function

Begin by clearly defining the set of all possible values x that the random variable X can take. This set, known as the support, must be finite or countably infinite.

Step 2: Check Non-negativity

Verify that the function $p(x)$ assigns non-negative probabilities to every value in the domain:

$$
\forall x,\quad p(x) \geq 0
$$

If any $p(x)$ is negative, the function cannot be a valid PMF.

Step 3: Verify Normalization Condition

Calculate the sum of the probabilities over all possible values:

$$
\sum_{x} p(x)
$$

If this sum equals 1, the normalization condition is satisfied. If not, the function is invalid as a PMF.

Step 4: Confirm Completeness of the Domain

Ensure that the domain includes all values where $p(x)$ is non-zero. Omitting any such values could lead to incorrect probability calculations.

Example: Verifying a Given Function as a PMF

Suppose we are given the following function $p(x)$:

x	1	2	3	4

$p(x)$	0.2	0.3	0.4	0.1

Let's verify whether this function is a valid PMF.

Step 1: Domain Identification

The domain is the set $\{1, 2, 3, 4\}$.

Step 2: Check Non-negativity

All $p(x)$ are non-negative:

$$
0.2, 0.3, 0.4, 0.1 \geq 0
$$

Confirmed.

Step 3: Check Normalization

Calculate the sum:

$$
0.2 + 0.3 + 0.4 + 0.1 = 1.0
$$

Sum equals 1, so normalization holds.

Step 4: Confirm Domain Completeness

All possible values with non-zero probabilities are included; the domain is complete.

Conclusion: Yes, $p(x)$ is a valid probability mass function.

Calculating Probabilities From a Valid PMF

Once the function is verified as a PMF, you can compute the probability of various events. Here are some common scenarios:

1. Probability of a Single Value

To find the probability that X equals a specific value x_0:

$$
P(X = x_0) = p(x_0)
$$

Using the previous example, $P(X=2) = 0.3$.

2. Probability of an Event (Set of Values)

For an event $A = \{ x_1, x_2, \ldots, x_k \}$, the probability is:

$$
P(A) = \sum_{x \in A} p(x)
$$

For example, the probability that X is either 2 or 3:

$$
P(X=2 \text{ or } 3) = p(2) + p(3) = 0.3 + 0.4 = 0.7
$$

3. Probability of Ranges or Intervals

If the domain is ordered (like integers), the probability that X falls within an interval:

$$
P(a \leq X \leq b) = \sum_{x=a}^{b} p(x)
$$

For example, probability $P(2 \leq X \leq 3)$:

$$
p(2) + p(3) = 0.3 + 0.4 = 0.7
$$

Determining Requested Probabilities

Suppose you are asked to find specific probabilities based on a given PMF. The process involves:

- Identifying the event of interest.
- Summing the probabilities of all values within that event.

- Ensuring the event is correctly specified to avoid errors.

Example: Calculating Probabilities in a Real-World Scenario

Imagine a quality control process where the number of defective items in a batch follows the PMF:

Number of Defective Items x	Probability $p(x)$
0	0.5
1	0.3
2	0.15
3	0.05

Questions:

a) What is the probability that there are no defective items?

b) What is the probability that there are at most one defective item?

c) What is the probability that there are more than one defective item?

Solutions:

a) $P(X=0) = 0.5$

b) $P(X \leq 1) = p(0) + p(1) = 0.5 + 0.3 = 0.8$

c) $P(X > 1) = p(2) + p(3) = 0.15 + 0.05 = 0.2$

Common Mistakes to Avoid When Verifying a PMF

Ensuring the validity of a PMF requires attention to detail. Here are some pitfalls to watch out for:

- **Ignoring negative probabilities:** Any negative $p(x)$ disqualifies the function.
- **Not summing to 1:** Even if all probabilities are non-negative, their sum must be exactly 1.
- **Misidentifying the domain:** Omitting possible values or including invalid values can lead to incorrect calculations.
- **Assuming the function is a PMF without verification:** Always perform the checks systematically.

Extensions and Related Concepts

While verifying a PMF is straightforward for discrete variables, understanding related concepts enhances your statistical toolkit:

Probability Generating Functions (PGF)

A PGF summarizes the distribution of a discrete random variable and can be used to compute moments and probabilities efficiently.

Expected Value and Variance

Once verified, you can compute the expectation:

$$
E[X] = \sum_{x} x \cdot p(x)
$$

and variance:

$$
Var(X) = E[X^2] - (E[X])^2
$$

Cumulative Distribution Function (CDF)

The CDF, $F(x)$, is the probability that X is less than or equal to x:

$$
F(x) = P(X \leq x) = \sum_{t \leq x} p(t)
$$

It provides a complete picture of the distribution.

Conclusion

Verifying that a function is a probability mass function is a fundamental skill in probability and statistics. It involves checking non-negativity, normalization, and domain completeness. Once validated, the PMF becomes a powerful tool for calculating the probabilities of various events, understanding the distribution's characteristics, and performing further statistical

Frequently Asked Questions

How do I verify if a given function is a valid probability mass function (pmf)?

To verify if a function is a valid pmf, ensure that all its values are non-negative and that the sum of all probabilities equals 1.

What steps should I take to determine the probability of a specific event using a pmf?

Identify the value corresponding to the event in the pmf and sum the probabilities if the event is a collection of outcomes; for a single outcome, simply use its pmf value.

If a function's total sum exceeds 1, what does that indicate about its status as a pmf?

It indicates that the function is not a valid pmf because the total probability must sum exactly to 1.

How can I compute the probability that a random variable is less than a certain value using a pmf?

Sum the pmf values for all outcomes less than the specified value to find the probability.

What is the significance of confirming that all probabilities in a pmf are between 0 and 1?

It ensures that probabilities are valid, as they cannot be negative or exceed 1, which maintains the integrity of the probability model.

How do I determine the probability that the variable falls within a range, say between a and b, using a pmf?

Sum the pmf values for all outcomes x where $a \le x \le b$ to find the probability of the variable falling within that range.

What should I do if the sum of probabilities in a pmf is less than 1?

The function is invalid as a pmf; you should check for missing probabilities or errors in the calculation, ensuring the total sums to exactly 1.

Additional Resources

Probability Mass Function (PMF): Verification and Application — An Expert Analysis

Understanding the behavior of discrete random variables is foundational in probability theory and statistical modeling. Among the core concepts is the Probability Mass Function (PMF), which assigns probabilities to the possible outcomes of a discrete random variable. Ensuring that a candidate function qualifies as a valid PMF and leveraging it to compute specific probabilities are essential skills for statisticians, data scientists, and researchers alike. This comprehensive review aims to explore the criteria for verifying a PMF, demonstrate the process using an example function, and guide you through calculating targeted probabilities with confidence.

Understanding the Probability Mass Function (PMF)

Before delving into verification techniques and calculations, it’s crucial to establish a clear conceptual foundation.

Definition and Properties

A Probability Mass Function is a function $p(x)$ that assigns probabilities to each possible value x of a discrete random variable X. Formally:

- Domain: The set of all possible outcomes x (often discrete and finite or countably infinite).
- Range: $p(x) \geq 0$ for all x.
- Total Probability: The sum over all possible outcomes must equal 1:

$$
\sum_{x} p(x) = 1
$$

These properties are fundamental; any function claiming to be a PMF must satisfy them.

Implications:
- No probability can be negative.
- The total probability across all outcomes is normalized to 1.
- Probabilities are assigned exclusively to outcomes in the support set.

Verifying a Candidate Function as a PMF

Suppose you are presented with a function $p(x)$, defined for various outcomes x. To verify whether $p(x)$ is a valid PMF, follow a systematic process.

Step 1: Confirm Non-negativity

- Check that $p(x) \geq 0$ for every outcome x in the domain.
- If any $p(x) < 0$, then $p(x)$ cannot be a PMF.

Step 2: Verify Total Sum Condition

- Sum the probabilities over all outcomes:

$$
\sum_{x} p(x)
$$

- For finite outcomes, sum explicitly.
- For infinite (countably infinite) outcomes, verify whether the series converges to 1.
- If the total sum equals 1, the function passes this criterion; if not, it is invalid.

Step 3: Confirm the Domain and Support

- Ensure the domain of $p(x)$ matches the set of outcomes where $p(x) > 0$.
- For some functions, the domain might be implicitly defined; clarify this before proceeding.

Example Verification Process

Suppose the function:

$$
p(x) = \frac{c}{x^2} \quad \text{for} \quad x=1,2,3,\dots
$$

and zero elsewhere. To verify:

- Check non-negativity: Since $c > 0$, and $x^2 > 0$, $p(x) \geq 0$.
- Find c such that the total sum equals 1:

$$
\sum_{x=1}^\infty p(x) = c \sum_{x=1}^\infty \frac{1}{x^2}
$$

- Knowing $\sum_{x=1}^\infty \frac{1}{x^2} = \frac{\pi^2}{6}$, set:

$$
c \cdot \frac{\pi^2}{6} = 1 \Rightarrow c = \frac{6}{\pi^2}
$$

- Since $c > 0$ and the sum equals 1, $p(x)$ is a valid PMF.

Applying the Verification to a Specific Function

Let's consider a concrete example and verify whether it qualifies as a PMF.

Given Function:

$$
p(x) = \frac{1}{6} \quad \text{for} \quad x=1,2,3,4,5,6
$$

and zero elsewhere.

Verification Process:

- Non-negativity: Each $p(x) = \frac{1}{6} \geq 0$. □
- Sum over all outcomes:

$$
\sum_{x=1}^6 p(x) = 6 \times \frac{1}{6} = 1
$$

- Domain: $\{1, 2, 3, 4, 5, 6\}$, consistent with a standard die.

Conclusion: Since both criteria are met, $p(x)$ is a valid PMF.

Calculating Probabilities from a Valid PMF

Once a function is confirmed as a PMF, it becomes a powerful tool for probability calculations.

Basic Probability Calculations

- Single outcomes: $P(X = x) = p(x)$
- Range of outcomes: For a subset $A \subseteq \text{support}$,

\[

$$
P(X \in A) = \sum_{x \in A} p(x)
$$

- Complement rule:

$$
P(X \notin A) = 1 - P(X \in A)
$$

Example: Calculating Specific Probabilities

Using the previous die example:

- Probability of rolling a 4:

$$
P(X=4) = p(4) = \frac{1}{6}
$$

- Probability of rolling an even number:

$$
A = \{2, 4, 6\}
$$

$$
P(X \in A) = p(2) + p(4) + p(6) = 3 \times \frac{1}{6} = \frac{1}{2}
$$

- Probability of rolling a number less than 3:

$$
A = \{1, 2\}
$$

$$
P(X \in A) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}
$$

Determining Requested Probabilities for a Custom Function

Now, consider a more complex, custom-distribution scenario. Suppose you are given a function:

$$
p(x) = \frac{2^{x}}{2^{n+1} - 2} \quad \text{for} \quad x=1, 2, \dots, n
$$

\]

where n is a positive integer. Your goal is to verify its legitimacy as a PMF and compute specific probabilities.

Verification Process

- Step 1: Non-negativity

Since $2^{x} > 0$ for all x, and the denominator $2^{n+1} - 2 > 0$ for $n \geq 1$, the entire fraction is positive. ✓

- Step 2: Sum over support

$$
\sum_{x=1}^n p(x) = \sum_{x=1}^n \frac{2^{x}}{2^{n+1} - 2}
$$

Factor numerator sum:

$$
\sum_{x=1}^n 2^{x} = 2^{n+1} - 2
$$

This identity results from the geometric series sum:

$$
\sum_{x=1}^n 2^{x} = 2 \frac{2^{n} - 1}{2 - 1} = 2 (2^{n} - 1) = 2^{n+1} - 2
$$

Therefore,

$$
\sum_{x=1}^n p(x) = \frac{2^{n+1} - 2}{2^{n+1} - 2} = 1
$$

- Conclusion: The sum of probabilities equals 1, and all probabilities are non-negative. Hence, $p(x)$ is a valid PMF.

Calculating Specific Probabilities in the Custom Distribution

Suppose $n=4$, and you want to find:

- The probability that \(X \leq 2 \):

\[
P(X \leq 2) = p(1) + p(2)
\]
\[
p(1) = \frac{2^{1}}{2^{5} - 2} = \frac{2}{32 - 2} = \frac{2}{30} = \frac{1}{15}
\]
\[
p(2) = \frac{2^{2}}{30} = \frac{4}{30} = \frac{2}{15}
\]
\[
P(X \leq 2) = \frac{1}{15}

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