Vrite A Slope -intercept Equation For A Line Passing Through The Point (2,7) That Is Parallel To Y=(2)/(5)x+5.

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Understanding the Slope-Intercept Form of a Line

When working with linear equations in algebra, the slope-intercept form is one of the most fundamental representations. It is written as:

$$ y = mx + b $$

where:
- m represents the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

This form makes it easy to graph lines and understand their behavior, especially when given specific points or slopes.

Determining the Slope of the Given Line

The problem states that we need to find the equation of a line passing through the point (2, 7) that is parallel to the line:

$$ y = \frac{2}{5}x + 5 $$

Since the lines are parallel, they share the same slope. Therefore, the slope m of our new line will be:

$$ m = \frac{2}{5} $$

Understanding this is crucial because parallel lines never intersect and always have identical slopes.

Formulating the Equation of the Parallel Line

Given:
- Point: (2, 7)
- Slope: $m = \frac{2}{5}$

Our goal is to find the line's equation in slope-intercept form, $y = mx + b$.

Step 1: Use the Point-Slope Form

The point-slope form of a line is:

$$ y - y_1 = m(x - x_1) $$

Plugging in the known point and slope:

$$ y - 7 = \frac{2}{5} (x - 2) $$

Step 2: Simplify to Slope-Intercept Form

Distribute the slope:

$$ y - 7 = \frac{2}{5}x - \frac{4}{5} $$

Add 7 to both sides to solve for y:

$$ y = \frac{2}{5}x - \frac{4}{5} + 7 $$

Express 7 as a fraction with denominator 5:

$$ y = \frac{2}{5}x - \frac{4}{5} + \frac{35}{5} $$

Combine the constants:

$$ y = \frac{2}{5}x + \frac{31}{5} $$

Final Equation of the Line

The slope-intercept form of the line passing through (2, 7) and parallel to $y = \frac{2}{5}x + 5$ is:

$$ y = \frac{2}{5}x + \frac{31}{5} $$

This equation can be used for graphing, solving related problems, or understanding the geometric relationship between lines.

Key Concepts and Tips

Understanding Parallel Lines

- Parallel lines have identical slopes.
- They never intersect, no matter how far extended.
- The only difference is their y-intercepts, which determine their vertical positions.

Finding a Line Equation Given a Point and Slope

- Use the point-slope form: $y - y_1 = m(x - x_1)$.
- Substitute the known point and slope.
- Simplify to get the slope-intercept form.

Working with Fractions in Equations

- When constants involve fractions, simplifying and combining like terms is essential.
- Convert mixed fractions or whole numbers to fractions with common denominators for easier calculation.

Applications of the Slope-Intercept Equation

Understanding how to derive the equation of a line passing through a specific point and parallel to another line has many practical applications, including:

- Graphing Lines Quickly: Using the slope-intercept form allows for rapid graphing.
- Predicting Values: Given an x-value, you can easily find the corresponding y-value.
- Solving Systems of Equations: Parallel lines help identify regions of no intersection.
- Real-world Modeling: Lines can model trends such as speed, growth, or decline in various fields.

Additional Examples and Practice Problems

Example 1: Find the equation of a line passing through (4, -3) and parallel to $y = -\frac{3}{4}x + 2$.

Solution:
- Parallel slope: $m = -\frac{3}{4}$
- Point: (4, -3)

Using point-slope form:

$$y - (-3) = -\frac{3}{4}(x - 4)$$

Simplify:

$$ y + 3 = -\frac{3}{4}x + 3 $$

Subtract 3 from both sides:

$$ y = -\frac{3}{4}x $$

Result: The equation is $y = -\frac{3}{4}x$.

Practice Problem: Find the equation of a line passing through (1, 4) and parallel to $y = \frac{5}{3}x - 7$.

Answer:
- Slope: $m = \frac{5}{3}$
- Equation:

$$ y - 4 = \frac{5}{3}(x - 1) $$

Simplify:

$$ y - 4 = \frac{5}{3}x - \frac{5}{3} $$

Add 4 (which is $\frac{12}{3}$) to both sides:

$$ y = \frac{5}{3}x - \frac{5}{3} + \frac{12}{3} = \frac{5}{3}x + \frac{7}{3} $$

Final answer: $y = \frac{5}{3}x + \frac{7}{3}$.

Conclusion

Mastering the process of deriving a slope-intercept equation for lines passing through specific points and parallel to given lines is essential in algebra and geometry. By understanding the slope, applying the point-slope form, and simplifying expressions, you can quickly formulate the equations of lines under various conditions. This skill not only enhances your mathematical proficiency but also prepares you for more complex topics like systems of equations, analytic geometry, and real-world data modeling.

Remember, the key steps include identifying the slope for parallel lines, using known points, and simplifying to the slope-intercept form. Practicing with different points and slopes will reinforce your understanding and ability to solve diverse problems involving linear equations.

Frequently Asked Questions

What is the slope of the line passing through (2,7) that is parallel to y = (2/5)x + 5?

The slope of the given line is 2/5. Since the new line is parallel, it will also have a slope of 2/5.

How do I find the y-intercept of the line passing through (2,7) with slope 2/5?

Use the point-slope form $y - y_1 = m(x - x_1)$ with the point (2,7) and slope 2/5, then solve for y to find the y-intercept.

What is the equation of the line passing through (2,7), parallel to y = (2/5)x + 5?

The equation is y = (2/5)x + b, where b is the y-intercept. To find b, substitute x=2 and y=7: 7 = (2/5)2 + b, so b = 7 - (4/5) = 7 - 0.8 = 6.2. Therefore, the equation is y = (2/5)x + 6.2.

How do I convert the point-slope form to slope-intercept form for this problem?

First, write the point-slope form: y - 7 = (2/5)(x - 2). Then expand and simplify to get y = (2/5)x + 6.2.

Can you explain why the new line has the same slope as y = (2/5)x + 5?

Yes, because parallel lines have identical slopes, so the new line must also have a slope of 2/5.

What steps are involved in writing the slope-intercept form of a line passing through a point and parallel to a given line?

Identify the slope from the given line, then substitute the point's coordinates into y = mx + b to solve for b, resulting in the line's equation.

Is the y-intercept of the new line greater or less than 5?

It's greater; specifically, the y-intercept is 6.2, which is more than 5.

How do I verify that the line passing through (2,7) is parallel to y = (2/5)x + 5?

Check that both lines have the same slope. Since both have slope 2/5, they are parallel.

What is the significance of the point (2,7) in deriving the line's equation?

The point (2,7) provides a specific coordinate through which the line passes, allowing us to solve for the y-intercept once the slope is known.

Can the slope of the new line be different from 2/5?

No, because the line is parallel to y = (2/5)x + 5, which has slope 2/5. Parallel lines share the same slope.

Additional Resources

Vrite A Slope-Intercept Equation For A Line Passing Through The Point (2,7) That Is Parallel To y = (2/5)x + 5

Introduction

Understanding the equation of a line is fundamental in algebra and analytical geometry, serving as the backbone for numerous applications across science, engineering, and everyday problem-solving. One particularly interesting problem involves determining the specific equation of a line that not only passes through a given point but also maintains a specific relationship—parallelism—to another line. This article delves into the process of deriving the slope-intercept form of such a line, focusing on the point (2,7) and the line y = (2/5)x + 5, which it must be parallel to.

The Importance of Slope-Intercept Form in Line Equations

Before we proceed, it’s crucial to understand why the slope-intercept form, written as y = mx + b, is so significant. This form provides an immediate visual of a line’s behavior:

- m: the slope of the line, indicating its steepness and direction (positive, negative, zero, or undefined).
- b: the y-intercept, where the line crosses the y-axis.

This form simplifies the process of graphing, analyzing, and manipulating linear equations. When given certain conditions—such as a point through which the line passes and a parallelism constraint—the slope-intercept form offers a straightforward path to the solution.

Understanding the Given Line and Its Characteristics

The line provided, y = (2/5)x + 5, is in slope-intercept form. Analyzing its components:

- Slope (m_1): 2/5

- Y-intercept (b_1): 5

This line has a positive slope, meaning it rises as it moves from left to right. Its steepness is relatively gentle due to the fractional slope.

Given that the new line must be parallel to this line, it is essential to understand the properties of parallel lines:

- Parallel lines have equal slopes.
- They never intersect, maintaining a constant distance apart.

Thus, the slope of the new line, which passes through the point (2,7), will be the same as the given line, $m_2 = 2/5$.

Deriving the Equation of the Parallel Line

Key Concept: Since the new line is parallel to $y = (2/5)x + 5$, it shares the same slope:

$$
m = \frac{2}{5}
$$

The goal is to find the y-intercept (b) of this line, given that it passes through the point (2, 7).

Step-by-Step Process

1. Write the general form of the line

Using the slope-intercept form:

$$
y = mx + b
$$

where:

- $m = \frac{2}{5}$
- $(x, y) = (2, 7)$

2. Substitute known values into the equation

Plugging the point (2, 7) into the equation:

$$
7 = \frac{2}{5} \times 2 + b
$$

Calculating:

$$
7 = \frac{4}{5} + b
$$

3. Solve for b:

Subtract $\frac{4}{5}$ from both sides:

$$
b = 7 - \frac{4}{5}
$$

Express 7 as a fraction with denominator 5:

$$
b = \frac{35}{5} - \frac{4}{5} = \frac{31}{5}
$$

Thus, the y-intercept:

$$
b = \frac{31}{5}
$$

Final Equation of the Line

Combining the slope and intercept, the equation of the line passing through (2,7) and parallel to y = (2/5)x + 5 is:

$$
\boxed{
y = \frac{2}{5}x + \frac{31}{5}
}
$$

This concise form encapsulates the line’s behavior, ensuring it passes through the specified point and maintains parallelism to the given line.

Analytical Insights and Implications

1. Geometric Interpretation

The derived line is visually parallel to the original, with both lines sharing the same slope of 2/5. The difference lies solely in their y-intercepts:

- Original line crosses the y-axis at (0, 5).
- New line crosses at (0, 31/5), approximately 6.2.

This shift indicates the new line is vertically displaced upwards, maintaining the same inclination.

2. Application Scenarios

Understanding how to derive such a line has practical implications:

- Engineering Design: Ensuring components or pathways run parallel at specified points.
- Navigation: Charting routes that maintain a certain bearing while passing through specific waypoints.
- Data Modeling: Fitting lines through data points with constraints on slopes and intercepts.

3. Mathematical Significance

The process underscores the importance of:

- Point-slope form: a flexible starting point for line equations.
- Parallelism constraints: translating geometric relationships into algebraic equations.
- Fractional slopes: handling non-integer slopes with care to avoid errors.

Extended Considerations

1. Beyond Parallel Lines: Perpendicularity and Other Conditions

While this article focuses on parallel lines, similar principles apply for perpendicular lines, where the slopes are negative reciprocals. For instance, if a line is perpendicular to $y = (2/5)x + 5$, its slope would be -5/2. Deriving the line passing through a point would involve similar steps but with this adjusted slope.

2. Impacts of Changing the Point or the Original Line

- If the point changes, the intercept calculation must be redone.
- If the original line has a different slope, the new line’s slope adapts accordingly.
- These adjustments demonstrate the flexibility and robustness of algebraic methods in geometric contexts.

Practical Tips for Students and Professionals

- Always verify the slope: When given a line and a point, confirm the slope before proceeding.
- Use fractions carefully: Keep fractions in their exact form during calculations to avoid rounding errors.
- Graphically verify: Plotting the lines can provide visual confirmation of correctness.
- Practice with varied scenarios: Derive lines passing through different points and with different constraints to solidify understanding.

Conclusion

The process of determining the slope-intercept equation of a line passing through a specific point and parallel to another is a foundational skill in algebra. By leveraging the properties of parallel lines and applying systematic algebraic methods, one can derive precise equations that model real-world problems with clarity and accuracy. The line passing through (2,7) and parallel to y = (2/5)x + 5 is given by:

\[
\boxed{
y = \frac{2}{5}x + \frac{31}{5}
}
\]

This exploration not only illuminates the mechanics behind the calculation but also emphasizes the interconnectedness of algebra, geometry, and practical applications, reinforcing the importance of mastering these concepts for academic success and professional proficiency.

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