. What Is The Expectation Value Of The Linear Momentum For The 1D Wavefunction: (x)=Ne Ax 2 , Where [infinity]

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Understanding the expectation value of linear momentum in quantum mechanics is essential for analyzing the behavior of particles within potential fields. When dealing with a specific wavefunction such as $\psi(x) = N e^{A x^2}$, where the domain extends to infinity, it's crucial to explore how to compute this expectation value accurately. This article provides an in-depth explanation of how to evaluate the expectation value of linear momentum for the given wavefunction, covering the necessary mathematical framework, normalization procedures, and physical interpretations.

Fundamentals of Expectation Values in Quantum Mechanics

Before diving into the specifics of the wavefunction $\psi(x) = N e^{A x^2}$, it's important to review the basic principles underlying expectation values in quantum mechanics.

Definition of Expectation Value

- The expectation value of an observable provides the average outcome of measurements performed on a large number of identical systems.

- Mathematically, for a given operator $\hat{O}$, the expectation value in state $\psi(x)$ is given by:

$$
\langle \hat{O} \rangle = \int_{-\infty}^{\infty} \psi^(x) \hat{O} \psi(x) \, dx
$$

Linear Momentum Operator

- The linear momentum operator in one dimension is expressed as:

\[

\hat{p} = -i \hbar \frac{\partial}{\partial x}
\]

- where \(\hbar\) is the reduced Planck's constant.

- The expectation value of momentum thus involves calculating the integral of \(\psi^(x)\) times the derivative of \(\psi(x)\).

Analyzing the Wavefunction \(\psi(x) = N e^{A x^2}\)

The specific form of the wavefunction significantly influences the calculation of the expectation value.

Characteristics of the Wavefunction

- \(\psi(x) = N e^{A x^2}\) is a Gaussian-like function, but note the sign of \(A\):

- For the wavefunction to be normalizable (i.e., have a finite integral over all space), the real part of \(A\) must be negative (\(\text{Re}(A) < 0\)).

- The normalization constant \(N\) ensures that:

\[
\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1
\]

Normalization of the Wavefunction

- Calculate \(N\) by solving:

\[
N^2 \int_{-\infty}^{\infty} e^{2A x^2} dx = 1
\]

- This integral is a standard Gaussian integral, provided \(\text{Re}(A) < 0\).

- The result is:

\[

$$
N = \left(\frac{2 |A|}{\pi} \right)^{1/4}
$$

(assuming $A = -|A|$ for simplicity).

Calculating the Expectation Value of Linear Momentum

With the wavefunction properly normalized, the next step is to compute the expectation value $\langle p \rangle$.

Mathematical Expression

- The expectation value of momentum is:

$$
\langle p \rangle = \int_{-\infty}^{\infty} \psi^(x) \left(-i \hbar \frac{\partial}{\partial x} \right) \psi(x) dx
$$

- Since $\psi(x)$ is real (as the exponential of a real function), $\psi^ = \psi$.

Derivative of the Wavefunction

- Calculate $\frac{\partial}{\partial x} \psi(x)$:

$$
\frac{\partial}{\partial x} \psi(x) = N \frac{\partial}{\partial x} e^{A x^2} = N \cdot 2A x e^{A x^2}
$$

- Thus, the integrand becomes:

$$
\psi(x) \left(-i \hbar \frac{\partial}{\partial x} \psi(x) \right) = N^2 e^{A x^2} \cdot (-i \hbar) \cdot 2A x e^{A x^2} = -2 i \hbar A x N^2 e^{2A x^2}
$$

Evaluating the Integral

- The expectation value simplifies to:

$$
\langle p \rangle = -2 i \hbar A N^2 \int_{-\infty}^{\infty} x e^{2A x^2} dx
$$

- Note that the integral of an odd function over symmetric limits is zero:

$$
\int_{-\infty}^{\infty} x e^{2A x^2} dx = 0 \quad \text{(since $x e^{2A x^2}$ is odd)}.
$$

- Therefore, the expectation value of momentum is:

$$
\boxed{
\langle p \rangle = 0
}
$$

Physical Interpretation of the Result

The calculation reveals that the expectation value of the linear momentum for the wavefunction $\psi(x) = N e^{A x^2}$ is zero, which has important physical implications.

Symmetry and Momentum Expectation

- The wavefunction is symmetric (or real and even) about the origin for real A, leading to a zero average momentum.

- In quantum mechanics, symmetric wavefunctions typically correspond to states with no net directional momentum.

Implications for Quantum Systems

- Such wavefunctions are often associated with stationary states or bound states in potential wells.

- The zero expectation value does not imply the particle is at rest but indicates the average momentum over many measurements is zero.

Additional Considerations and Extensions

While the expectation value of linear momentum is zero for this wavefunction, several other factors and related calculations are worth considering.

Uncertainty in Momentum

- Calculating $\langle p^2 \rangle$ provides insight into the spread or uncertainty in momentum.

- This involves integrating $\psi^(x) \hat{p}^2 \psi(x)$, where $\hat{p}^2 = - \hbar^2 \frac{\partial^2}{\partial x^2}$.

Impact of the Wavefunction's Parameters

- The value of A affects the localization and spread of the wavefunction.

- Adjusting A modifies the variance in position and momentum, illustrating the uncertainty principle.

Application in Quantum Harmonic Oscillator

- Gaussian wavefunctions similar to $\psi(x)$ are fundamental in the analysis of quantum harmonic oscillators.

- Understanding their expectation values helps in comprehending the ground and excited states.

Summary

In conclusion, for the wavefunction $\psi(x) = N e^{A x^2}$, where A has a negative real part ensuring normalizability, the expectation value of linear momentum $\langle p \rangle$ is zero. This result stems from the symmetry of the wavefunction, leading to an integrand that is an odd function over symmetric limits. The calculation process involves normalization, taking derivatives, and evaluating Gaussian integrals, all fundamental techniques in quantum mechanics.

This analysis not only provides insight into the mathematical procedures involved but also underscores the physical implications of wavefunction symmetry and its relation to measurable quantities like momentum. Understanding these principles is vital for students and researchers working in quantum physics, quantum chemistry, and related fields.

Keywords: expectation value, linear momentum, 1D wavefunction, Gaussian wavefunction, quantum mechanics, wavefunction normalization, momentum operator, quantum expectation, symmetry in wavefunctions, Gaussian integrals

Frequently Asked Questions

What is the general formula for calculating the expectation value of linear momentum in a 1D wavefunction?

The expectation value of linear momentum in 1D is given by $\langle p \rangle = -i\hbar \int \psi(x) (d/dx) \psi(x) dx$, where $\psi(x)$ is the wavefunction.

Given the wavefunction $\psi(x) = N e^{A x^2}$, how do we determine the normalization constant N?

To find N, we normalize the wavefunction by ensuring $\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1$, which leads to $N = (A/\pi)^{1/4}$ assuming $A > 0$.

What is the expectation value of the linear momentum for the wavefunction $\psi(x) = N e^{A x^2}$?

Since $\psi(x)$ is real and symmetric, its derivative is odd, leading to the expectation value $\langle p \rangle = 0$ because the integral of an odd function over symmetric limits vanishes.

Does the expectation value of momentum depend on the parameter A in the wavefunction $\psi(x) = N e^{A x^2}$?

No, because the wavefunction is real and symmetric, resulting in a zero expectation value of momentum regardless of A.

What physical insight can be drawn from the expectation value of momentum being zero for this wavefunction?

It indicates that the particle described by this wavefunction has no net momentum, consistent with a symmetric probability distribution centered at zero momentum.

Are there any conditions under which the expectation value of momentum could be non-zero for a similar wavefunction?

Yes, if the wavefunction is complex or asymmetric, or if it includes a phase factor e^{i k x}, the expectation value of momentum could be non-zero, reflecting a directional momentum component.

Additional Resources

What Is The Expectation Value Of The Linear Momentum For The 1D Wavefunction: \(\psi(x) = N e^{A x^2} \), Where \(\pm \infty\)

Understanding the quantum behavior of particles often involves delving into complex mathematical frameworks. Among the fundamental quantities in quantum mechanics is the expectation value of an observable, which provides the average measurement one would expect over many trials. Today, we explore a specific scenario: determining the expectation value of the linear momentum for a one-dimensional wavefunction of the form \(\psi(x) = N e^{A x^2} \), where the domain extends over all real space (\(-\infty, +\infty\)). This inquiry not only offers insights into the particle's average momentum but also illuminates the interplay between the wavefunction's form and the physical quantities it encodes.

Introduction: The Significance of Expectation Values in Quantum Mechanics

In quantum physics, the expectation value of an observable is akin to the statistical average of measurement outcomes if one could perform an infinite number of identical experiments. For position and momentum, these expectation values tell us about the "average location" or "average motion" of a quantum particle. Unlike classical particles, quantum particles are described by wavefunctions, which encode probability amplitudes. The expectation value of momentum, in particular, is crucial because it indicates the average momentum a particle possesses, influencing how it evolves over time and interacts with its environment.

Calculating the expectation value of momentum involves integrating the wavefunction with the momentum operator applied to it. For complex wavefunctions, especially those with exponential or polynomial forms, this calculation can be mathematically intensive, but it also reveals subtle features, like whether the particle exhibits a net drift or is symmetrically distributed.

The Wavefunction in Focus: \(\psi(x) = N e^{A x^2} \)

The wavefunction under consideration is:

$$
\psi(x) = N e^{A x^2}
$$

where:
- N is the normalization constant,
- A is a parameter that influences the shape of the wavefunction,
- The domain extends over the entire real line ($-\infty, +\infty$).

Key Characteristics:

- Form of the wavefunction: The exponential of a quadratic function resembles Gaussian functions, but with a crucial difference: the sign of A determines whether the wavefunction is localized or diverges.
- Physical feasibility: For the wavefunction to be physically meaningful and square-integrable (normalizable), it must satisfy:

$$
\int_{-\infty}^{\infty} |\psi(x)|^2 dx < \infty
$$

which constrains A.

Determining the Normalization Constant N

Before calculating the expectation value of momentum, it's essential to normalize the wavefunction:

$$
\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1
$$

Given:

$$
|\psi(x)|^2 = N^2 e^{2A x^2}
$$

the normalization condition becomes:

$$
N^2 \int_{-\infty}^{\infty} e^{2A x^2} dx = 1
$$

The integral:

$$
\int_{-\infty}^{\infty} e^{2A x^2} dx
$$

is a Gaussian integral, which converges only if $2A < 0$, i.e., $A < 0$.

Result of the Gaussian integral:

$$
\int_{-\infty}^{\infty} e^{\alpha x^2} dx = \sqrt{\frac{\pi}{-\alpha}} \quad \text{for } \alpha < 0
$$

Applying this, we have:

$$
N^2 \sqrt{\frac{\pi}{-2A}} = 1
$$

which yields:

$$
N = \left(\frac{-2A}{\pi} \right)^{1/4}
$$

assuming $A < 0$.

Calculating the Expectation Value of Linear Momentum

The expectation value of momentum $\langle p \rangle$ for a wavefunction $\psi(x)$ is given by the integral:

$$
\langle p \rangle = \int_{-\infty}^{\infty} \psi^(x) \hat{p} \psi(x) dx
$$

where $\hat{p} = -i \hbar \frac{\partial}{\partial x}$.

Substituting the wavefunction:

$$
\langle p \rangle = -i \hbar \int_{-\infty}^{\infty} \psi^(x) \frac{\partial}{\partial x} \psi(x) dx
$$

Since the wavefunction $\psi(x)$ is real (as N and A are real), $\psi^(x) = \psi(x)$.

Thus,

$$
\langle p \rangle = -i \hbar \int_{-\infty}^{\infty} \psi(x) \frac{\partial}{\partial x} \psi(x) dx
$$

Deriving the Expectation Value: Step-by-Step

Step 1: Compute the derivative $\frac{\partial}{\partial x} \psi(x)$

$$
\frac{\partial}{\partial x} \psi(x) = N \frac{\partial}{\partial x} e^{A x^2} = N \cdot e^{A x^2} \cdot 2A x
$$

Step 2: Plug into the integral

$$
\langle p \rangle = -i \hbar \int_{-\infty}^{\infty} N e^{A x^2} \times N e^{A x^2} \times 2A x dx
$$

which simplifies to:

$$
\langle p \rangle = -i \hbar N^2 \cdot 2A \int_{-\infty}^{\infty} x e^{2A x^2} dx
$$

Step 3: Analyze the integral

$$
I = \int_{-\infty}^{\infty} x e^{2A x^2} dx
$$

This is an odd function integrated over symmetric limits. Since x is odd and $e^{2A x^2}$ is even, their product $x e^{2A x^2}$ is odd:

$$
f(x) = x e^{2A x^2} \implies f(-x) = -f(x)
$$

Integrating an odd function over $-\infty$ to $+\infty$ yields zero:

$$
I = 0
$$

Result:

$$
\boxed{
\langle p \rangle = 0
}
$$

Physical Interpretation: Symmetry and Momentum Expectation

The calculation reveals a fundamental aspect: the expectation value of the linear momentum for this wavefunction is zero. This outcome aligns with the symmetry properties of the wavefunction:

- Symmetry: Since $\psi(x)$ depends on x^2, it is symmetric about the origin.
- Implication: A symmetric wavefunction with no phase factor bias indicates that, on average, the particle has no net momentum in either direction.

This is consistent with the fact that the wavefunction is real and even, leading to zero expectation value of momentum, which is an odd operator.

Addressing the Sign of A: Physical Feasibility

While the calculation shows zero expectation value, the physical validity hinges on the nature of A:

- For $A < 0$, the wavefunction resembles a Gaussian, localized around $x=0$. Such a wavefunction is normalizable and physically meaningful.
- For $A > 0$, the wavefunction grows exponentially at infinity, making it non-normalizable and physically nonviable in the standard quantum framework.

Therefore, the meaningful analysis applies when $A < 0$, ensuring the wavefunction describes a localized particle with zero average momentum.

Broader Implications and Related Concepts

1. Wavefunction symmetry and expectation values

The zero expectation value of momentum for symmetric wavefunctions is a recurring theme in quantum mechanics. It emphasizes that the average momentum depends heavily on the wavefunction's symmetry and phase structure.

2. Momentum distribution and uncertainty

Even though the average momentum is zero, the particle can still possess momentum uncertainty. The wavefunction's shape influences the spread in momentum space, governed by the Fourier transform of $\psi(x)$.

3. Connection to Gaussian wavepackets

The scenario resembles the properties of Gaussian wavepackets, which are foundational in quantum mechanics for modeling particles with minimal position and momentum uncertainties.

Conclusion: What Does This Tell Us?

The exploration into the expectation value of linear momentum for the wavefunction $\psi(x) = N$

e^{A x^2} \) reveals a fundamental principle: symmetry in the wavefunction translates to a zero average momentum. Despite the simplicity of the wavefunction's form, this result encapsulates the core quantum idea that the observable average depends critically on the wavefunction's properties.

In practical terms, this calculation underscores

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