Which Linear Inequality Is Represented By The Graph?y One-halfx + 2y One-halfx + 2y One-thirdx + 2y One-thirdx

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Understanding the relationship between algebraic inequalities and their graphical representations is fundamental in mathematics. When analyzing a graph, one of the key tasks is to identify the inequality it represents. This process involves interpreting the boundary line, the shaded region, and the slope-intercept form of the associated equations. In this comprehensive guide, we will explore how to determine which linear inequality corresponds to a given graph, focusing on the expressions such as $y \leq \frac{1}{2}x + 2$, $y \geq \frac{1}{2}x + 2$, $y \leq \frac{1}{3}x + 2$, and $y \geq \frac{1}{3}x + 2$.

Understanding Linear Inequalities and Their Graphs

What Is a Linear Inequality?

A linear inequality is an inequality that involves a linear expression—meaning the highest power of the variable is 1. These inequalities can be written in forms such as:

- $y < mx + b$
- $y \leq mx + b$
- $y > mx + b$
- $y \geq mx + b$

where:
- m is the slope of the boundary line
- b is the y-intercept, the point where the line crosses the y-axis

The solution set of a linear inequality is a region on the coordinate plane, either above or below the boundary line, depending on the inequality symbol.

Graphing Linear Inequalities

When graphing a linear inequality:

1. Draw the boundary line: This line corresponds to the associated linear equation (e.g., $y = \frac{1}{2}x + 2$).
2. Decide whether the boundary is solid or dashed:
- Solid line if the inequality is ≤ or ≥ (inclusive inequalities).

- Dashed line if the inequality is < or > (strict inequalities).
3. Shade the appropriate region:
- Shade above the line if the inequality is y > mx + b or y ≥ mx + b.
- Shade below the line if the inequality is y < mx + b or y ≤ mx + b.

Analyzing the Given Expressions

The expressions provided are:

- y ≤ ½x + 2
- y ≥ ½x + 2
- y ≤ ⅓x + 2
- y ≥ ⅓x + 2

Each of these inequalities has a different slope (m) and the same y-intercept (b = 2). The key to identifying the correct inequality from a graph is to analyze the boundary line's position, slope, and the shaded region.

Understanding the Slopes and Their Graphical Implications

Slope of ½ (or 0.5)

- The line y = ½x + 2 has a moderate positive slope.
- It rises ½ unit vertically for every 1 unit it moves horizontally.
- The boundary line is less steep compared to a slope of 1, but more steep than ⅓.

Slope of ⅓ (~0.333)

- The line y = ⅓x + 2 has a gentler positive slope.
- It rises ⅓ unit vertically for every 1 unit horizontally.
- The line appears flatter compared to y = ½x + 2.

Implications for the Graph

- The steeper the slope, the more tilted the line appears.

- The less steep the slope, the flatter the line appears.
- The position of the shaded region relative to the boundary line indicates the inequality symbol.

How to Identify Which Inequality Corresponds to the Graph

Step-by-Step Approach

1. Locate the Boundary Line:
- Determine whether the line is solid or dashed.
- Identify the line's equation if possible.

2. Examine the Slope:
- Approximate the slope visually or by calculating two points on the boundary line.
- Compare with the known slopes (½ or ⅓).

3. Observe the Shaded Region:
- Is the region above or below the line?
- Use test points (like the origin (0,0)) to verify which side is shaded.

4. Match the Inequality:
- If the shading is above the boundary line and the line is solid, the inequality is $y \geq mx + b$.
- If the shading is below and the line is dashed, the inequality is $y < mx + b$.

Applying the Process to the Given Inequalities

Case 1: Graph with a Steeper Slope (~½)

- The boundary line appears steeper.
- The shaded region is above the line.
- The line is solid.

Identification: The inequality is $y \geq \frac{1}{2}x + 2$.

Case 2: Graph with a Gentle Slope (~⅓)

- The boundary line appears flatter.
- The shaded region is below the line.
- The line is dashed.

Identification: The inequality is $y < \frac{1}{3}x + 2$.

Common Mistakes to Avoid

- Confusing the slope with the y-intercept: Remember that the slope determines tilt, while the y-intercept is where the line crosses the y-axis.
- Misinterpreting the shading: Always verify whether the shaded area is above or below the boundary line.
- Ignoring whether the boundary line is solid or dashed: This determines whether the inequality includes equality (≥, ≤) or not (> , <).

Practice Examples

Example 1: The boundary line passes through points (0, 2) and (2, 3), and the shaded region is above the line with a solid boundary.

- Slope calculation:
- Slope $m = (3 - 2) / (2 - 0) = 1/2$
- Since the shading is above and the line is solid, the inequality is $y \geq \frac{1}{2}x + 2$.

Example 2: The boundary line passes through (0, 2) and (3, 3), and the shaded region is below the line with a dashed boundary.

- Slope calculation:
- $m = (3 - 2) / (3 - 0) = 1/3$
- Since shading is below and the line is dashed, the inequality is $y < \frac{1}{3}x + 2$.

Summary: Which Inequality Does Your Graph Represent?

Graph Characteristics	Corresponding Inequality	Explanation

Line with slope ~½, solid, shaded above	y ≥ ½x + 2	Inclusive, above line
Line with slope ~½, dashed, shaded below	y < ½x + 2	Strict, below line
Line with slope ~⅓, solid, shaded above	y ≥ ⅓x + 2	Inclusive, above line
Line with slope ~⅓, dashed, shaded below	y < ⅓x + 2	Strict, below line

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Conclusion

Identifying which linear inequality is represented by a graph requires a systematic approach:

- Determine the slope and intercept of the boundary line.
- Observe whether the line is solid or dashed.
- Look at which side of the line is shaded.
- Use these clues to match the graph with the correct inequality.

By applying these steps diligently, you can accurately interpret graphs of linear inequalities and understand the relationship between algebraic expressions and their graphical representations. Whether dealing with y ≥ ½x + 2, y ≤ ⅓x + 2, or other forms, mastering this skill enhances your overall understanding of coordinate geometry and inequality solutions.

Remember: Practice with various graphs and inequalities to develop confidence in identifying the correct algebraic expressions from visual representations.

Frequently Asked Questions

What does the graph of the inequality y > (1/2)x + 2 represent?

It represents all the points above the line y = (1/2)x + 2, where the inequality is strict (greater than).

How can you identify the correct linear inequality from its graph?

By observing whether the shaded region is above or below the line and whether the line is solid (≤ or ≥) or dashed (< or >).

What does a dashed line indicate in the graph of a linear inequality?

A dashed line indicates a strict inequality (< or >), meaning the points on the line are not included in the solution set.

If a graph shows a region below the line $y = (1/3)x + 2$, what is the inequality?

The inequality is $y < (1/3)x + 2$.

How do you write the inequality if the shaded region is above the line $y = (1/2)x + 2$?

The inequality would be $y \geq (1/2)x + 2$ if the line is solid, or $y > (1/2)x + 2$ if dashed.

What role does the slope play in determining the inequality?

The slope indicates the incline of the boundary line, which helps determine the inequality's direction based on the shaded region.

Can the inequality $y = (1/3)x + 2$ be represented directly from the graph?

Yes, if the line is solid and the shaded region indicates the solution set, then $y \geq (1/3)x + 2$ or $y \leq (1/3)x + 2$ depending on the shading.

What is the significance of the y-intercept in the linear inequality graph?

The y-intercept shows where the line crosses the y-axis, helping to determine the equation of the boundary line.

How do you determine which linear inequality matches a given graph with multiple lines?

By analyzing the shading region relative to each line, the line's style (solid or dashed), and matching the boundary line equation to the slope and intercept observed.

Additional Resources

Linear Inequality Analysis: Decoding the Graph Representation

When delving into the world of algebra and coordinate geometry, understanding how inequalities are represented graphically is fundamental. The question at hand—Which linear inequality is represented by the graph?—invites us to explore the intricate relationship between linear equations and their inequality counterparts. The provided expression, y one-half x + 2 y one-half x + 2 y one-third x + 2 y one-third x, appears complex at first glance, but with careful analysis, we can decipher its meaning and determine the corresponding inequality.

In this article, we will thoroughly analyze the components of this expression, interpret its structure, and guide you through the process of identifying the linear inequality it embodies, just as an expert would explain a sophisticated product feature.

Understanding the Components of the Expression

Before attempting to identify the inequality, it's essential to dissect the given expression:

y one-half x + 2 y one-half x + 2 y one-third x + 2 y one-third x

At first glance, this appears to be a sequence of terms involving x and y, each multiplied by fractional coefficients. The phrasing suggests some possible formatting issues or shorthand notation, but based on typical algebraic conventions, we can interpret it as:

Possible intended form:

- (1/2) x + 2 y
- (1/2) x + 2 y
- (1/3) x + 2 y
- (1/3) x + 2 y

This seems to imply two terms involving (1/2) x + 2 y and two involving (1/3) x + 2 y, perhaps suggesting multiple lines or segments on a graph.

Key observations:

- The coefficients are fractional: 1/2 and 1/3.
- The constant term is 2 y, which suggests the presence of multiple linear expressions.
- The repetition implies the graph could involve boundary lines for inequalities involving these expressions.

Rephrased as algebraic terms:

- $\frac{1}{2}x + 2 y$
- $\frac{1}{3}x + 2 y$

Interpreting the Graphical Representation

When analyzing the inequality represented by a graph, the key is to understand which boundary lines are drawn, and which side of those lines is shaded.

Typical process:

1. Identify the boundary lines: These are the lines corresponding to the equations involved, such as \(

$y = m x + c$.

2. Determine the inequality sign: Whether the region includes the boundary (≤ or ≥) or excludes it (< or >).

3. Analyze the shading: Which side of the boundary line is shaded? This indicates the solution set for the inequality.

Given the expression components, the graph might involve multiple lines with equations derived from the expressions:

- $y = \frac{1}{2}x + \text{constant}$
- $y = \frac{1}{3}x + \text{constant}$

Given the coefficients, these are lines with slopes 1/2 and 1/3.

Reconstructing the Inequality

Given the terms, a plausible interpretation is that the graph involves the following boundary lines:

- $y = \frac{1}{2}x + 2$
- $y = \frac{1}{3}x + 2$

or perhaps inequalities involving these boundary lines.

Possible inequalities:

- $y \leq \frac{1}{2}x + 2$
- $y \geq \frac{1}{3}x + 2$

or a combination such as:

- $y \leq \frac{1}{2}x + 2 \quad \text{and} \quad y \geq \frac{1}{3}x + 2$

which would describe the region between two lines with different slopes but the same y-intercept.

Visualizing the Graph

Imagine plotting the two lines:

Line 1: $y = \frac{1}{2}x + 2$

- Slope: 0.5
- Y-intercept: 2

Line 2: $y = \frac{1}{3}x + 2$

- Slope: approximately 0.333
- Y-intercept: 2

The lines are parallel since they share the same y-intercept but have different slopes (1/2 and 1/3). The region between these lines is a band where the inequalities could be:

- y between $\frac{1}{3}x + 2$ and $\frac{1}{2}x + 2$.

Possible combined inequality:

$$\frac{1}{3}x + 2 \leq y \leq \frac{1}{2}x + 2$$

This describes the strip of points lying between the two lines.

Formulating the Final Inequality

Based on the above interpretation, the inequality represented by the graph is likely:

Between the two lines:

$$\frac{1}{3}x + 2 \leq y \leq \frac{1}{2}x + 2$$

which can be expressed as a compound inequality:

$$\boxed{
\frac{1}{3}x + 2 \leq y \leq \frac{1}{2}x + 2
}$$

This indicates the region between the two boundary lines, inclusive of the lines if the inequalities are ≤ and ≥.

Implications and Additional Considerations

1. Boundary lines:

- If the graph shows solid lines, the inequality signs are inclusive (≤ or ≥).
- If dashed lines are used, the inequalities are strict (< or >).

2. Shading region:

- If the shaded region is between the two lines, the inequality is as above.
- If the shade is above the lines, then the inequalities would be $y \geq \frac{1}{2}x + 2$ and $y \geq \frac{1}{3}x + 2$, or similar.

3. Multiple inequalities:

- Sometimes, the graph may involve more than one inequality, possibly forming a bounded or unbounded region.

Summary of the Key Findings

- The core of the expression involves two linear equations with fractional slopes: $\frac{1}{2}x + 2$ and $\frac{1}{3}x + 2$.
- The graph likely depicts the region between these two lines.
- The corresponding inequality, assuming the region between the two lines is shaded, is:

$$\boxed{
\frac{1}{3}x + 2 \leq y \leq \frac{1}{2}x + 2
}$$

- This compound inequality describes all points lying between the two boundary lines.

Conclusion: Decoding the Graph and Inequality

In essence, the original expression hints at a set of linear equations with fractional slopes, and the graph's visual cues—such as the boundary lines' positions and shading—help determine the precise inequality. Recognizing the slopes, intercepts, and the shaded region allows us to confidently identify the inequality as the set of points lying between the two lines:

$$
\frac{1}{3}x + 2 \leq y \leq \frac{1}{2}x + 2
$$

This comprehensive analysis exemplifies how algebraic expressions translate into geometric representations, providing clarity and insight into the underlying inequalities. Whether you're solving for particular regions or exploring the relationships between lines, understanding these concepts is fundamental to mastering linear inequalities and their graphical counterparts.

Which Linear Inequality Is Represented By The Graph Y One Halfx 2y One Halfx 2y One Thirdx 2y One Thirdx

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