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2. (a) Find The Derivative Y. Given: (i) Y = $(x^2 + 1)$ arctan x − x (ii) Y = sinh(x log x). (b) Using Logarithmic Differentiation

In calculus, differentiation is a fundamental concept used to determine the rate at which a function changes with respect to its variable. When dealing with complex functions, especially those involving products, quotients, or compositions, differentiation can become intricate. Logarithmic differentiation is a powerful technique that simplifies the process, especially for functions involving exponents, products, or quotients. This article provides a comprehensive guide to finding derivatives of the given functions, employing standard differentiation rules and logarithmic differentiation where appropriate, ensuring clarity and depth for learners and practitioners alike.

Understanding the Problem

Part (a): Differentiating Y = $(x^2 + 1)$ arctan x − x

This function combines polynomial, inverse trigonometric, and linear components. The differentiation involves applying the product rule and chain rule, considering that arctan x is a composite function.

Part (b): Differentiating Y = sinh(x log x)

This function involves hyperbolic sine and a logarithmic function inside its argument, making it an ideal candidate for logarithmic differentiation to simplify the process.

Part (a): Derivative of Y = $(x^2 + 1)$ arctan x − x

Step 1: Break Down the Function

The function can be viewed as two separate terms:
- \(Y_1 = (x^2 + 1) \arctan x\)
- \(Y_2 = - x\)

Our goal is to find \(\frac{dY}{dx} = \frac{dY_1}{dx} + \frac{dY_2}{dx}\).

Step 2: Differentiate \(Y_1 = (x^2 + 1) \arctan x\)

Since \(Y_1\) is a product of two functions, apply the product rule:

\[
\frac{dY_1}{dx} = \frac{d}{dx}(x^2 + 1) \times \arctan x + (x^2 + 1) \times \frac{d}{dx}(\arctan x)
\]

Calculate each derivative:

- \(\frac{d}{dx}(x^2 + 1) = 2x\)
- \(\frac{d}{dx}(\arctan x) = \frac{1}{1 + x^2}\)

Thus,

\[
\frac{dY_1}{dx} = 2x \times \arctan x + (x^2 + 1) \times \frac{1}{1 + x^2}
\]

Notice that \(\frac{x^2 + 1}{1 + x^2} = 1\), simplifying the second term:

\[
\frac{dY_1}{dx} = 2x \arctan x + 1
\]

Step 3: Differentiate \(Y_2 = -x\)

\[
\frac{dY_2}{dx} = -1
\]

Step 4: Combine Results

\[
\frac{dY}{dx} = \frac{dY_1}{dx} + \frac{dY_2}{dx} = (2x \arctan x + 1) - 1 = 2x \arctan x
\]

Final Derivative for Part (a):

\[

$$
\boxed{\frac{dY}{dx} = 2x \arctan x}
$$

This derivative reflects the rate of change of the original function with respect to x, combining the effects of both the product and linear terms.

Part (b): Derivative of Y = sinh(x log x) Using Logarithmic Differentiation

Introduction to Logarithmic Differentiation

Logarithmic differentiation is particularly useful for functions where the variable appears in both the base and exponent or in complex compositions. The key idea involves taking the natural logarithm of both sides to simplify differentiation, then solving for the derivative.

Step 1: Rewrite the Function

Given:

$$
Y = \sinh(x \log x)
$$

Recall that:

$$
\sinh u = \frac{e^u - e^{-u}}{2}
$$

but directly differentiating the hyperbolic sine function is straightforward, especially with the chain rule. However, to simplify the process and handle the composite argument $x \log x$, we employ logarithmic differentiation.

Step 2: Take the Natural Logarithm of Y

$$
\ln Y = \ln \sinh(x \log x)
$$

Using properties of logarithms:

$$
\ln Y = \ln \sinh u, \quad \text{where } u = x \log x
$$

Note that:

$$
\frac{d}{dx} (\ln \sinh u) = \frac{1}{\sinh u} \times \frac{d}{dx} (\sinh u)

But $\frac{d}{dx} (\sinh u) = \cosh u \times \frac{du}{dx}$.

Therefore,

$$
\frac{d}{dx} (\ln Y) = \frac{\cosh u}{\sinh u} \times \frac{du}{dx} = \coth u \times \frac{du}{dx}
$$

Now, differentiate $u = x \log x$:

$$
\frac{du}{dx} = \log x + x \times \frac{1}{x} = \log x + 1
$$

Note: The derivative of $x \log x$ uses the product rule:

$$
\frac{d}{dx} (x \log x) = 1 \times \log x + x \times \frac{1}{x} = \log x + 1
$$

Step 3: Express $\frac{dy}{dx}$ in terms of Y

From the logarithmic derivative:

$$
\frac{1}{Y} \frac{dY}{dx} = \coth u \times (\log x + 1)
$$

Therefore,

$$
\frac{dY}{dx} = Y \times \coth u \times (\log x + 1)
$$

Recall that $Y = \sinh u$. So,

$$
\boxed{
\frac{dY}{dx} = \sinh (x \log x) \times \coth (x \log x) \times (\log x + 1)
}
$$

Alternatively, since $\coth u = \frac{\cosh u}{\sinh u}$, the derivative simplifies to:

$$
\frac{dY}{dx} = \cosh (x \log x) \times (\log x + 1)
$$

because:

$$
\sinh u \times \coth u = \cosh u
$$

Final concise expression:

$$
\boxed{
\frac{d}{dx} \sinh(x \log x) = \cosh (x \log x) \times (\log x + 1)
}
$$

This result elegantly captures the derivative of the composite hyperbolic sine function with respect to x, utilizing the logarithmic differentiation technique.

Summary and Key Takeaways

- Applying the product rule is essential when differentiating functions like $(x^2 + 1) \arctan x$.
- Recognizing that $\frac{x^2 + 1}{1 + x^2} = 1$ simplifies the derivative calculation.
- Logarithmic differentiation is particularly effective for functions like $\sinh(x \log x)$, where the variable appears in both base and exponent.
- The derivative of $\sinh u$ is $\cosh u \times \frac{du}{dx}$, which facilitates differentiation of hyperbolic functions.
- Expressing derivatives in terms of hyperbolic functions ($\sinh$, $\cosh$, and $\coth$) offers concise and elegant results.

Conclusion

Differentiation techniques such as the product rule, chain rule, and logarithmic differentiation are fundamental tools in calculus. They enable the systematic and efficient computation of derivatives for complex functions. In this article, we demonstrated how to differentiate two challenging functions—one involving a product with an inverse tangent function, and the other involving a hyperbolic sine with a composite argument—using these methods. Mastery of these techniques enhances problem-solving skills and deepens understanding of calculus concepts, which are essential for advanced studies and practical applications in science, engineering, and mathematics.