A Coil Has 500 Turns And Self-inductance 7.50 MH.The Current In The Coil Varies With Time According Toi=(680mA)cos[t/(0.0250s)].Part

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Understanding the behavior of electrical coils, especially in the context of inductance and time-varying currents, is fundamental in electromagnetic theory and practical applications such as transformers, inductors, and electrical circuits. In this article, we explore the characteristics of a coil with specific parameters, analyze its inductance, current variation, and the resulting electromotive force (EMF), along with relevant calculations and implications.

Fundamental Concepts of Inductance and Coil Behavior

What Is Self-Inductance?

Self-inductance is the property of a coil or inductor that opposes changes in the current flowing through it. It is defined as the ratio of the magnetic flux linkage to the current causing it:

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L = Φ / I
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where:
- L is the inductance (measured in henries, H),
- Φ is the magnetic flux linkage (weber-turns),
- I is the current (amperes).

A higher inductance indicates a stronger opposition to changes in current, leading to more significant induced EMF when the current varies.

Parameters of the Given Coil

The specific parameters provided are:
- Number of turns, N = 500
- Self-inductance, L = 7.50 millihenries = 7.50×10^{-3} H
- Current variation with time: i(t) = (680 mA) cos [t / (0.0250 s)]

These parameters form the basis for analyzing the coil's behavior under the specified current variation.

Analysis of the Current Variation

Current as a Function of Time

The current in the coil is given by:

$$i(t) = 680 \text{ mA} \times \cos [t / 0.0250 \text{ s}]$$

Expressed in amperes:

$$i(t) = 0.680 \text{ A} \times \cos [t / 0.0250 \text{ s}]$$

This describes a sinusoidal current with an angular frequency:

$$\omega = 1 / 0.0250 \text{ s} = 40 \text{ rad/sec}$$

which implies the period:

$$T = 2\pi / \omega \approx 2\pi / 40 \approx 0.157 \text{ s.}$$

Key observations:
- The current oscillates between +0.680 A and -0.680 A.
- The cosine function indicates a smooth, periodic change in current over time.

Implications of the Current Variation

Since the current varies sinusoidally, the coil experiences an alternating magnetic flux, which induces an EMF opposing the change in current — a fundamental principle of electromagnetic induction.

Calculating the Induced EMF

Faraday's Law of Electromagnetic Induction

The induced EMF (voltage) in the coil is given by:

$$\varepsilon(t) = -L \times (d\, i(t) / dt)$$

where:
- $d\, i(t) / dt$ is the time derivative of the current.

Derivative of the Current

Given:

$$i(t) = 0.680 \text{ A} \times \cos(\omega t)$$

then:

$$d\, i(t) / dt = -0.680 \text{ A} \times \omega \times \sin(\omega t)$$

Substituting $\omega = 40$ rad/sec:

$$d\, i(t) / dt = -0.680 \text{ A} \times 40 \times \sin(40 t) \approx -27.2 \text{ A} \times \sin(40 t)$$

Maximum Induced EMF

The maximum EMF occurs when $\sin(\omega t) = \pm 1$:

$$|\varepsilon_{max}| = L \times \omega \times 0.680 \text{ A}$$

Calculating:

$$|\varepsilon_{max}| = 7.50 \times 10^{-3} \text{ H} \times 40 \text{ rad/sec} \times 0.680 \text{ A} \approx 0.205 \text{ V}$$

Interpretation:
- The coil induces an EMF with a maximum magnitude of approximately 0.205 volts.
- The phase difference between current and induced EMF is 90°, typical of inductive circuits.

Power and Energy Considerations

Instantaneous Power

The instantaneous power delivered or absorbed by the coil:

$$p(t) = i(t) \times \varepsilon(t)$$

Substituting:

$$p(t) = (0.680 \cos \omega t) \times (-L \omega) \times (0.680 \sin \omega t)$$

which simplifies to:

$$p(t) = -L \omega \times (0.680)^2 \times \cos \omega t \times \sin \omega t$$

Using the identity:

$$\cos \omega t \times \sin \omega t = (1/2) \times \sin 2 \omega t$$

we get:

$$p(t) = -(1/2) L \omega \times (0.680)^2 \times \sin 2 \omega t$$

Maximum power:

$$|p_{max}| = (1/2) \times L \times \omega \times (0.680)^2$$

Calculating:

$$|p_{max}| = 0.5 \times 7.50 \times 10^{-3}\ H \times 40 \times (0.680)^2 \approx 0.5 \times 7.50 \times 10^{-3} \times 40 \times 0.462 \approx 0.055\ W$$

This indicates the coil exchanges energy with the circuit periodically but does not dissipate power as heat, assuming ideal conditions.

Applications and Practical Implications

In Electrical Circuits

Understanding the behavior of coils with specified inductance and current variation is essential in designing:

1. Transformers for voltage regulation
2. Inductive filters in electronic circuits
3. Wireless power transfer systems
4. Inductive sensors and coils in measurement devices

In Signal Processing

The sinusoidal current and induced EMF are critical in AC circuit analysis, allowing engineers to design systems with predictable phase relationships and power flow characteristics.

In Power Systems

Large inductors are used to control current flow and smooth out fluctuations, essential for maintaining stability in power grids.

Additional Calculations and Considerations

Energy Stored in the Magnetic Field

The energy stored in the magnetic field of the inductor at any instant:

$$U = (1/2)\ L\ I^2$$

At peak current I = 0.680 A:

$$U = 0.5 \times 7.50 \times 10^{-3}\ H \times (0.680\ A)^2 \approx 0.5 \times 7.50 \times 10^{-3} \times 0.462 \approx 1.73 \times 10^{-3}\ J$$

This energy is cyclically stored and released during each oscillation.

Effect of Resistance

While the problem assumes an ideal inductor, real coils have resistance which causes energy dissipation as heat. The presence of resistance affects:
- The amplitude of current oscillations
- The phase relationship
- The quality factor (Q) of the coil

If resistance R is significant, the circuit becomes a damped oscillator, and the maximum current and EMF would be lower.

Summary and Key Takeaways

- The coil's inductance of 7.50 mH opposes rapid changes in current, inducing an EMF proportional to the rate of change.
- The sinusoidal current with a maximum of 0.680 A oscillates with a period of approximately 0.157 seconds, driven by the angular frequency ω = 40 rad/sec.
- The maximum induced EMF in the coil is approximately 0.205 volts, opposing the change in current per Faraday’s law.
- Energy is cyclically stored in the magnetic field, with a maximum of around 1.73 millijoules during each oscillation.
- Understanding these parameters is crucial in designing and analyzing AC circuits, transformers, and inductive components.

In conclusion, analyzing a coil with specific parameters provides valuable insights into electromagnetic phenomena, enabling engineers and scientists to optimize the performance of electrical devices and systems that rely on inductance and time-varying currents.

Frequently Asked Questions

What is the total number of turns in the coil?

The coil has 500 turns.

What is the self-inductance of the coil?

The self-inductance is 7.50 millihenries (mH).

How does the current in the coil vary with time?

The current varies according to $i(t) = (680 \text{ mA}) \cos[t / (0.0250 \text{ s})]$.

What is the maximum current flowing through the coil?

The maximum current is 680 mA, as given by the amplitude of the cosine function.

What is the angular frequency of the current variation?

The angular frequency ω is $1 / 0.0250 \text{ s} = 40 \text{ rad/s}$.

How do you calculate the induced emf in the coil at any time?

The induced emf is given by $\varepsilon = -L (di/dt)$, where L is the inductance and di/dt is the derivative of current with respect to time.

What is the value of the derivative di/dt at t = 0?

At $t = 0$, $di/dt = - (680 \text{ mA}) (1 / 0.0250 \text{ s}) \sin(0) = 0$, since $\sin(0) = 0$.

How does the self-inductance influence the coil's behavior with changing current?

The self-inductance opposes changes in current, inducing emf that resists the variation in current over time.

Calculate the maximum induced emf in the coil.

Maximum emf = $L \omega I_{max} = (7.50 \times 10^{-3} \text{ H})\ 40 \text{ rad/s}\ 0.68 \text{ A} \approx 0.204 \text{ V}$.

What is the significance of the phase difference between current and emf in an inductor?

In an ideal inductor, the emf leads the current by 90 degrees, meaning the emf reaches its maximum before the current does.

Additional Resources

Understanding the Self-Inductance of a Coil with Varying Current: A Comprehensive Analysis

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Introduction to Inductance and Its Significance

The concept of inductance is fundamental in electromagnetism and electrical engineering. It describes the property of a coil or a circuit component that opposes changes in the electric current flowing through it. When current varies with time, the coil's magnetic field also changes, inducing an electromotive force (emf) that opposes the change in current, according to Lenz's Law.

In our case, we are examining a coil with the following parameters:

- Number of turns, $N = 500$
- Self-inductance, $L = 7.50\,\text{mH}$ (millihenrys)
- Current variation, $i(t) = (680\,\text{mA}) \cos \left[\frac{t}{0.0250\,\text{s}}\right]$

This analysis aims to explore the physical meaning, mathematical implications, and practical consequences of these parameters, as well as to examine the dynamic behavior of the coil in response to the time-varying current.

Fundamentals of Self-Inductance

Definition and Physical Interpretation

Self-inductance is a measure of a coil's ability to induce an emf within itself due to a change in its own current. It is mathematically expressed as:

$$
\mathcal{E} = -L \frac{d i(t)}{dt}
$$

where:

- $\mathcal{E}$ is the emf induced (in volts),
- L is the self-inductance (in henrys),
- $\frac{d i(t)}{dt}$ is the rate of change of current with respect to time.

The negative sign indicates that the emf opposes the change in current, consistent with Lenz's Law.

Physical meaning: The higher the inductance L, the greater the emf generated for a given rate of change of current, effectively resisting rapid changes in current.

Relation Between Turns and Inductance

The number of turns in a coil influences its inductance significantly. For a solenoid, the inductance L is related to N by:

$$
L = \mu_0 \mu_r \frac{N^2 A}{l}
$$

where:

- μ_0 is the permeability of free space ($4\pi \times 10^{-7} \,\text{H/m}$),
- μ_r is the relative permeability of core material (assumed to be 1 for air core),
- A is the cross-sectional area,
- l is the length of the coil.

Given $N = 500$ turns and $L = 7.50\,\text{mH}$, one can analyze the coil's physical dimensions or core properties to understand the inductance.

Mathematical Analysis of the Given Current Function

Expressing the Current as a Time-Dependent Function

The current in the coil is given by:

$$
i(t) = (680\,\text{mA}) \cos \left[\frac{t}{0.0250\,\text{s}}\right]
$$

which can be rewritten as:

$$
i(t) = 0.68\,\text{A} \cos(\omega t)
$$

where:

- $\omega = \frac{1}{0.0250\,\text{s}} = 40\,\text{rad/s}$

This indicates a simple harmonic oscillation with angular frequency $\omega = 40\,\text{rad/s}$.

Key observations:

- The amplitude of current oscillation is $0.68\,\text{A}$,
- The period of oscillation, T, is:

$$
T = \frac{2 \pi}{\omega} = \frac{2 \pi}{40} \approx 0.157\,\text{s}
$$

- The frequency, f, is:

$$
f = \frac{1}{T} \approx 6.37\,\text{Hz}
$$

This is a low-frequency oscillation, typical in many practical circuits involving inductors.

Calculating the Rate of Change of Current

The emf induced within the coil depends on $\frac{d i(t)}{dt}$. Differentiating $i(t)$:

$$
\frac{d i(t)}{dt} = -0.68\,\text{A} \times \omega \sin(\omega t) = -0.68 \times 40 \sin(40 t)
$$

which simplifies to:

$$
\frac{d i(t)}{dt} = -27.2 \sin(40 t) \quad \text{A/s}
$$

This variation indicates the emf will oscillate sinusoidally, with maximum magnitude when $\sin(40 t) = \pm 1$.

Induced emf and Power Considerations

Calculating the Instantaneous emf

Using the relation:

$$
\mathcal{E}(t) = -L \frac{d i(t)}{dt}
$$

and substituting $L = 7.50\,\text{mH} = 7.50 \times 10^{-3}\,\text{H}$:

$$
\mathcal{E}(t) = - (7.50 \times 10^{-3}) \times (-27.2 \sin(40 t))
$$

$$
\mathcal{E}(t) = 0.204\, \text{V} \times \sin(40 t)
$$

Key points:

- The maximum emf magnitude is approximately 0.204 V.
- The emf oscillates sinusoidally, in phase with the derivative of current.

Power Dissipation and Energy Storage

The coil stores energy in its magnetic field, given by:

$$
U(t) = \frac{1}{2}L i^2(t)
$$

At any instant:

$$
U(t) = \frac{1}{2} \times 7.50 \times 10^{-3} \times (0.68 \cos 40 t)^2
$$

$$
U(t) = 3.75 \times 10^{-3} \times 0.4624 \cos^2 40 t \approx 1.73 \times 10^{-3} \cos^2 40 t\, \text{J}
$$

The maximum stored energy:

$$
U_{max} \approx 1.73 \times 10^{-3}\, \text{J}
$$

This energy oscillates with time, reflecting the energy transfer between the magnetic field and the electric current.

Physical and Practical Implications of the Parameters

Effect of the Number of Turns and Inductance

The high number of turns (500) contributes significantly to the coil's inductance. The inductance value of 7.50 mH indicates:

- The coil can oppose rapid changes in current.
- It can act as a low-frequency filter in circuits.
- It stores a small amount of magnetic energy compared to larger inductors.

The combination of the turns and physical dimensions determines the coil's inductance, influencing how it interacts with AC signals.

Impact of the Oscillating Current

Given that the current varies sinusoidally at approximately 6.37 Hz:

- The coil experiences periodic energy storage and release.
- The induced emf oscillates accordingly.
- The coil exhibits reactive behavior, mainly characterized by inductive reactance:

$$
X_L = \omega L = 40 \times 7.50 \times 10^{-3} = 0.3\, \Omega
$$

This indicates a relatively small inductive reactance at this frequency, implying minimal opposition to current flow.

Implications:

- The circuit behaves predominantly as a resistor with some reactive properties.
- The phase difference between voltage and current is 90° in an ideal inductor, but the low reactance here hints at near-resistive behavior.

Resonance and Circuit Behavior

In a circuit involving this coil, the effect of inductance at the specified frequency influences the overall impedance:

$$
Z = R + j X_L
$$

where R is resistance (not specified, so assumed negligible here), and j is the imaginary unit.

- The coil's impedance at this frequency is primarily reactive.
- If combined with a resistor, the circuit could be tuned to resonance, maximizing current or voltage as desired.

Real-World Applications and Considerations

Inductive Reactance in AC Circuits

In practice, coils with these parameters are used in:

- Filters: To block or pass specific frequency components.
- Transformers: Although this specific coil isn't a transformer, multiple turns and inductance are key.
- Oscillators: The frequency of current oscillation can be adjusted

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