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Understanding wave equations is fundamental in physics, especially when analyzing wave motion in various mediums. The given wave equation:

$$ y = 0.2 \sin (0.4 \pi (x - 60 t)) $$

encapsulates essential information about the wave's amplitude, wavelength, frequency, and speed. In this article, we will thoroughly analyze this wave equation to determine the wave's speed, velocity, and frequency, providing a comprehensive understanding grounded in wave physics principles.

Understanding the Wave Equation

The general form of a sinusoidal wave traveling along a medium can be written as:

$$ y = A \sin (k x - \omega t + \phi) $$

where:
- A is the amplitude,
- k is the wave number,
- ω is the angular frequency,
- x is the position,
- t is the time,
- ϕ is the phase constant (which can be zero in simple wave equations).

In the given wave equation:

$$ y = 0.2 \sin (0.4 \pi (x - 60 t)) $$

we notice that the argument of the sine function is of the form:

$$ \theta = 0.4 \pi (x - 60 t) $$

which can be rewritten as:

$$ y = 0.2 \sin [k x - \omega t] $$

by comparing terms, where:

- $k x = 0.4 \pi x$
- $\omega t = 0.4 \pi \times 60 t$

From this, we identify:

$$ k = 0.4 \pi $$
$$ \omega = 0.4 \pi \times 60 $$

Calculating the Wave Number (k)

The wave number k relates to the wavelength λ as:

$$ k = \frac{2 \pi}{\lambda} $$

Given:

$$ k = 0.4 \pi $$

we can solve for λ:

$$ 0.4 \pi = \frac{2 \pi}{\lambda} $$

Dividing both sides by π:

$$ 0.4 = \frac{2}{\lambda} $$

Rearranging:

$$ \lambda = \frac{2}{0.4} $$

Calculating:

$$ \lambda = 5 \text{ centimeters} $$

Therefore, the wavelength λ is 5 centimeters.

Calculating the Angular Frequency (ω)

The angular frequency ω is derived from the coefficient of t:

$$ \omega = 0.4 \pi \times 60 $$

Calculating:

$$ \omega = 0.4 \times 3.1416 \times 60 $$

$$ \omega = 0.4 \times 188.495 $$

$$ \omega \approx 75.398 \text{ radians per second} $$

Thus, the angular frequency ω is approximately 75.4 rad/sec.

Calculating the Wave Speed (v)

Wave speed v is related to wavelength λ and frequency f:

$$ v = \lambda f $$

Alternatively, in terms of angular frequency:

$$ v = \frac{\omega}{k} $$

Using the values calculated:

$$ v = \frac{75.4}{0.4 \pi} $$

Recall:

$$ 0.4 \pi \approx 0.4 \times 3.1416 \approx 1.2566 $$

Therefore:

$$ v = \frac{75.4}{1.2566} $$

$$ v \approx 60 \text{ centimeters per second} $$

The wave speed v is approximately 60 cm/sec.

Calculating the Frequency (f)

Frequency is related to angular frequency:

$$ f = \frac{\omega}{2 \pi} $$

Using the calculated ω:

$$ f = \frac{75.4}{2 \pi} $$

$$ 2 \pi \approx 6.2832 $$

Calculating:

$$ f = \frac{75.4}{6.2832} $$

$$ f \approx 12 \text{ Hz} $$

The wave frequency f is approximately 12 Hz.

Summary of Calculations

Parameter	Value	Explanation
Wavelength (λ)	5 centimeters	Derived from wave number k
Angular frequency (ω)	75.4 radians/sec	Derived from coefficient of t in the wave equation
Wave speed (v)	60 centimeters/sec	Calculated as $v = \omega / k$
Frequency (f)	12 Hz	Derived from $\omega = 2 \pi f$

Key Concepts in Wave Physics

Understanding the relationships between wave parameters is crucial in physics:

Wave Number (k): Indicates how many radians the wave varies per unit length. It relates directly to the wavelength.

Angular Frequency (ω): Represents how many radians the wave oscillates per second; it is linked to the frequency.

Wave Speed (v): The rate at which the wave propagates through the medium, determined by the ratio of ω to k.

Frequency (f): The number of wave cycles passing a point per second; measured in Hertz (Hz).

Practical Applications of Wave Calculations

Calculating wave speed, wavelength, and frequency has numerous real-world applications:

- Communication Technologies: Understanding wave parameters helps in designing antennas and transmission systems.
- Medical Imaging: Ultrasound waves rely on precise wave calculations for accurate imaging.
- Seismology: Analyzing seismic waves involves similar calculations to determine Earth's internal structure.
- Music and Acoustics: Sound wave properties influence instrument design and acoustic treatments.

Conclusion

In summary, the given wave equation describes a sinusoidal wave traveling along a medium with specific characteristics. By analyzing the wave equation, we determined:

- The wavelength (λ) as 5 centimeters.
- The wave's angular frequency (ω) as approximately 75.4 radians/sec.
- The wave speed (v) as approximately 60 centimeters/sec.
- The frequency (f) as approximately 12 Hz.

These calculations demonstrate the fundamental relationships between wave parameters and emphasize the importance of wave physics in both theoretical and practical contexts. Mastery of these concepts enables scientists and engineers to analyze and manipulate wave phenomena across various fields, enhancing technological innovations and scientific understanding.

Keywords: wave equation, wave speed, wavelength, frequency, wave velocity, sinusoidal wave, wave analysis, wave physics, wave parameters, wave motion

Frequently Asked Questions

What is the general form of the wave equation given, and what do the parameters represent?

The wave equation is $y = 0.2 \sin 0.4\pi (x - 60t)$, where 0.2 is the amplitude, 0.4π is the angular wave number, and 60 is the angular frequency related to time. Distances are in centimeters and time in seconds.

How do you determine the wave's speed from the wave equation $y = 0.2 \sin 0.4\pi (x - 60t)$?

The wave speed v is given by $v = \omega / k$, where ω is the angular frequency and k is the wave number. From the equation, $k = 0.4\pi$ and $\omega = 60$.

What is the value of the wave number k in the given wave equation?

The wave number k is 0.4π radians per centimeter.

How do you calculate the wave's frequency from the given wave equation?

The frequency f is calculated as $f = \omega / (2\pi)$. Given $\omega = 60$, so $f = 60 / (2\pi) \approx 9.55$ Hz.

What is the velocity of the wave described by the equation $y = 0.2 \sin 0.4\pi (x - 60t)$?

The wave speed $v = \omega / k = 60 / (0.4\pi) \approx 47.75$ centimeters per second.

Can you summarize the key wave parameters derived from the equation?

Yes. The wave speed is approximately 47.75 cm/s, the frequency is about 9.55 Hz, and the wave number is 0.4π radians/cm.

Additional Resources

A wave is represented by the equation y = 0.2 Sin 0.4 pi (x - 60t), where all distances are measured in centimeters and time in seconds. Calculate the speed of the wave, its velocity, and frequency.

Introduction

Waves are fundamental to understanding many physical phenomena, from sound and light to water ripples and seismic activity. The mathematical representation of a wave provides crucial information about its behavior, including how fast it propagates, how often it oscillates, and its overall dynamics. In this article, we analyze the given wave equation:

$$ y = 0.2 \sin (0.4 \pi (x - 60t)) $$

where all spatial measurements are in centimeters, and time is in seconds. Our goal is to extract and compute three key properties:

- The speed of the wave (how fast the wave travels through space)
- The velocity (which, in wave context, is essentially the same as speed but sometimes distinguished in vector form)
- The frequency (how many oscillations occur per second)

By dissecting this equation, we'll understand the underlying physics governing the wave's behavior.

Understanding the Wave Equation

The general form of a sinusoidal wave traveling along a medium is:

$$ y(x,t) = A \sin [k(x - vt)] $$

where:

- A is the amplitude (maximum displacement)
- k is the wave number
- v is the wave speed
- x is the position
- t is the time

Alternatively, the wave can be expressed as:

$$ y(x,t) = A \sin (kx - \omega t) $$

where:

- ω is the angular frequency

In our case, the given wave equation is:

$$ y = 0.2 \sin (0.4 \pi (x - 60t)) $$

Notice that the argument of the sine function is:

$$ 0.4 \pi (x - 60t) $$

which can be expanded as:

$$ (0.4 \pi) x - (0.4 \pi)(60) t $$

This directly relates to the standard form $k x - \omega t$, allowing us to identify the wave number k and angular frequency ω.

Step 1: Calculating the Wave Number k

The wave number k indicates the number of wave cycles per unit distance and is given by the coefficient of x:

$$ k = 0.4 \pi \quad \text{(radians per centimeter)} $$

Calculating:

$$ k = 0.4 \pi \approx 0.4 \times 3.1416 \approx 1.2566 \, \text{rad/cm} $$

This means each complete wave cycle spans a distance related to the wave number, which we'll determine next.

Step 2: Determining the Angular Frequency ω

The angular frequency ω corresponds to the coefficient of t in the phase:

$$ \omega = (0.4 \pi) \times 60 $$

Calculating:

$$ \omega = 0.4 \pi \times 60 = (0.4 \times 3.1416) \times 60 $$
$$ \omega \approx 1.2566 \times 60 \approx 75.396 \, \text{rad/sec} $$

Angular frequency ω describes how rapidly the wave oscillates in time.

Step 3: Calculating the Wave Speed v

The wave propagates through space with a certain speed, which relates to the wave number and angular frequency:

$$ v = \frac{\omega}{k} $$

Substituting the calculated values:

$$ v = \frac{75.396}{1.2566} \approx 60 \, \text{cm/sec} $$

This indicates that the wave travels at approximately 60 centimeters per second.

Step 4: Finding the Frequency f

Frequency f tells us how many complete oscillations occur per second. It relates to the angular frequency by:

$$ \omega = 2\pi f $$

Thus:

$$ f = \frac{\omega}{2\pi} $$

Calculating:

$$ f = \frac{75.396}{2 \times 3.1416} \approx \frac{75.396}{6.2832} \approx 12 \, \text{Hz} $$

So, the wave oscillates roughly 12 times per second.

Summarizing the Key Properties

Property	Calculation / Value	Explanation
Wave Number k	approximately 1.2566 rad/cm	Number of radians per centimeter
Angular Frequency ω	approximately 75.396 rad/sec	How rapidly the wave oscillates in time
Wave Speed v	approximately 60 cm/sec	How fast the wave propagates through space
Frequency f	approximately 12 Hz	Number of oscillations per second

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The Significance of These Calculations

Understanding these properties enables us to predict the behavior of the wave under various conditions. For example:

- Wave Speed (60 cm/sec): This tells us how quickly disturbances travel along the medium. A higher speed would mean the wave reaches distant points faster.
- Frequency (12 Hz): This indicates how many peaks pass a fixed point each second. Higher frequency waves are associated with higher energy and, in sound waves, higher pitch.
- Wave Number (1.2566 rad/cm): This relates to the wavelength λ, since:

$$\lambda = \frac{2\pi}{k}$$

Calculating:

$$\lambda = \frac{6.2832}{1.2566} \approx 5 \, \text{cm}$$

which means each wave cycle spans approximately 5 centimeters.

Connecting the Concepts

The relationships among wave properties are fundamental in physics:

- The wave speed v relates to both frequency f and wavelength λ:

$$v = f \lambda$$

Using the calculated wavelength:

$$v = 12 \times 5 = 60 \, \text{cm/sec}$$

which confirms our earlier calculation.

- These interrelationships illustrate how changing one property affects others. For instance, increasing the frequency while keeping wave speed constant would decrease the wavelength.

Practical Implications

Understanding wave characteristics is essential across various fields:

- Engineering: Designing materials and structures that can withstand or manipulate wave propagation, such as in acoustics or telecommunications.
- Physics: Analyzing wave phenomena in different media, from water to electromagnetic waves.
- Medical Imaging: Ultrasound waves rely on precise knowledge of wave speed and frequency to generate images.

- Seismology: Studying how seismic waves travel through Earth's layers helps in earthquake analysis.

Final Thoughts

The mathematical dissection of the wave equation reveals a wealth of information about its physical behavior. From the wave number and frequency to the wave speed, each parameter plays a crucial role in describing how the wave moves and interacts with its environment. The ability to extract these properties from a simple sinusoidal equation exemplifies the power of mathematical physics in understanding the natural world.

In our case, the wave travels at approximately 60 centimeters per second, oscillates about 12 times per second, and has a wavelength of roughly 5 centimeters. These insights not only deepen our understanding of specific wave phenomena but also reinforce foundational principles applicable across physics and engineering disciplines.

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