Calculate The Integral Of F(x,y)=7x Over The Region D Bounded Above By Y=x(2-x) And Below By X=y(2- Y).Hint:Apply

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Understanding the Problem: Calculating a Double Integral Over a Specific Region

In the realm of multivariable calculus, one common problem involves evaluating double integrals over regions bounded by curves or lines. Here, the task is to compute the integral of the function F(x, y) = 7x over a region D that is bounded above by the curve Y = x(2 - x) and below by the curve X = y(2 - y). The problem hints at applying certain techniques, likely substitutions or transformations, to simplify the integration process.

This article provides a comprehensive step-by-step guide to solving this problem, including understanding the region, setting up the integral, choosing appropriate methods, and executing the calculations. Our goal is to make the process clear and understandable, even for those new to double integrals and region transformations.

Step 1: Analyzing and Visualizing the Region D

1.1 Understanding the bounding curves

- Upper boundary: $y = x(2 - x)$

- This is a quadratic function in terms of x. It opens downward (since the coefficient of x^2 is negative).
- It can be rewritten as: $y = -x^2 + 2x$.
- The parabola intersects the x-axis at $x = 0$ and $x = 2$.

- Lower boundary: $x = y(2 - y)$

- Similarly, this is a quadratic in terms of y: $x = - y^2 + 2 y$.

1.2 Symmetry and the region D

- Notice the symmetry: the upper boundary is expressed as y in terms of x, while the lower boundary expresses x in terms of y.
- The region D is bounded above by the parabola in the xy-plane and below by the inverse relation.

1.3 Graphical interpretation

- Plotting these curves helps visualize the region D:
- The parabola $y = x(2 - x)$ peaks at $x=1$, where $y=1$.
- The parabola $x = y(2 - y)$ peaks at $y=1$, where $x=1$.

- The curves intersect at points where each is equal to the other or where they cross:
- Set $y = x(2 - x)$ and $x = y(2 - y)$.
- Substituting $y = x(2 - x)$ into the second:

$$
x = [x(2 - x)] (2 - x(2 - x))
$$

but this can quickly become complex; instead, look for intersection points directly.

1.4 Finding the intersection points

- From the symmetry:

- At $x = 0$:

$$
y = 0(2 - 0) = 0
$$

- At $x=2$:

$$
y= 2(2 - 2)= 0
$$

- Similarly, at $y=0$:

$$
x= 0(2 - 0)= 0
$$

- At $y=2$:

$$
x= 2(2 - 2)=0
$$

- The key point is at $(1,1)$:

$$
y = 1(2 - 1)=1
$$

$$
x= 1(2-1)=1
$$

- Thus, the region is symmetric about $(1,1)$, bounded between $x=0$ to $x=2$ and $y=0$ to $y=2$.

Step 2: Choosing the Method of Integration

Given the symmetry and nature of the boundaries, there are two main approaches:

2.1 Integrate with respect to y first, then x

- For each fixed x, y ranges from the lower boundary $x = y(2 - y)$ to the upper boundary $y = x(2 - x)$.

- But the boundaries are given in different forms, making direct integration complicated.

2.2 Transform variables to simplify the region

- The problem hints at applying a substitution, which suggests transforming (x, y) into new variables (u, v) that align with the boundaries.

- Since the boundaries involve quadratic forms, a natural substitution is to consider:

$$
u = x + y
$$
$$
v = x - y
$$

- Alternatively, considering the symmetry, defining:

$$
u = x + y
$$

\]

$$
v = x - y
$$

can help transform the region into a more manageable shape, such as a rectangle.

Step 3: Applying Variable Transformation

3.1 Defining the substitution

- Let:

$$
u = x + y
$$

$$
v = x - y
$$

- Solving for x and y:

$$
x = \frac{u + v}{2}
$$

$$
y = \frac{u - v}{2}
$$

3.2 Transforming the boundaries

- Upper boundary: $y = x(2 - x)$.

- Substituting:

$$
y = \frac{u - v}{2}
$$

$$
x = \frac{u + v}{2}
$$

- The boundary becomes:

$$
\frac{u - v}{2} = \left(\frac{u + v}{2} \right) \left(2 - \frac{u + v}{2} \right)
$$

- Simplify RHS:

$$
\left(\frac{u + v}{2} \right) \left(2 - \frac{u + v}{2} \right) = \frac{u + v}{2} \times \left(2 - \frac{u + v}{2} \right)
$$

$$
= \frac{u + v}{2} \times \frac{4 - (u + v)}{2} = \frac{(u + v)(4 - u - v)}{4}
$$

- Therefore, the boundary condition:

$$
\frac{u - v}{2} = \frac{(u + v)(4 - u - v)}{4}
$$

Multiply both sides by 4:

$$
2(u - v) = (u + v)(4 - u - v)
$$

- Expand RHS:

$$
(u + v)(4 - u - v) = u(4 - u - v) + v(4 - u - v)
$$

$$
= 4u - u^2 - uv + 4v - uv - v^2
$$

$$
= 4u + 4v - u^2 - 2uv - v^2
$$

- The boundary condition becomes:

$$
2u - 2v = 4u + 4v - u^2 - 2uv - v^2
$$

- Bring all to one side:

$$
0 = 4u + 4v - u^2 - 2uv - v^2 - 2u + 2v
$$

$$
0 = (4u - 2u) + (4v + 2v) - u^2 - 2uv - v^2
$$

$$
0 = 2u + 6v - u^2 - 2uv - v^2
$$

- Rearranged as:

$$
u^2 + 2uv + v^2 = 2u + 6v
$$

3.3 Similar transformation for the lower boundary

- The lower boundary is $x = y(2 - y)$.

- Substituting:

$$
x = \frac{u + v}{2}
$$

$$
y = \frac{u - v}{2}
$$

- The boundary:

\[

\frac{u + v}{2} = \left(\frac{u - v}{2} \right) (2 - \frac{u - v}{2})

\]

- Simplify RHS:

\[

\frac{u - v}{2} \times \left(2 - \frac{u - v}{

Frequently Asked Questions

How do I set up the double integral for the function F(x,y) = 7x over the region D bounded above by y = x(2 - x) and below by y = y(2 - y)?

To set up the double integral, first identify the region D bounded by the curves y = x(2 - x) and y = y(2 - y). Then determine their intersection points to find the limits for x and y. The integral is typically expressed as $\iint_D$ 7x dy dx or dy dx, depending on the order of integration.

What is the first step in applying the method to evaluate the integral of F(x,y) over the given region?

The first step is to find the intersection points of the curves y = x(2 - x) and y = y(2 - y) to define the limits of integration accurately.

How can I simplify the region bounded by y = x(2 - x) and y = y(2 - y)

for easier integration?

You can analyze and sketch the curves to understand their intersection points and shape. Often, converting the region into a suitable coordinate order—either x then y or y then x—helps simplify the limits and integration process.

Why is it helpful to apply substitution methods when calculating this integral?

Applying substitution can simplify complicated boundary equations, especially if the curves are quadratic, by transforming the region into a more manageable shape or coordinates, making the integration process easier.

How do I determine the limits of integration after finding the intersection points?

Once the intersection points are identified, you set the limits for x and y based on their respective bounds: for example, x from x_min to x_max, and y between the curves $y = x(2 - x)$ and $y = y(2 - y)$, depending on the chosen order of integration.

Are there any common pitfalls to watch out for when calculating this integral over a region bounded by these curves?

Yes, common pitfalls include incorrect determination of intersection points, misreading the curves as inequalities, and choosing an inappropriate order of integration. Careful sketching and verifying the limits can help avoid these errors.

Additional Resources

Calculating the Double Integral of $F(x,y) = 7x$ Over a Complex Region D

When tackling the problem of evaluating a double integral, especially over a region bounded by intricate curves, the key lies in understanding the geometry of the region and choosing an effective method of integration. Here, we examine the integral of the function F(x,y) = 7x over a region D bounded above by y = x(2 - x) and below by x = y(2 - y). This problem provides an excellent opportunity to explore advanced integration techniques, including variable transformations and region analysis, to simplify the computation process.

Understanding the Geometry of Region D

Before diving into the calculus, it’s crucial to visualize and understand the region D over which the integration occurs. The region is described by two bounding curves:

- Upper boundary: $y = x(2 - x)$
- Lower boundary: $x = y(2 - y)$

Let's analyze these curves individually.

Analyzing the Upper Boundary: $y = x(2 - x)$

This is a quadratic function in x. Rewriting:

$$
y = 2x - x^2
$$

which is a parabola opening downward with vertex at:

$$
x = \frac{-b}{2a} = \frac{-2}{-2} = 1
$$

and

$$
y_{\text{max}} = 2(1) - (1)^2 = 2 - 1 = 1
$$

Thus, the parabola reaches its maximum at point $(1, 1)$. The parabola intersects the x-axis when $y=0$:

$$
0 = 2x - x^2 \Rightarrow x(2 - x) = 0 \Rightarrow x=0, 2
$$

So, the parabola passes through points $(0, 0)$ and $(2, 0)$.

Analyzing the Lower Boundary: $x = y(2 - y)$

Similarly, this is a quadratic in y:

$$
x = 2y - y^2
$$

which is symmetric to the previous parabola but in the y-direction. Its vertex is at:

$$
y = \frac{-b}{2a} = \frac{-2}{-2} = 1
$$

with maximum x:

$$
x_{\text{max}} = 2(1) - (1)^2 = 2 - 1 = 1
$$

intersecting the y-axis when $x=0$:

$$
0 = 2y - y^2 \Rightarrow y=0, 2
$$

It passes through $(0,0)$ and $(0, 2)$.

Region D Sketch and Symmetry

From the analysis, both curves intersect at the origin:

$$
(0,0)
$$

and each reaches a maximum at $(1,1)$. The region D is bounded above by the parabola $y = x(2 - x)$ and below by $x = y(2 - y)$, which are symmetric with respect to the line $y=x$.

This symmetry hints that a substitution involving $u = x + y$ or $u = x - y$ could simplify the problem, but for the current problem, directly setting up the integral with proper bounds is an essential first step.

Setting Up the Double Integral

The goal is to compute:

$$
\iint_D 7x \, dA
$$

Given the region's description, the most straightforward approach is to express the bounds in terms of y for x, or vice versa.

Choosing the Order of Integration

Because the region is bounded between two curves, a natural choice is to integrate with respect to x first, then y:

- For a fixed y, determine the corresponding x-values between the two curves.

Alternatively, integrating with respect to y first could be considered, but analyzing the bounds suggests that integrating with respect to x first is more straightforward.

Expressing the Bounds in Terms of y

From the curves:

- The upper boundary: $y = x(2 - x) \Rightarrow y = 2x - x^2$

- The lower boundary: $x = y(2 - y)$

For a fixed y, find the corresponding x-values:

$$
x_{\text{upper}} = \text{from } y = 2x - x^2
$$

Rearranged:

$$
x^2 - 2x + y = 0
$$

which is quadratic in x:

$$
x^2 - 2x + y = 0
$$

Discriminant:

$$
\Delta_x = 4 - 4 y
$$

Real solutions exist when:

$$
4 - 4 y \geq 0 \Rightarrow y \leq 1
$$

Similarly, solving for x:

$$
x = \frac{2 \pm \sqrt{4 - 4 y}}{2} = 1 \pm \sqrt{1 - y}
$$

Since the parabola opens downward, the relevant x-values for a fixed y are:

$$
x \in [1 - \sqrt{1 - y},\, 1 + \sqrt{1 - y}]
$$

The lower boundary $x = y(2 - y)$ gives the minimal x at each y, which, by symmetry, matches the lower limit for the x-integral.

Defining the Integration Limits

Based on the above, the integral becomes:

\[

\int_{y=0}^{1} \int_{x = y(2 - y)}^{x_{\text{upper}}(y)} 7x \, dx \, dy

\]

where:

\[

x_{\text{upper}}(y) = 1 + \sqrt{1 - y}

\]

and

\[

x_{\text{lower}}(y) = y(2 - y)

\]

Noticing that the curves intersect at \(y=0 \) and \(y=1 \), these bounds are consistent.

Performing the Integration

Now, we can write the integral explicitly:

\[

\boxed{

\iint_D 7x \, dA = \int_{0}^{1} \left[\int_{x=y(2 - y)}^{x=1 + \sqrt{1 - y}} 7x \, dx \right] dy

}

\]

Inner Integral in \(x \)

Compute:

\[

I(y) = \int_{x=y(2 - y)}^{1 + \sqrt{1 - y}} 7x \, dx

\]

The antiderivative:

\[

\frac{7x^2}{2}

\]

Evaluate at the bounds:

\[

I(y) = \frac{7}{2} \left[(1 + \sqrt{1 - y})^2 - (y(2 - y))^2 \right]

\]

Simplifying the Inner Integral

Compute each part:

1. $(1 + \sqrt{1 - y})^2$:

$$
= 1 + 2 \sqrt{1 - y} + (1 - y) = 2 + 2 \sqrt{1 - y} - y
$$

2. $y(2 - y)$:

$$
= 2y - y^2
$$

3. $(y(2 - y))^2$:

$$
= (2y - y^2)^2 = 4 y^2 - 4 y^3 + y^4
$$

Putting it all together:

$$
I(y) = \frac{7}{2} \left[2 + 2 \sqrt{1 - y} - y - (4 y^2 - 4 y^3 + y^4) \right]
$$

which simplifies to:

\[

I(y) = \frac{7}{2} \left[2 + 2 \sqrt{1 - y} - y - 4 y^2 + 4 y^3 - y^4 \right]

\]

Outer Integral in \(y \)

Now, the total integral:

\[

\boxed{

\int_0^1 I(y) \, dy = \frac{7}{2} \int_0^1 \left[2 + 2 \sqrt{1 - y} - y - 4 y^2 + 4 y^3 - y^4 \right] dy

}

\]

Break into separate integrals:

\[

\frac{7}{2} \left[\int_0^1 2 \

Calculate The Integral Of F X Y 7x Over The Region D Bounded Above By Y X 2 X And Below By X Y 2 Y Hint Apply

Calculate The Integral Of F X Y 7x Over The Region D Bounded Above By Y X 2 X And Below By X Y 2 Y Hint Apply

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