Determine the equation of the parabola that opens up, has vertex(−1,−4), and a focal diameter of 28

Determine the equation of the parabola that opens up, has vertex (−1,−4), and a focal diameter of 28 is a common problem in conic sections that combines understanding of parabola properties, standard forms, and geometric relationships. Whether you're a student working on a math assignment or a math enthusiast exploring conic sections, understanding how to derive a parabola's equation from given parameters is essential. This article will guide you through the step-by-step process to determine the specific parabola that fits these criteria, providing clarity on key concepts like vertex form, focus, directrix, and focal diameter.

Understanding the Basic Properties of Parabolas

Before diving into the calculations, it’s important to grasp the fundamental properties of parabolas, especially those relevant to this problem.

What is a parabola?

A parabola is a symmetric, U-shaped curve that can open upward, downward, left, or right. It is defined as the set of all points equidistant from a fixed point called the focus and a fixed line called the directrix.

The vertex

The vertex of a parabola is its highest or lowest point (for upward or downward opening parabolas) and serves as a key reference point in its equation.

Focus and directrix

The focus is a fixed point inside the parabola, and the directrix is a line outside it. The parabola is the locus of points equidistant from the focus and the directrix.

Focal diameter

The focal diameter (also called the latus rectum) is the length of the chord passing through the focus and perpendicular to the axis of symmetry. It measures how wide the parabola is at the level of the focus.

Given Parameters and Their Significance

The problem provides three pieces of information:

- Vertex at $(-1,-4)$
- Parabola opens upward
- Focal diameter of 28

Let's analyze what each of these means for our parabola:

Vertex at $(-1,-4)$

This point indicates the parabola’s minimum point since it opens upward. The vertex form of a parabola with a vertex at (h, k) is typically written as:
$$ y = a(x - h)^2 + k $$

Parabola opening upward

This suggests the parabola's axis of symmetry is vertical, and the parabola's focus lies above the vertex.

Focal diameter of 28

This length relates to the parabola's width at the focus and is connected to the parameter p (distance from the vertex to the focus).

Understanding how these parameters interrelate is crucial to deriving the equation.

Step-by-Step Process to Find the Equation

To determine the parabola's equation, follow these logical steps:

Step 1: Write the vertex form of the parabola

Since the parabola opens upward with vertex at $(-1,-4)$, the general form is:
$$ y = a(x + 1)^2 - 4 $$

Our goal is to find the value of a.

Step 2: Connect focal diameter to the parabola's parameters

The focal diameter D is related to the parameter p (the distance from vertex to focus) by:
$$D = 4p$$
Given $D = 28$, then:
$$4p = 28$$
$$p = 7$$

This means the focus is 7 units above the vertex, since the parabola opens upward.

Step 3: Determine the focus point

The focus's y-coordinate is:
$$y_{focus} = y_{vertex} + p = -4 + 7 = 3$$
The focus's x-coordinate remains the same as the vertex's x-coordinate (since the axis of symmetry is vertical):
$$x_{focus} = -1$$
Thus, focus point:
$$F(-1, 3)$$

Step 4: Find a using the focus and the parabola's equation

The focus lies on the parabola, so substitute $x = -1$ and $y = 3$ into the parabola's equation:
$$y = a(x + 1)^2 - 4$$
$$3 = a(0)^2 - 4$$
$$3 = -4$$
This does not hold unless a is undefined, indicating a need to revisit the approach.

Instead, recognize that for a parabola opening upward with vertex at (h, k), the standard form can be written as:
$$(x - h)^2 = 4p(y - k)$$
where p is the distance from the vertex to the focus (positive for upward opening).

Using this form:

- $h = -1$
- $k = -4$
- $p = 7$

Plug into the standard form:
$$(x + 1)^2 = 4 \times 7 \times (y + 4)$$
$$(x + 1)^2 = 28(y + 4)$$

This is the equation of the parabola.

Final Equation and Summary

Based on the above steps, the equation of the parabola that opens upward with a vertex at $(-1,-4)$ and a focal diameter of 28 is:

$$\boxed{(x + 1)^2 = 28(y + 4)}$$

This form clearly expresses the parabola's geometric properties and provides a straightforward equation for graphing or further analysis.

Additional Insights and Applications

Understanding how to derive the parabola's equation from given parameters has many practical applications:

- **Graphing parabolas:** Using the vertex form simplifies plotting points and sketching the curve accurately.
- **Physics problems:** Parabolas describe projectile trajectories where vertex and focus relate to maximum height and focal points.
- **Engineering designs:** Parabolic shapes are used in satellite dishes and headlights for focusing signals or light.

Summary of Key Concepts

- The vertex form of a parabola opens upward/downward based on the sign of the coefficient.
- The focal diameter relates to the parameter p via $D = 4p$.
- The standard form $(x - h)^2 = 4p(y - k)$ is particularly useful for parabolas opening vertically.
- Coordinate substitution helps verify the focus and ensure the derived equation matches given parameters.

By mastering these steps, you can confidently determine the equation of any parabola given specific properties like vertex, focal diameter, and direction of opening.

In conclusion, the parabola with vertex $(-1,-4)$, opening upward, and focal diameter 28 is described by the equation:

$$\boxed{(x + 1)^2 = 28(y + 4)}$$

This comprehensive approach not only solves the problem but also deepens your understanding of parabola properties and equations, enhancing your skills in conic sections analysis.

Frequently Asked Questions

How do you find the equation of a parabola that opens upward with a given vertex and focal diameter?

To find the equation, use the vertex form $y = a(x - h)^2 + k$, where (h, k) is the vertex. Determine 'a' using the focal diameter, which relates to the length of the latus rectum, then write the equation accordingly.

What is the significance of the focal diameter in determining a parabola's equation?

The focal diameter is the length of the latus rectum, the chord passing through the focus and perpendicular to the axis. It helps find the value of 'a' in the parabola's equation, since the focal diameter equals 4a.

Given the vertex (-1, -4) and focal diameter 28, how do we find the value of 'a' in the parabola's equation?

Since the focal diameter equals 4a, divide 28 by 4 to get $a = 7$. This 'a' value is used in the vertex form to write the parabola's equation.

What is the complete equation of the parabola that opens upward with vertex (-1, -4) and focal diameter 28?

Using the vertex (-1, -4) and $a = 7$, the equation is $y = 7(x + 1)^2 - 4$.

How can you verify that the parabola's equation is correct based on the given focal diameter and vertex?

Calculate the focus and directrix from the equation and check that the distance between the focus and the directrix matches the focal diameter of 28, ensuring the equation correctly models the parabola.

What is the focus of the parabola with vertex (-1, -4) and focal diameter 28?

The focus is located at (-1, -4 + a), which is (-1, -4 + 7) = (-1, 3).

Additional Resources

Determine the equation of the parabola that opens up, has vertex (−1,−4), and a focal diameter of 28

Understanding the shape and position of parabolas is fundamental in various fields, from physics to engineering. Whether designing satellite dishes, reflecting telescopes, or analyzing projectile motion, the mathematical representation of a parabola provides essential insights. In this article, we will explore how to determine the equation of a specific parabola—one that opens upward, with a vertex at (−1,−4), and a focal diameter of 28. This detailed guide aims to demystify the process, offering clear explanations and step-by-step procedures suitable for students and enthusiasts alike.

Fundamentals of Parabolas: Key Concepts and Terminology

Before diving into the specifics of this problem, it's crucial to understand some fundamental properties and terminology related to parabolas.

What is a Parabola?

A parabola is a symmetric, U-shaped curve that can be represented mathematically as a quadratic function. It has several defining features, such as:

- Vertex: The highest or lowest point of the parabola, depending on its orientation.
- Focus: A fixed point located inside the parabola, used to define the parabola's shape.
- Directrix: A fixed line outside the parabola, perpendicular to the axis of symmetry, used in the parabola's geometric definition.
- Axis of symmetry: A vertical or horizontal line passing through the vertex and focus, dividing the parabola into mirror images.

Standard Forms of a Parabola

Depending on its orientation, a parabola can be expressed in different forms:

- Vertical parabola (opening up or down):

$$
(x - h)^2 = 4p(y - k)
$$

- Horizontal parabola (opening left or right):

$$
(y - k)^2 = 4p(x - h)
$$

Where:

- (h, k) is the vertex.
- p is the distance from the vertex to the focus (and to the directrix).

Focal Diameter (Latus Rectum)

The focal diameter, also called the latus rectum, is the length of the chord passing through the focus and perpendicular to the axis of symmetry. It measures how "wide" the parabola is at the focus.

- For a parabola $(x - h)^2 = 4p(y - k)$, the focal diameter is:

$$
\text{Focal diameter} = |4p|
$$

- The focal diameter is always twice the focal distance from the vertex to the focus:

$$
\text{Focal diameter} = 2|p|
$$

Understanding these concepts helps us connect the given information to the parabola's equation.

Step 1: Analyzing the Given Data

The problem states:

- The parabola opens upward.
- The vertex is at $(-1, -4)$.

- The focal diameter is 28.

Let's interpret each piece:

Orientation: Opens Upward

Since the parabola opens upward, its standard form will be:

$$
(x - h)^2 = 4p(y - k)
$$

with $p > 0$.

Vertex Coordinates

$$
(h, k) = (-1, -4)
$$

Focal Diameter

$$
\text{Focal diameter} = 28
$$

Given that the focal diameter equals $|4p|$:

$$
|4p| = 28
$$

Since the parabola opens upward, $p > 0$, so:

$$
4p = 28
$$

which yields:

$$
p = 7
$$

This value of p indicates the focus is located 7 units above the vertex.

Step 2: Determining the Focus and Directrix

Knowing the vertex and p, we can locate the focus and directrix, which are essential in understanding the parabola's shape.

Focus Coordinates

The focus lies along the axis of symmetry, which, for a parabola opening upward, is a vertical line passing through the vertex.

- The focus is p units above the vertex:

$$
\text{Focus} = (h, k + p) = (-1, -4 + 7) = (-1, 3)
$$

Directrix Equation

The directrix is a horizontal line p units below the vertex:

$$
\text{Directrix}: y = k - p = -4 - 7 = -11
$$

These points and lines align with the geometric definition of the parabola, where any point on the parabola is equidistant from the focus and the directrix.

Step 3: Formulating the Equation of the Parabola

With the vertex $(-1, -4)$, $p = 7$, and the parabola opening upward, the standard form is:

$$
(x - h)^2 = 4p(y - k)
$$

Plugging in the known values:

$$
(x + 1)^2 = 4 \times 7 \times (y + 4)
$$

Simplifying:

$$
(x + 1)^2 = 28(y + 4)
$$

This quadratic equation fully describes the parabola's shape and position.

Step 4: Verifying and Visualizing the Parabola

It's essential to verify that the derived equation aligns with the given parameters and understand how the parabola looks.

Verification of Focal Diameter

Recall that the focal diameter is 28, which matches the $4p$ in the equation. Since $4p = 28$, the calculations are consistent.

Key points on the parabola:

- Vertex: $(-1, -4)$
- Focus: $(-1, 3)$
- Directrix: $y = -11$
- Shape: U-shaped, opening upward, symmetric about the line $x = -1$.

Graphical interpretation:

Plotting these points and lines provides a visual confirmation of the parabola's shape, ensuring the equation accurately models the specified parameters.

Additional Considerations and Applications

Understanding how to derive the parabola's equation from given features is invaluable across multiple disciplines.

Engineering and Design

Designing satellite dishes and reflecting telescopes relies on precise parabolic equations to ensure signals or light are directed accurately.

Physics and Motion

Projectile motion often models the path of an object as a parabola, where knowledge of vertex and focal length helps predict landing points and trajectories.

Mathematics Education

Solving such problems enhances problem-solving skills, reinforcing understanding of conic sections, coordinate geometry, and algebra.

Summary and Final Remarks

To determine the equation of a parabola given specific features, the process involves:

1. Recognizing the parabola's orientation.
2. Using the vertex to establish the standard form.
3. Connecting the focal diameter to the focal length p.
4. Calculating the focus and directrix positions.
5. Writing the explicit quadratic equation.

In this case, the parabola that opens upward with vertex at $(-1, -4)$ and a focal diameter of 28 has the equation:

$$
\boxed{
(x + 1)^2 = 28(y + 4)
}
$$

This equation encapsulates the parabola's geometric properties and provides a foundation for further analysis, whether for academic purposes or practical applications.

Final thought: Mastering the process of deriving the parabola's equation from its defining features enhances not only mathematical competence but also provides a toolkit for solving real-world problems involving conic sections.

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