Determine The Unit Vector Normal To The Plane When A And B Are Equal To (7)i (8)j (2)k And 9i 4j 5k Respectively.

Determine The Unit Vector Normal To The Plane When A And B Are Equal To (7)i (8)j (2)k And 9i 4j 5k Respectively.

Understanding how to find the unit vector normal to a plane is a fundamental concept in vector calculus and geometry. When given two vectors lying on the same plane, such as vectors A and B, it becomes essential to determine the vector that is perpendicular (normal) to that plane. This process involves calculating the cross product of the two vectors and then normalizing the result to obtain the unit vector. This article provides a comprehensive guide to determining the unit vector normal to a plane when vectors A and B are specified as (7)i + 8j + 2k and 9i + 4j + 5k respectively.

Introduction to Vectors and Planes

Before delving into the calculation process, it is crucial to understand the basic concepts related to vectors and planes:

- Vectors: Quantities having both magnitude and direction, typically represented in component form, such as $A = a_1i + a_2j + a_3k$.
- Plane: A flat, two-dimensional surface extending infinitely in all directions within three-dimensional space.
- Normal Vector: A vector perpendicular to the surface of the plane, which is vital in various applications including physics, engineering, and computer graphics.

Given Data and Objective

The problem provides:

- Vector A = 7i + 8j + 2k
- Vector B = 9i + 4j + 5k

Objective: To determine the unit vector normal to the plane containing vectors A and B.

Step-by-Step Methodology

The process involves several key steps:

1. Identify the vectors: Clearly write down the vectors A and B in component form.
2. Calculate the cross product: Find the vector that is perpendicular to both A and B.
3. Determine the magnitude of the cross product: To normalize the vector.
4. Normalize the cross product: Divide the cross product vector by its magnitude to get the unit vector.

Step 1: Expressing the Vectors

Given:

- A = 7i + 8j + 2k → A = (7, 8, 2)
- B = 9i + 4j + 5k → B = (9, 4, 5)

This component form simplifies calculations in the subsequent steps.

Step 2: Calculating the Cross Product

The cross product of vectors A = (a_1, a_2, a_3) and B = (b_1, b_2, b_3) is given by:

$$
\mathbf{A} \times \mathbf{B} =
\begin{vmatrix}
i & j & k \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3 \\
\end{vmatrix}
$$

Expanding the determinant:

$$
\mathbf{A} \times \mathbf{B} = i(a_2b_3 - a_3b_2) - j(a_1b_3 - a_3b_1) + k(a_1b_2 - a_2b_1)
$$

Plugging in the values:

- $a_1 = 7$, $a_2 = 8$, $a_3 = 2$

- $b_1 = 9, b_2 = 4, b_3 = 5$

Calculations:

- $a_2b_3 - a_3b_2 = 8 \times 5 - 2 \times 4 = 40 - 8 = 32$
- $a_1b_3 - a_3b_1 = 7 \times 5 - 2 \times 9 = 35 - 18 = 17$
- $a_1b_2 - a_2b_1 = 7 \times 4 - 8 \times 9 = 28 - 72 = -44$

Thus, the cross product vector:

$$
\mathbf{A} \times \mathbf{B} = 32i - 17j - 44k
$$

or in component form:

$$
\boxed{\mathbf{N} = (32, -17, -44)}
$$

This vector is perpendicular to the plane containing vectors A and B.

Step 3: Calculating the Magnitude of the Cross Product

The magnitude (length) of vector N = (32, -17, -44):

$$
|\mathbf{N}| = \sqrt{(32)^2 + (-17)^2 + (-44)^2}
$$

Calculations:

- $32^2 = 1024$
- $-17)^2 = 289$
- $-44)^2 = 1936$

Sum:

$$
1024 + 289 + 1936 = 3249
$$

Magnitude:

$$
|\mathbf{N}| = \sqrt{3249} \approx 57.02
$$

Step 4: Normalizing the Vector to Obtain the Unit Vector

To find the unit vector $\hat{\mathbf{N}}$, divide each component of N by its magnitude:

$$
\hat{\mathbf{N}} = \frac{1}{|\mathbf{N}|} \times \mathbf{N}
$$

Calculations:

- $\frac{32}{57.02} \approx 0.561$
- $\frac{-17}{57.02} \approx -0.298$
- $\frac{-44}{57.02} \approx -0.771$

Thus, the unit vector normal to the plane is approximately:

$$
\boxed{
\hat{\mathbf{N}} \approx (0.561, -0.298, -0.771)
}
$$

Implications and Applications

Understanding how to determine the unit normal vector to a plane is essential in various fields:

- Physics: Calculating forces, torques, and electric/magnetic fields perpendicular to surfaces.
- Engineering: Designing structures, analyzing stress distributions, and surface orientations.
- Computer Graphics: Rendering realistic images by calculating surface normals for lighting and shading.
- Mathematics and Geometry: Solving problems involving planes, angles, and intersections.

Additional Tips for Accurate Calculation

- Always double-check the component values before computing the cross product.

- Use precise calculations for the magnitude to avoid rounding errors.
- Remember that the direction of the normal vector can be reversed; both are valid normals.
- When working with large or complex vectors, consider using computational tools or calculators to minimize errors.

Summary

In conclusion, to determine the unit vector normal to the plane when given vectors A and B:

1. Calculate the cross product of vectors A and B to find a normal vector.
2. Find the magnitude of this normal vector.
3. Normalize the vector by dividing each component by its magnitude to obtain the unit normal vector.

For the given vectors, the unit normal vector to the plane is approximately (0.561, -0.298, -0.771). This vector is crucial in understanding the orientation of the plane and is widely applicable across scientific and engineering disciplines.

Conclusion

Mastering the process of finding the unit vector normal to a plane is fundamental in vector calculus and related fields. Whether you're solving geometric problems, analyzing physical systems, or creating computer graphics, this skill provides a strong foundation for understanding spatial relationships and surface orientations. Remember to carefully perform each step, verify your calculations, and appreciate the significance of the normal vector in various applications.

Keywords for SEO Optimization: vector calculus, unit vector normal to a plane, cross product calculation, normal vector of a plane, vector mathematics, plane orientation, physics and engineering, 3D geometry, surface normal, vector operations

Frequently Asked Questions

How do you find the unit vector normal to a plane given

two vectors A and B?

To find the unit vector normal to the plane, first compute the cross product of vectors A and B to get the normal vector, then divide this vector by its magnitude to normalize it.

What are the components of vector A in the given problem?

Vector A components are (7, 8, 2) corresponding to 7i + 8j + 2k.

What are the components of vector B in the given problem?

Vector B components are (9, 4, 5) corresponding to 9i + 4j + 5k.

How do you compute the cross product of vectors A and B?

The cross product is calculated using the determinant of a matrix formed by the unit vectors i, j, k and the components of A and B:
|i j k|
|7 8 2|
|9 4 5|.

What is the cross product of vectors A and B in this problem?

The cross product is calculated as (85 - 24)i - (75 - 29)j + (74 - 89)k, which simplifies to (40 - 8)i - (35 - 18)j + (28 - 72)k, resulting in 32i - 17j - 44k.

How do you find the magnitude of the normal vector obtained from the cross product?

The magnitude is found using the formula: $\sqrt{32^2 + (-17)^2 + (-44)^2} = \sqrt{1024 + 289 + 1936} = \sqrt{3249}$.

What is the magnitude of the vector 32i - 17j - 44k?

The magnitude is $\sqrt{1024 + 289 + 1936} = \sqrt{3249} \approx 57.02$.

What is the unit vector normal to the plane in this problem?

The unit vector normal to the plane is obtained by dividing each component of the cross product by its magnitude: (32/57.02)i + (-17/57.02)j + (-44/57.02)k, approximately (0.56)i - (0.30)j - (0.77)k.

Why is it important to normalize the normal vector when determining the unit vector normal to the plane?

Normalizing ensures the vector has a magnitude of 1, making it a true 'unit' vector that only indicates direction, which is essential for many applications such as calculating angles or reflections.

Additional Resources

Determine The Unit Vector Normal To The Plane When A And B Are Equal To (7)i + 8j + 2k And 9i + 4j + 5k Respectively

Understanding the geometric and algebraic principles behind vectors and planes is fundamental in vector calculus and physics. When tasked with determining the unit vector normal to a plane defined by two vectors, A and B, it is essential to leverage the properties of vector operations, particularly the cross product. This comprehensive guide walks through each step of this process, exploring the core concepts, calculations, and interpretations involved.

Introduction to Vectors and Planes

Before delving into the specifics of the problem, it is crucial to review some foundational concepts.

What Are Vectors?

- Vectors are quantities that possess both magnitude and direction.
- They are represented in component form, typically in three dimensions as (x, y, z), or in unit vector notation such as i, j, k.

The Concept of a Plane in Vector Form

- A plane in 3D space can often be described using two non-parallel vectors originating from a common point (often the origin).
- These vectors, A and B, can be interpreted as spanning the plane, meaning every point on the plane can be expressed as a linear combination of A and B.

Normal Vector to a Plane

- A normal vector is a vector perpendicular to the plane.
- The unit normal vector is the normalized version of this vector, having a length of 1.
- It is vital in various applications, including calculating angles between planes, reflections, and determining equations of planes.

Given Data and Problem Restatement

Let's restate the problem with explicit clarity:

> Given two vectors:
> - \(\mathbf{A} = 7\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\)
> - \(\mathbf{B} = 9\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}\)
>
> Determine the unit vector normal to the plane spanned by these vectors.

Step 1: Understanding the Approach

To find the unit vector normal to the plane:

1. Identify the normal vector as the cross product of vectors A and B.
2. Normalize this vector to make it a unit vector.

This approach hinges on the fact that:

> The cross product of two vectors yields a vector perpendicular to both, hence normal to the plane they define.

Step 2: Computing the Cross Product of Vectors A and B

2.1 Cross Product Formula

Given vectors:

\[
\mathbf{A} = a_1 \mathbf{i} + a_2 \mathbf{j} + a_3 \mathbf{k}
\]
\[
\mathbf{B} = b_1 \mathbf{i} + b_2 \mathbf{j} + b_3 \mathbf{k}
\]

The cross product \(\mathbf{A} \times \mathbf{B}\) is given by the determinant:

\[
\mathbf{A} \times \mathbf{B} =
\begin{vmatrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_1 & a_2 & a_3 \\
b_1 & b_2 & b_3
\end{vmatrix}
\]

Expanding this determinant:

\[
\mathbf{A} \times \mathbf{B} = \mathbf{i}(a_2 b_3 - a_3 b_2) - \mathbf{j}(a_1 b_3 - a_3 b_1) + \mathbf{k}(a_1 b_2 - a_2 b_1)
\]

2.2 Substituting the Values

Given:

- \(\mathbf{A} = (7, 8, 2)\)
- \(\mathbf{B} = (9, 4, 5)\)

Calculating each component:

- i-component:

\[
a_2 b_3 - a_3 b_2 = (8)(5) - (2)(4) = 40 - 8 = 32
\]

- j-component:

\[
a_1 b_3 - a_3 b_1 = (7)(5) - (2)(9) = 35 - 18 = 17
\]

- k-component:

\[
a_1 b_2 - a_2 b_1 = (7)(4) - (8)(9) = 28 - 72 = -44
\]

The cross product:

\[
\mathbf{A} \times \mathbf{B} = 32 \mathbf{i} - 17 \mathbf{j} - 44 \mathbf{k}
\]

In component form:

\[
\mathbf{N} = (32, -17, -44)
\]

This vector $\mathbf{N}$ is perpendicular to the plane spanned by A and B.

Step 3: Normalizing the Normal Vector to Obtain the Unit Vector

3.1 Calculating the Magnitude of $\mathbf{N}$

The magnitude (or length) of $\mathbf{N}$ is:

$$
|\mathbf{N}| = \sqrt{(32)^2 + (-17)^2 + (-44)^2}
$$

Calculations:

- $32^2 = 1024$
- $(-17)^2 = 289$
- $(-44)^2 = 1936$

Sum:

$$
1024 + 289 + 1936 = 3249
$$

Therefore:

$$
|\mathbf{N}| = \sqrt{3249}
$$

Using a calculator or approximation:

$$
|\mathbf{N}| \approx 57.02
$$

3.2 Deriving the Unit Normal Vector

The unit vector in the direction of $\mathbf{N}$ is obtained by dividing each component by $|\mathbf{N}|$:

$$
\hat{\mathbf{n}} = \frac{\mathbf{N}}{|\mathbf{N}|} = \left(\frac{32}{57.02}, \frac{-17}{57.02}, \frac{-44}{57.02} \right)
$$

Approximate values:

- $\frac{32}{57.02} \approx 0.560$
- $\frac{-17}{57.02} \approx -0.298$
- $\frac{-44}{57.02} \approx -0.770$

3.3 Final Expression for the Unit Vector

$$
\boxed{
\hat{\mathbf{n}} \approx 0.560\mathbf{i} - 0.298\mathbf{j} - 0.770\mathbf{k}
}
$$

Additional Insights and Considerations

Orientation of the Normal Vector

- The normal vector's direction is not unique; multiplying it by -1 yields another valid normal vector pointing in the opposite direction.
- The choice between the two depends on the context, such as the orientation of the plane or application-specific conventions.

Significance in Geometric and Physical Contexts

- This normal vector is instrumental in deriving the equation of the plane.
- It plays a critical role in calculating angles between planes or vectors, reflection phenomena, and in physics, such as torque or force analysis.

Applications and Further Calculations

1. Equation of the Plane

Given a point P_0 on the plane and the normal vector $\hat{\mathbf{n}}$, the plane's equation can be expressed as:

$$
\hat{\mathbf{n}} \cdot (\mathbf{r} - \mathbf{r}_0) = 0
$$

Where:

- \(\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\) (any point on the plane)
- \(\mathbf{r}_0 = x_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k}\) (a specific point on the plane)

---

2. Angle Between the Plane and Other Vectors

Using the normal vector, one can determine the angle \(\theta\) between the plane and other vectors or planes by applying the dot product formula:

\[
\cos \theta = \frac{\mathbf{N} \cdot \mathbf{V}}{|\mathbf{N}| |\mathbf{V}|}
\]

Where \(\mathbf{V}\) is the vector in question.

Conclusion and Summary

In summary, the process of determining the unit vector normal to the plane spanned by vectors A and B involves:

- Computing the cross product of A and B to find a perpendicular vector.
- Calculating the magnitude of this vector.
- Normalizing the vector to convert it into a unit vector.

For the given vectors:

\[
\mathbf{A} = 7\

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