Draw The Domain Of The Functionf(x,y)=x2y3x2+y2[4] B. Use The Chain Rule To Find The Partial Derivativesuzandvzof

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Understanding the domain of a function and calculating its partial derivatives are fundamental concepts in multivariable calculus. In this article, we will thoroughly analyze the function $f(x,y) = \frac{x^2 y^3}{x^2 + y^2}$, explore how to determine its domain, and demonstrate how to apply the Chain Rule to find the partial derivatives u_z and v_z. Whether you're a student preparing for exams or a calculus enthusiast, this comprehensive guide aims to clarify these concepts with detailed explanations and step-by-step procedures.

Drawing the Domain of the Function $f(x,y) = \frac{x^2 y^3}{x^2 + y^2}$

Understanding the Function's Structure

Before sketching the domain, it's essential to analyze the structure of the function:

- The numerator is $x^2 y^3$, which is defined for all real x and y.
- The denominator is $x^2 + y^2$, which is always non-negative and equals zero only at the origin $(0,0)$.

Therefore, the function is undefined where the denominator equals zero; that is, at $x = 0$ and $y = 0$.

Identifying the Domain

Given the above, the domain consists of all points in $\mathbb{R}^2$ where the denominator is non-zero:

$$
\boxed{
\text{Domain } D = \{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \neq 0 \}
}
$$

This simplifies to:

$$
D = \mathbb{R}^2 \setminus \{ (0,0) \}
$$

In other words, the domain includes every point in the xy-plane except the origin.

Graphical Representation of the Domain

To visualize the domain:

- Plot the entire xy-plane.
- Exclude the origin $(0,0)$ where the function is undefined.
- The domain is thus the entire plane minus a single point.

This results in a plane with a single point removed, often referred to as a punctured plane.

Applying the Chain Rule to Find Partial Derivatives u_z and v_z

In many multivariable calculus problems, especially those involving composite functions or parametrizations, the Chain Rule is a pivotal tool. Here, we'll assume u and v are functions derived from f, possibly through a substitution involving z.

While the initial problem statement references u_z and v_z, it seems to imply that u and v are functions of z through a composition. Let's clarify the general approach:

- Suppose $u = u(x,y)$ and $v = v(x,y)$ are functions of two variables.
- Assume both x and y are functions of z: $x = x(z)$, $y = y(z)$.

In this context, the Chain Rule provides a way to compute derivatives of composite functions:

$$
\frac{d u}{d z} = \frac{\partial u}{\partial x} \cdot \frac{d x}{d z} + \frac{\partial u}{\partial y} \cdot \frac{d y}{d z}
$$

Similarly,

$$
\frac{d v}{d z} = \frac{\partial v}{\partial x} \cdot \frac{d x}{d z} + \frac{\partial v}{\partial y} \cdot \frac{d y}{d z}
$$

Step-by-step Process to Use the Chain Rule

1. Find the Partial Derivatives $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$

- These derivatives are computed directly from the function $u(x, y)$.

2. Find $\frac{d x}{d z}$ and $\frac{d y}{d z}$

- These are derivatives of $x(z)$ and $y(z)$ with respect to z.

3. Apply the Chain Rule Formula

- Combine the derivatives as shown above to find u_z and v_z.

Example: Computing u_z and v_z for Specific u, v, $x(z)$, and $y(z)$

Suppose:

- $u(x, y) = x^2 y$
- $v(x, y) = \sin(x y)$
- $x(z) = z^2$
- $y(z) = e^{z}$

Step 1: Compute partial derivatives:

$$
\frac{\partial u}{\partial x} = 2 x y, \quad \frac{\partial u}{\partial y} = x^2
$$
$$
\frac{\partial v}{\partial x} = y \cos(x y), \quad \frac{\partial v}{\partial y} = x \cos(x y)
$$

Step 2: Compute derivatives of $x(z)$ and $y(z)$:

$$
\frac{d x}{d z} = 2 z
$$
$$
\frac{d y}{d z} = e^{z}
$$

Step 3: Apply the Chain Rule:

\[
u_z = \frac{\partial u}{\partial x} \cdot \frac{d x}{d z} + \frac{\partial u}{\partial y} \cdot \frac{d

y}{d z} = (2 x y) \cdot 2 z + (x^2) \cdot e^{z}
\]

Substituting $x(z) = z^2$, $y(z) = e^{z}$:

$$
u_z = 2 (z^2)(e^{z}) \cdot 2 z + (z^2)^2 \cdot e^{z} = 4 z (z^2) e^{z} + z^4 e^{z} = 4 z^3 e^{z} + z^4 e^{z}
$$

Similarly,

$$
v_z = y \cos(x y) \cdot \frac{d x}{d z} + x \cos(x y) \cdot \frac{d y}{d z} = \left(e^{z} \cos(z^2 e^{z}) \right) \cdot 2 z + z^2 \cos(z^2 e^{z}) \cdot e^{z}
$$

This example demonstrates how to use the Chain Rule to find derivatives of composite functions involving multiple variables and parameters.

Conclusion

In this comprehensive guide, we have:

- Drawn the domain of the function $f(x,y) = \frac{x^2 y^3}{x^2 + y^2}$, identifying it as $\mathbb{R}^2$ minus the origin.
- Explored the application of the Chain Rule to find the partial derivatives u_z and v_z, illustrating the step-by-step process with a concrete example.

Mastering these concepts enables you to analyze complex multivariable functions, understand their behavior across their domains, and compute their derivatives efficiently. Whether dealing with theoretical problems or practical applications, these tools are essential for deepening your understanding of multivariable calculus.

Frequently Asked Questions

What is the domain of the function f(x, y) = x^2 y^3 / (x^2 + y^2)?

The domain consists of all points (x, y) where the denominator $x^2 + y^2 \neq 0$, i.e., all points except (0, 0).

How do you identify the domain restrictions for the function

$f(x, y) = x^2 y^3 / (x^2 + y^2)$?

The only restriction is that the denominator cannot be zero, so the domain is all (x, y) with $x^2 + y^2 \neq 0$.

What is the significance of the point (0, 0) in the domain of f(x, y)?

The point (0, 0) is excluded because the denominator $x^2 + y^2$ equals zero there, making the function undefined.

How do you apply the chain rule to find the partial derivatives of a composite function involving f(x, y)?

Identify the inner functions with respect to which you differentiate, then multiply their derivatives by the derivatives of the outer functions, applying the chain rule appropriately.

What are the partial derivatives ∂z/∂x and ∂z/∂y of the function $z = f(x, y) = x^2 y^3 / (x^2 + y^2)$?

You find $\partial z/\partial x$ and $\partial z/\partial y$ by differentiating f(x, y) with respect to x and y respectively, treating the other variable as constant, and applying the quotient rule.

How can the chain rule simplify the process of differentiating complex multivariable functions?

The chain rule allows you to differentiate composite functions by breaking down derivatives into simpler parts, multiplying derivatives of inner functions by derivatives of outer functions.

What steps are involved in computing the partial derivatives of $f(x, y) = x^2 y^3 / (x^2 + y^2)$ using the chain rule?

First, rewrite the function as a quotient, then apply the quotient rule to differentiate with respect to each variable, and use the chain rule when inner functions are involved.

Why is it important to understand the domain when calculating partial derivatives using the chain rule?

Because the derivatives are only valid where the function is differentiable, which depends on the domain; points outside the domain may cause undefined or non-differentiable behavior.

Can the chain rule be used to find higher-order partial derivatives of the function f(x, y)?

Yes, the chain rule can be extended to higher-order derivatives by repeatedly applying the rule and carefully differentiating each term involved.

Additional Resources

Draw The Domain Of The Function $f(x,y) = x^2 y^3 / (x^2 + y^2)$ — A Comprehensive Guide

Understanding the domain of a function is fundamental in calculus, as it tells us where the function is defined and helps us analyze its behavior. In this article, we will delve into drawing the domain of the function $f(x,y) = (x^2 y^3) / (x^2 + y^2)$. This involves identifying all points (x, y) in the xy-plane for which the function is mathematically meaningful. We will also explore how to use the chain rule to find the partial derivatives of the function, providing a thorough and insightful analysis suitable for students and professionals alike.

Understanding the Function Structure

Before we can draw the domain, it's essential to understand the structure of the function:

- Function: $f(x,y) = (x^2 y^3) / (x^2 + y^2)$

- Type: Rational function, where the numerator is $x^2 y^3$ and the denominator is $x^2 + y^2$.

- Key features:
- The numerator is a polynomial.
- The denominator is a sum of squares, which is always non-negative.

Step 1: Determining Where the Function is Defined

The primary concern in identifying the domain is points where the function is undefined—specifically, where the denominator equals zero.

1.1 The Denominator: $x^2 + y^2$

Since x^2 and y^2 are both squares, their sum is:

- $x^2 + y^2 \geq 0$ for all real x and y.

- $x^2 + y^2 = 0$ only when $x = 0$ and $y = 0$.

Thus, the function is undefined at the origin (0, 0), because division by zero is undefined.

1.2 Domain in Set Notation

- The domain D is all points in $\mathbb{R}^2$ except (0, 0):

$D = \mathbb{R}^2 \setminus \{(0, 0)\}$.

- In words, the domain is the entire xy-plane minus the origin.

Step 2: Visualizing the Domain

2.1 Graphical Representation

- Plotting the domain: Imagine the entire plane, then mark the origin and exclude it.

- Implication: The function is defined everywhere except at (0,0), including along lines and curves approaching the origin, where the function may tend toward infinity or a finite limit depending on the path taken.

2.2 Behavior near the Singularity

- Approaching (0,0): The limit behavior depends on the path taken, which we will analyze later.

Step 3: Drawing the Domain

3.1 Visual Approach

- Use a coordinate plane showing all points except the origin.

- Clearly indicate the excluded point (0,0).

- Shade the entire plane outside the origin to emphasize the domain.

3.2 Additional Features

- To better understand the function's behavior, consider plotting level curves or contours of the function where possible, but keep in mind the domain excludes the origin.

Part 2: Using the Chain Rule to Find Partial Derivatives of f(x,y)

Next, we turn our attention to calculating the partial derivatives of the function with respect to x and y, which involves applying the chain rule carefully.

Step 4: Expressing the Function for Differentiation

The function:

f(x,y) = $\frac{x^2 y^3}{x^2 + y^2}$.

We recognize this as a quotient, so the quotient rule is useful, but since we are asked specifically to use the chain rule, we can consider the numerator and denominator as functions involving x and y.

4.1 Alternative Representation

Let’s denote:

- Numerator: \(N(x,y) = x^2 y^3\).

- Denominator: \(D(x,y) = x^2 + y^2\).

So:

f(x,y) = N(x,y)/D(x,y).

Step 5: Calculating the Partial Derivatives

5.1 Partial Derivative with respect to x (∂f/∂x)

Applying the quotient rule:

\[
\frac{\partial f}{\partial x} = \frac{\frac{\partial N}{\partial x} \cdot D - N \cdot \frac{\partial D}{\partial x}}{D^2}
\]

Calculations:

- \(\frac{\partial N}{\partial x} = 2x y^3\) (since y is treated as constant).

- \(\frac{\partial D}{\partial x} = 2x\).

Substituting:

\[
\frac{\partial f}{\partial x} = \frac{(2x y^3)(x^2 + y^2) - (x^2 y^3)(2x)}{(x^2 + y^2)^2}
\]

Simplify numerator:

\[
= \frac{2x y^3 (x^2 + y^2) - 2x x^2 y^3}{(x^2 + y^2)^2}
\]

Factor out common terms:

\[
= \frac{2x y^3 [x^2 + y^2 - x^2]}{(x^2 + y^2)^2} = \frac{2x y^3 y^2}{(x^2 + y^2)^2}
\]

Result:

\[
\boxed{

\frac{\partial f}{\partial x} = \frac{2x y^5}{(x^2 + y^2)^2}
}
\]

5.2 Partial Derivative with respect to y (∂f/∂y)

Similarly, use the quotient rule:

\[
\frac{\partial f}{\partial y} = \frac{\frac{\partial N}{\partial y} \cdot D - N \cdot \frac{\partial D}{\partial y}}{D^2}
\]

Calculations:

- \(\frac{\partial N}{\partial y} = 3 x^2 y^2\).

- \(\frac{\partial D}{\partial y} = 2 y\).

Substitute:

\[
\frac{\partial f}{\partial y} = \frac{3 x^2 y^2 (x^2 + y^2) - x^2 y^3 (2 y)}{(x^2 + y^2)^2}
\]

Simplify numerator:

\[
= \frac{3 x^2 y^2 (x^2 + y^2) - 2 x^2 y^4}{(x^2 + y^2)^2}
\]

Factor numerator:

\[
= \frac{x^2 y^2 [3(x^2 + y^2) - 2 y^2]}{(x^2 + y^2)^2}
\]

Simplify inside brackets:

\[
3 x^2 + 3 y^2 - 2 y^2 = 3 x^2 + y^2
\]

Thus,

\[
\frac{\partial f}{\partial y} = \frac{x^2 y^2 (3 x^2 + y^2)}{(x^2 + y^2)^2}
\]

Final Expressions for Partial Derivatives

- Partial derivative with respect to x:

$$
\boxed{
\frac{\partial f}{\partial x} = \frac{2 x y^5}{(x^2 + y^2)^2}
}
$$

- Partial derivative with respect to y:

$$
\boxed{
\frac{\partial f}{\partial y} = \frac{x^2 y^2 (3 x^2 + y^2)}{(x^2 + y^2)^2}
}
$$

Additional Insights: Using the Chain Rule in Multivariable Calculus

6.1 Chain Rule Overview

The chain rule in multivariable calculus allows us to differentiate composite functions. For functions of several variables, it is essential when the variables themselves are functions of other variables, or when dealing with functions expressed in terms of other functions.

6.2 Application to Our Function

In our case, the function is explicitly expressed in x and y, but the process of differentiating numerator and denominator involves the chain rule because:

- Each term involves powers of x and y.

- When differentiating these powers, we treat y as constant when differentiating with respect to x, and vice versa.

Summary and Key Takeaways

- Domain: The function $f(x, y) = \frac{x^2 y^3}{x^2 + y^2}$ is defined for all points in $\mathbb{R}^2$ except at the origin (0, 0), where the denominator becomes zero.

- Visualization: The domain covers the entire plane minus a single point; graphical representation involves shading all space except the origin.

- Partial derivatives:

- \(\frac{\partial f}{\partial x} = \frac{2 x y^5}{(x^2 + y^2)^2}\).

- \(\frac{\partial f}{\partial y} = \frac{x^2 y^

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