Find An Equation Of The Line Tangent To The Curve At The Point Corresponding To The Given Value Of T:x=cost+tsint,

Find An Equation Of The Line Tangent To The Curve At The Point Corresponding To The Given Value Of T: x = cos t + t sin t

Understanding how to find the tangent line to a curve at a particular point is a fundamental concept in calculus. When the curve is given parametrically, and the point corresponds to a specific parameter value t, the process involves differentiating the parametric equations and applying the point-slope form of a line. This article provides a comprehensive guide to deriving the equation of the tangent line to the curve defined by $x = \cos t + t \sin t$.

Introduction to Parametric Curves and Tangent Lines

Before delving into the specifics of the curve $x = \cos t + t \sin t$, it's important to understand the general principles involved in finding tangent lines to parametric curves.

What Are Parametric Equations?

- Parametric equations express the coordinates of points on a curve as functions of a parameter t.
- For example, a curve in the plane might be given by:
$$
x = f(t), \quad y = g(t)
$$
- The parameter t can represent time, angle, or any other variable.

Why Find the Tangent Line?

- The tangent line at a point provides the best linear approximation of the curve near that point.
- It is essential in understanding the behavior of the curve, optimizing functions, and solving geometric problems.

Key Concepts in Finding the Tangent Line

- Derivatives of parametric equations: $\frac{dx}{dt}$ and $\frac{dy}{dt}$.
- Slope of the tangent line: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.
- Equation of the tangent line: using point-slope form, $y - y_0 = m (x - x_0)$, where (x_0, y_0) is the point on the curve.

Understanding the Given Curve: $x = \cos t + t \sin t$

The specified curve is given by the parametric equation:
$$
x = \cos t + t \sin t

\]
This expression involves both trigonometric functions and the parameter t, making the process of finding the tangent line interesting and instructive.

Analyzing the Curve

- The function is defined in terms of t.
- To find the tangent at a particular point, we need both the coordinate x and the derivative information at that point.

Additional Information Needed

- Typically, for parametric curves, the y-coordinate is also provided. Since the problem only specifies x, it suggests that the curve may be one-dimensional or that the focus is on the x-coordinate with respect to t.
- If the problem involves a more complex curve, additional equations or context might be necessary. For now, we'll assume the goal is to find the tangent line to the curve at a specific t-value, which includes the point $(x(t), y(t))$ if y is given or can be derived.

Step-by-Step Process to Find the Equation of the Tangent Line

The process involves several key steps:

1. Determine the Point Corresponding to the Given t

- Calculate $x(t)$ at the specific parameter value.
- If $y(t)$ is given, evaluate it at the same t.

2. Find the Derivative $\frac{dx}{dt}$

- Differentiate $x(t)$ with respect to t.

3. Find $\frac{dy}{dt}$ (if $y(t)$ exists)

- Differentiate $y(t)$ with respect to t.

4. Compute $\frac{dy}{dx}$

- Use the chain rule:
$$
\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}
$$

5. Evaluate the derivative at the specific t

- Find the slope $m = \frac{dy}{dx}$ at the parameter t.

6. Write the Equation of the Tangent Line

- Use the point-slope form:
$$
y - y_0 = m (x - x_0)
$$
- Here, (x_0, y_0) is the point on the curve corresponding to the given t.

Calculating the Derivative $\frac{dx}{dt}$ for $x = \cos t + t \sin t$

Let's perform the differentiation step-by-step:

Derivative of $\cos t$

- $\frac{d}{dt} (\cos t) = -\sin t$

Derivative of $t \sin t$

- Apply the product rule:
$$
\frac{d}{dt} (t \sin t) = \sin t + t \cos t
$$
- Explanation:
- $\frac{d}{dt} (t) = 1$
- $\frac{d}{dt} (\sin t) = \cos t$

Combined derivative $\frac{dx}{dt}$

- Summing the derivatives:
$$
\frac{dx}{dt} = -\sin t + (\sin t + t \cos t) = t \cos t
$$
- Simplified, the derivative of x with respect to t is:
$$
\boxed{\frac{dx}{dt} = t \cos t}
$$

Finding the Corresponding y-Coordinate (If Available)

Since the problem statement only provides $x = \cos t + t \sin t$, it suggests that the curve may be one-dimensional or that the focus is on the x-coordinate behavior.

However, in many cases, the parametric form involves both $x(t)$ and $y(t)$;

- If $y(t)$ is provided, follow similar steps as for $x(t)$: differentiate $y(t)$, evaluate at the specific t, and proceed.

- If the problem is only concerned with the x-coordinate (e.g., in a curve defined implicitly or in a parametric form with y implicitly known), then the tangent line's slope may involve additional context.

Assumption: To fully analyze the tangent line, we need the y-coordinate at the point. For illustration, assume $y(t)$ is known or given; otherwise, the problem might be only about the x-direction.

Example: Finding the Tangent Line at a Specific t Value

Let's assume a specific value of t, say $t = t_0$, is given.

Step 1: Compute $x(t_0)$:
$$
x_0 = \cos t_0 + t_0 \sin t_0
$$

Step 2: Compute $\frac{dx}{dt}$ at $t = t_0$:
$$
\left.\frac{dx}{dt}\right|_{t=t_0} = t_0 \cos t_0
$$

Step 3: Find $y(t)$ and its derivative (if available).

Step 4: Calculate $\frac{dy}{dx}$ at t_0:
$$
m = \left.\frac{dy}{dt}\right|_{t=t_0} \big/ \left.\frac{dx}{dt}\right|_{t=t_0}
$$

Step 5: Write the tangent line using the point-slope form:
$$
y - y(t_0) = m (x - x(t_0))
$$

Special Case: When Only $x(t)$ Is Known

If the problem involves only the x-coordinate, and the goal is to find the tangent line in the x-direction, the process simplifies but also indicates the limitation—without a y-coordinate, the tangent line cannot be fully determined in the plane.

In such cases:
- The derivative $\frac{dx}{dt}$ indicates the rate of change of x.
- If considering a parametric curve in the x-direction, the tangent's slope can be viewed as the rate of change of x with respect to t.

Conclusion and Summary

Finding the equation of the tangent line to a parametric curve like $x = \cos t + t \sin t$ involves

Frequently Asked Questions

How do you find the slope of the tangent line to the curve x = cos t + t sin t at a specific value of t?

You differentiate x with respect to t to find dx/dt, which gives the slope of the tangent line at that point.

What is the process to determine the equation of the tangent line to the curve at a given t?

First, find dx/dt and y in terms of t, then evaluate these at the given t to get the point and slope. Use the point-slope form to write the tangent's equation.

How do I find dy/dt if y is not explicitly given in the problem?

If y is given as a function of t, differentiate it with respect to t. If y is implicit, use implicit differentiation as needed.

Can you provide a step-by-step method to find the tangent line when x = cos t + t sin t?

Yes. Step 1: Differentiate x with respect to t to find dx/dt. Step 2: Find the point coordinates by substituting the given t. Step 3: Calculate the slope using dx/dt at that t. Step 4: Write the tangent line equation using point-slope form.

What is the significance of the parameter t in finding the tangent line of the curve?

The parameter t defines a point on the curve and allows us to compute derivatives with respect to t, which are essential for finding slopes and tangent lines.

How do I handle the trigonometric functions when differentiating x = cos t + t sin t?

Differentiate term by term: d/dt(cos t) = -sin t, and d/dt(t sin t) = sin t + t cos t, using the product rule.

If the given value of t is t0, how do I find the corresponding point on the curve?

Substitute t0 into the original parametric equations to find the coordinates (x(t0), y(t0)).

What is the final formula to write the equation of the tangent line at t = t0?

Using point (x0, y0) and slope m = dx/dt at t0, the tangent line equation is y - y0 = m (x - x0).

Additional Resources

Find An Equation Of The Line Tangent To The Curve At The Point Corresponding To The Given Value Of T: x=cos t + t sin t

When working with parametric curves, a common problem is to find the equation of the tangent line to the curve at a specific point corresponding to a given parameter value t. In particular, when given a parametric form such as $x = \cos t + t \sin t$, the challenge involves not only determining the coordinates of the point on the curve at that t, but also calculating the slope of the tangent line at that point. This process combines differentiation techniques and algebraic manipulation, ultimately leading to the equation of the tangent line. This comprehensive guide will walk you through the steps to find the tangent line to the curve at a specific t, providing clarity for students and professionals alike.

Understanding the Problem

Before diving into calculations, it's important to understand the problem's core components:

- Curve Definition: The curve is given parametrically as:

$$
x(t) = \cos t + t \sin t
$$

Typically, parametric equations involve both $x(t)$ and $y(t)$, but in this case, only $x(t)$ is provided. To find the tangent line, we need the corresponding $y(t)$. Often, in problems like this, $y(t)$ is either implicitly given or not explicitly provided, but for a complete tangent line, both x and y are necessary.

Note: If only $x(t)$ is given, the problem might be asking for the tangent line in the x-direction or perhaps a misprint. Alternatively, the full parametric form might include $y(t)$, which could be specified elsewhere.

For the purpose of this guide, we will assume the curve is given as:

$$
\begin{cases}
x(t) = \cos t + t \sin t \\
y(t) = \sin t - t \cos t
\end{cases}
$$

This is a common parametric representation of a curve known as the cycloid or related to it, but the specific $y(t)$ can vary depending on the problem.

- Given Parameter t: The task is to find the tangent line at the point corresponding to a specific $t = t_0$.

- Key Goal: Derive the equation of the tangent line at $t = t_0$.

Step 1: Find the Coordinates of the Point on the Curve

Given the parameter t, the first step is to determine the point $(x(t), y(t))$ on the curve:

$$
\boxed{
\begin{aligned}

x(t) &= \cos t + t \sin t \\
y(t) &= \sin t - t \cos t
\end{aligned}
}
\]

At the specific value \(t = t_0 \), the coordinates are:

\[
\boxed{
\begin{aligned}
x_0 &= \cos t_0 + t_0 \sin t_0 \\
y_0 &= \sin t_0 - t_0 \cos t_0
\end{aligned}
}
\]

Step 2: Find the Derivatives \(\frac{dx}{dt} \) and \(\frac{dy}{dt} \)

To determine the slope of the tangent line at \(t = t_0 \), we need the derivatives of \(x(t) \) and \(y(t) \):

Derivative of \(x(t) \):

\[
x(t) = \cos t + t \sin t
\]

Applying differentiation term-by-term:

\[
\frac{dx}{dt} = -\sin t + \sin t + t \cos t
\]

Simplify:

\[
\frac{dx}{dt} = (-\sin t + \sin t) + t \cos t = 0 + t \cos t = t \cos t
\]

Derivative of \(y(t) \):

\[
y(t) = \sin t - t \cos t
\]

Differentiating:

\[
\frac{dy}{dt} = \cos t - \left(\cos t - t \sin t \right) = \cos t - \cos t + t \sin t = t \sin t
\]

Step 3: Find the Slope of the Tangent Line

The slope of the tangent line at \(t = t_0 \) is given by:

$$
m = \frac{dy/dt}{dx/dt} = \frac{t_0 \sin t_0}{t_0 \cos t_0}
$$

Provided $t_0 \neq 0$, simplifying:

$$
m = \frac{\sin t_0}{\cos t_0} = \tan t_0
$$

Special case: If $t_0 = 0$, then $dx/dt = 0$, and the slope is undefined, indicating a vertical tangent line.

Step 4: Write the Equation of the Tangent Line

Using point-slope form:

$$
y - y_0 = m (x - x_0)
$$

Substituting the known values:

$$
\boxed{
y - \left(\sin t_0 - t_0 \cos t_0 \right) = \tan t_0 \left(x - \left(\cos t_0 + t_0 \sin t_0 \right) \right)
}
$$

This is the equation of the tangent line at the point corresponding to $t = t_0$.

Summary of the Procedure

1. Identify the parametric equations $x(t)$ and $y(t)$.
2. Calculate the point (x_0, y_0) at $t = t_0$.
3. Differentiate $x(t)$ and $y(t)$ with respect to t.
4. Evaluate derivatives at t_0 to get dx/dt and dy/dt.
5. Determine the slope $m = \frac{dy/dt}{dx/dt}$ at t_0.
6. Write the equation of the tangent line using the point-slope form.

Additional Considerations

When is the tangent line vertical?

If $dx/dt = 0$ at $t = t_0$, the slope is undefined, and the tangent line is vertical. For our derivatives:

$$
dx/dt = t_0 \cos t_0
$$

So, the tangent is vertical when:

$$
t_0 \cos t_0 = 0

\]

which occurs if:

- \(t_0 = 0 \), or
- \(\cos t_0 = 0 \Rightarrow t_0 = \frac{\pi}{2} + n \pi, \quad n \in \mathbb{Z} \)

In these cases, the tangent line equation simplifies to:

\[
x = x(t_0)
\]

Final Remarks

Finding the equation of the tangent line to a parametric curve involves a blend of calculus and algebraic skills. The key steps involve computing derivatives to find the slope, evaluating the point coordinates at the given parameter, and then applying the point-slope form of a line. Mastery of these techniques allows you to analyze a wide range of curves and their tangents, which are fundamental in fields like physics, engineering, and computer graphics.

By understanding the process detailed here, you can confidently approach similar problems involving parametric equations and tangent lines, whether in academic exercises or real-world applications.

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