Find Sin(2x), Cos(2x), And Tan(2x) From The Given Information.cos(x) = 4/5 , Csc(x) < 0sin(2x)=cos(2x)=tan(2x)=

Find Sin(2x), Cos(2x), And Tan(2x) From The Given Information. cos(x) = 4/5, csc(x) < 0, sin(2x) = cos(2x) = tan(2x) =

Understanding how to find the double angle trigonometric functions—sin(2x), cos(2x), and tan(2x)—when given certain information about an angle x is an essential skill in trigonometry. In this article, we will explore the step-by-step process of determining these double angle functions based on the provided data: cos(x) = 4/5 and the condition csc(x) < 0. We will analyze the problem thoroughly, interpret the given conditions, and use fundamental identities to arrive at the solutions. This detailed guide aims to enhance your understanding of double angle formulas and their applications, especially in cases involving specific quadrants and sign conventions.

Understanding the Given Data and Conditions

Before diving into calculations, let’s clarify what the given information implies:

- cos(x) = 4/5: The cosine of angle x is positive, with a value of 4/5.
- csc(x) < 0: The cosecant of x is negative, indicating that sin(x) is negative because csc(x) = 1/sin(x).

Implication of the Given Values

From cos(x) = 4/5, which is positive, and csc(x) < 0, which indicates sin(x) < 0, we can deduce the quadrant in which x lies:

- Since cos(x) > 0 and sin(x) < 0, x resides in the Fourth Quadrant, where cosine is positive and sine is negative.

Step 1: Find sin(x)

Given cos(x) = 4/5, we can find sin(x) using the Pythagorean theorem:

\[
\sin^2(x) + \cos^2(x) = 1
\]

Plugging in the known value:

\[

\sin^2(x) + \left(\frac{4}{5}\right)^2 = 1
\]
\[
\sin^2(x) + \frac{16}{25} = 1
\]
\[
\sin^2(x) = 1 - \frac{16}{25} = \frac{25}{25} - \frac{16}{25} = \frac{9}{25}
\]

Taking the square root:

\[
\sin(x) = \pm \frac{3}{5}
\]

Since we know csc(x) < 0, which implies sin(x) < 0, the negative root applies:

\[
\sin(x) = -\frac{3}{5}
\]

Summary:

- \(\sin(x) = -\frac{3}{5}\)
- \(\cos(x) = \frac{4}{5}\)

Step 2: Find \(\sin(2x)\), \(\cos(2x)\), and \(\tan(2x)\)

Now that we know \(\sin(x)\) and \(\cos(x)\), we can use double angle formulas to find the respective functions.

Double Angle Formulas:

- \(\sin(2x) = 2 \sin x \cos x\)
- \(\cos(2x) = \cos^2 x - \sin^2 x\)
- \(\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}\)

Step 3: Calculate \(\sin(2x)\)

Using the double angle formula:

\[
\sin(2x) = 2 \sin x \cos x
\]

Substitute the known values:

\[
\sin(2x) = 2 \times \left(-\frac{3}{5}\right) \times \frac{4}{5} = 2 \times -\frac{3}{5}
\times \frac{4}{5}
\]

Calculate step-by-step:

\[
\sin(2x) = 2 \times -\frac{3}{5} \times \frac{4}{5} = -2 \times \frac{3}{5} \times
\frac{4}{5}
\]
\[
= -2 \times \frac{12}{25} = - \frac{24}{25}
\]

Result:

\[
\boxed{\sin(2x) = -\frac{24}{25}}
\]

Step 4: Calculate \(\cos(2x)\)

Using the double angle formula:

\[
\cos(2x) = \cos^2 x - \sin^2 x
\]

Plugging in the known values:

\[
\cos(2x) = \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2
\]
\[
= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}
\]

Result:

\[
\boxed{\cos(2x) = \frac{7}{25}}
\]

Step 5: Calculate \(\tan(2x)\)

Using the tangent double angle formula:

\[
\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}
\]

First, find \(\tan x\):

\[
\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{5} \times \frac{5}{4} = -\frac{3}{4}
\]

Now, compute \(\tan(2x)\):

\[
\tan(2x) = \frac{2 \times -\frac{3}{4}}{1 - \left(-\frac{3}{4}\right)^2} = \frac{-\frac{6}{4}}{1 - \frac{9}{16}}
\]

Simplify numerator:

\[
-\frac{6}{4} = -\frac{3}{2}
\]

Calculate denominator:

\[
1 - \frac{9}{16} = \frac{16}{16} - \frac{9}{16} = \frac{7}{16}
\]

Therefore:

\[
\tan(2x) = \frac{-\frac{3}{2}}{\frac{7}{16}} = -\frac{3}{2} \times \frac{16}{7} = -\frac{3 \times 16}{2 \times 7} = -\frac{48}{14} = -\frac{24}{7}
\]

Result:

\[
\boxed{\tan(2x) = -\frac{24}{7}}
\]

Summary of Results

Function	Value
\(\sin(2x)\)	\(-\frac{24}{25}\)
\(\cos(2x)\)	\(\frac{7}{25}\)

$\tan(2x)$	$-\frac{24}{7}$

Additional Considerations and Key Takeaways

Quadrant and Sign Analysis

- Since $\cos(x) > 0$ and $\sin(x) < 0$, x is in the Fourth Quadrant.
- Double angle functions' signs depend on the quadrant of $2x$. Because x is in the fourth quadrant, $2x$ could be in either the first, second, third, or fourth quadrant depending on the specific angle measures, but with the provided signs, the double angle functions reflect the signs computed.

Significance of the Negative Signs

- $\sin(2x)$ is negative because $\sin x$ is negative.
- $\cos(2x)$ is positive as per the calculation.
- $\tan(2x)$ is negative, consistent with the signs of $\sin(2x)$ and $\cos(2x)$.

Practical Applications

Understanding how to compute double angle functions from known values is critical in various fields, including physics, engineering, and computer graphics, where wave functions and rotational transformations frequently involve double angles.

Final Thoughts

Mastering the process of finding $\sin(2x)$, $\cos(2x)$, and $\tan(2x)$ from given trigonometric ratios involves:

- Recognizing the quadrant based on sign conventions.
- Using Pythagorean identities to find missing values.
- Applying double angle formulas carefully.
- Simplifying fractions and rational expressions.

By following these systematic steps, you can confidently solve similar problems involving double angles and given trigonometric ratios, strengthening your overall understanding of trigonometric identities and their applications.

Frequently Asked Questions (FAQs)

1. Why is $\sin(x)$ negative in this problem?

Because the given condition $csc(x) < 0$ implies $\sin(x) < 0$, and the quadrant analysis shows that x is in the Fourth Quadrant where sine is negative.

2. How do I choose

Frequently Asked Questions

Given that $\cos(x) = 4/5$ and $\csc(x) < 0$, what is $\sin(x)$?

Since $\csc(x) < 0$ and $\cos(x) > 0$, x is in the fourth quadrant. Using $\sin^2(x) + \cos^2(x) = 1$: $\sin^2(x) = 1 - (4/5)^2 = 1 - 16/25 = 9/25$. Since $\sin(x) < 0$ in the fourth quadrant, $\sin(x) = -3/5$.

How do you find $\sin(2x)$ given $\sin(x) = -3/5$ and $\cos(x) = 4/5$?

Using the double angle formula: $\sin(2x) = 2 \sin(x) \cos(x)$. Plugging in: $2 (-3/5) (4/5) = -24/25$.

What is $\cos(2x)$ if $\cos(x) = 4/5$ and $\sin(x) = -3/5$?

Using the double angle formula: $\cos(2x) = \cos^2(x) - \sin^2(x)$. Substituting: $(4/5)^2 - (-3/5)^2 = 16/25 - 9/25 = 7/25$.

How do you compute $\tan(2x)$ with $\sin(x) = -3/5$ and $\cos(x) = 4/5$?

Using the double angle formula: $\tan(2x) = 2 \tan(x) / (1 - \tan^2(x))$. First, find $\tan(x) = \sin(x)/\cos(x) = (-3/5) / (4/5) = -3/4$. Then, $\tan(2x) = 2 (-3/4) / (1 - (-3/4)^2) = (-6/4) / (1 - 9/16) = (-6/4) / (7/16) = (-6/4) (16/7) = (-6\ 16) / (4\ 7) = (-96) / (28) = -24/7$.

Why is $\sin(2x)$ negative in this problem?

Because $\sin(x)$ is negative (x is in the fourth quadrant), and $\sin(2x) = 2 \sin(x) \cos(x)$. Since $\sin(x) < 0$ and $\cos(x) > 0$, their product is negative, making $\sin(2x)$ negative.

Are $\cos(2x)$ and $\tan(2x)$ positive or negative given these values?

$\cos(2x) = 7/25$ is positive, and $\tan(2x) = -24/7$ is negative, based on the sine and cosine values provided.

What is the significance of the quadrant in determining the signs of the trigonometric functions?

The quadrant determines the signs of sine and cosine: in the fourth quadrant, $\cos(x) > 0$ and $\sin(x) < 0$. This affects the signs of their double angle counterparts accordingly.

Can you verify the values of sin(2x), cos(2x), and tan(2x) using identities?

Yes. Using identities: sin(2x) = 2 sin(x) cos(x) = -24/25; cos(2x) = 7/25; tan(2x) = sin(2x)/cos(2x) = (-24/25) / (7/25) = -24/7, confirming the calculated values.

What are the key steps to find sin(2x), cos(2x), and tan(2x) from given cos(x) and quadrant info?

First, determine sin(x) using Pythagoras and quadrant info. Then, apply double angle formulas for sin(2x), cos(2x), and tan(2x). Verify signs based on the quadrant to ensure correct results.

Additional Resources

Find Sin(2x), Cos(2x), And Tan(2x) From The Given Information. cos(x) = 4/5, csc(x) < 0

Understanding the intricate relationships between trigonometric functions is fundamental in mathematics, especially when solving problems involving angles and their multiple identities. In this comprehensive review, we explore the process of determining the exact values of sin(2x), cos(2x), and tan(2x) given specific initial conditions: that cos(x) = 4/5 and csc(x) < 0. These conditions imply important information about the angle x’s quadrant and the behavior of the trigonometric functions involved.

Introduction to the Problem

The problem at hand involves finding the double-angle functions—sin(2x), cos(2x), and tan(2x)—using the provided cosine value and the inequality csc(x) < 0. These identities are essential in many fields, including physics, engineering, and advanced mathematics, where precise angle calculations are necessary.

The key data points are:

- cos(x) = 4/5
- csc(x) < 0

From this, we can infer the quadrant of x and understand how the signs of the sine and cosine functions influence the calculations.

Understanding the Given Information

Deciphering $\cos(x) = 4/5$

The cosine of an angle x is given as 4/5. This ratio suggests that the adjacent side over the hypotenuse in a right triangle is 4 units over 5 units, respectively. It indicates a right triangle where:

- The adjacent side (adj) = 4
- The hypotenuse (hyp) = 5

Using the Pythagorean theorem, we can find the length of the opposite side (opp):

$$
opp = \sqrt{hyp^2 - adj^2} = \sqrt{5^2 - 4^2} = \sqrt{25 - 16} = \sqrt{9} = 3
$$

This setup is standard in trigonometric calculations, providing the fundamental values needed to determine other functions.

Implication of $\csc(x) < 0$

The cosecant function $\csc(x) = 1/\sin(x)$. Given that $\csc(x) < 0$, it follows that $\sin(x)$ must also be negative because:

$$
csc(x) = \frac{1}{sin(x)} < 0 \Rightarrow sin(x) < 0
$$

This sign indicates that x lies in either the third or fourth quadrant, where sine is negative.

Determining the Sign and Value of sin(x)

Given the initial cosine value and the sign of sine, we can accurately locate the quadrant of x and compute sin(x).

Calculating sin(x)

From the right triangle constructed earlier:

\[
sin(x) = \frac{opp}{hyp} = \frac{3}{5}
\]

But since csc(x) < 0, and csc(x) = 1/sin(x), the sine must be negative:

\[
sin(x) = -\frac{3}{5}
\]

Thus, the value of sin(x) is -3/5, and the angle x lies in the third or fourth quadrant. Because cos(x) = 4/5 is positive, but sine is negative, x must be in the fourth quadrant. The standard Cartesian plane indicates:

- Quadrant III: sin < 0, cos < 0
- Quadrant IV: sin < 0, cos > 0

Since cos(x) = 4/5 > 0, we conclude x is in the fourth quadrant.

Calculating Double-Angle Functions

With the values of sin(x) and cos(x), we are prepared to compute the double-angle functions:

\[
\sin(2x), \quad \cos(2x), \quad \tan(2x)
\]

These are fundamental identities:

- \(\sin(2x) = 2 \sin x \cos x\)
- \(\cos(2x) = \cos^2 x - \sin^2 x\)
- \(\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}\)

Let's analyze each in turn.

Calculating sin(2x)

Using the double-angle identity:

\[
\sin(2x) = 2 \sin x \cos x

\]

Substituting the known values:

\[
\sin(2x) = 2 \times \left(-\frac{3}{5}\right) \times \frac{4}{5} = 2 \times -\frac{3}{5} \times \frac{4}{5}
\]

Calculating:

\[
\sin(2x) = 2 \times -\frac{12}{25} = -\frac{24}{25}
\]

Result:

\[
\boxed{
\sin(2x) = -\frac{24}{25}
}
\]

The negative sign aligns with the quadrant information, confirming that the double angle also falls in a quadrant where sine is negative.

Calculating cos(2x)

Using the identity:

\[
\cos(2x) = \cos^2 x - \sin^2 x
\]

Substitute the known values:

\[
\cos(2x) = \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}
\]

Result:

\[
\boxed{
\cos(2x) = \frac{7}{25}
}

\]

This positive value suggests the double angle is in the first or fourth quadrant, consistent with the initial quadrant.

Calculating tan(2x)

To find \(\tan(2x)\), it is most straightforward to use the tangent double-angle identity:

\[
\tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}
\]

First, compute \(\tan x\):

\[
\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}
\]

Now substitute into the double-angle formula:

\[
\tan(2x) = \frac{2 \times \left(-\frac{3}{4}\right)}{1 - \left(-\frac{3}{4}\right)^2} =
\frac{-\frac{6}{4}}{1 - \frac{9}{16}} = \frac{-\frac{3}{2}}{\frac{7}{16}}
\]

Simplify numerator and denominator:

\[
\tan(2x) = -\frac{3}{2} \times \frac{16}{7} = -\frac{3 \times 16}{2 \times 7} = -
\frac{48}{14} = -\frac{24}{7}
\]

Result:

\[
\boxed{
\tan(2x) = -\frac{24}{7}
}
\]

The negative value aligns with the quadrant considerations, confirming consistency across calculations.

Summary of Results

Based on the given data and detailed calculations, the exact values of the double-angle trigonometric functions are:

- sin(2x) = -24/25
- cos(2x) = 7/25
- tan(2x) = -24/7

These results are coherent with the initial conditions, the quadrant location of x, and the fundamental identities of trigonometry.

Conclusion and Significance

This problem exemplifies the elegance and interconnectedness of trigonometric identities. Starting from a single known value (cos(x) = 4/5) and the inequality csc(x) < 0, we navigated through quadrant identification, sign analysis, and the application of core identities to derive the double-angle functions. Understanding such relationships is critical not only in academic contexts but also in practical applications involving wave mechanics, oscillations, and signal processing.

The process underscores the importance of careful sign analysis, geometric interpretation, and algebraic manipulation in solving trigonometric problems. As mathematical tools, these identities and methods form the backbone of more complex analyses in science and engineering, demonstrating the timeless relevance of foundational trigonometry.

In essence, the calculated double-angle functions are:

\[
\boxed{
\sin(2x) = -\frac{24}{25}, \quad \cos(2x) = \frac{7}{25}, \quad \tan(2x) = -\frac{24}{7}
}
\]

These values not only solve the problem but also reinforce the importance of methodical reasoning in mathematical problem-solving.

Find Sin 2x Cos 2x And Tan 2x From The Given Information Cos X 4 5 Csc X Lt 0sin 2x Cos 2x Tan 2x

Find Sin 2x Cos 2x And Tan 2x From The Given Information Cos X 4 5 Csc X Lt 0sin 2x Cos 2x Tan 2x

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