Find Tan , Sin , And Seco, Where 0 Is The Angle Shown In The Figure.Give Exact Values, Not Decimal Approximations.

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Understanding how to determine the exact values of tangent (tan), sine (sin), and secant (seco) for a given angle is fundamental in trigonometry. These functions are interconnected through the properties of right triangles and the unit circle. The goal of this article is to provide a comprehensive guide on how to find these values precisely for a given angle, emphasizing exact expressions rather than decimal approximations. Although the problem references a figure with an angle labeled 0, we will assume it is a generic angle θ (or 0°), and the methods described will be applicable to any specific angle provided in a figure.

Understanding Basic Trigonometric Functions

1. The Sine Function (sin)

The sine of an angle in a right triangle is the ratio of the length of the side opposite the angle to the length of the hypotenuse:

$$
\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}
$$

On the unit circle, $\sin \theta$ corresponds to the y-coordinate of the point where the terminal side of the angle intersects the circle.

2. The Cosine Function (cos)

While the problem emphasizes sine, tangent, and secant, cosine is often used in calculations:

$$
\cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}
$$

On the unit circle, it corresponds to the x-coordinate.

3. The Tangent Function (tan)

Tangent is the ratio of sine to cosine:

$$
\boxed{\tan \theta = \frac{\sin \theta}{\cos \theta}}

\]
It can also be interpreted as the ratio of the opposite to the adjacent side in a right triangle:
\[
\text{If } \text{Opposite} = a, \text{Adjacent} = b, \text{Hypotenuse} = c,\quad
\tan \theta = \frac{a}{b}
\]

4. The Secant Function (sec)

Secant is the reciprocal of cosine:
\[
\boxed{\sec \theta = \frac{1}{\cos \theta}}
\]
It is undefined where \(\cos \theta = 0\).

Approach to Finding Exact Values

1. Identifying the Angle and Coordinates

- If the figure provides a right triangle with known side lengths, use those lengths directly.
- If the figure shows a point on the unit circle, note the coordinates \((x, y)\):
- \(\sin \theta = y\)
- \(\cos \theta = x\)
- \(\tan \theta = \frac{y}{x}\)
- \(\sec \theta = \frac{1}{x}\)

2. Using Special Angles

- For common angles such as \(0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ \), the exact values are well-known and can be memorized or derived using special triangles.

3. Applying the Pythagorean Theorem

- If the side lengths are known or can be determined, use the Pythagorean theorem to find missing sides:
\[
a^2 + b^2 = c^2
\]
- Use these lengths to compute the ratios for sine, cosine, and tangent.

4. Using Symmetry and Unit Circle Properties

- Recognize the symmetry in the unit circle to find sine, cosine, and tangent for angles in different quadrants.

Calculating Exact Values for Common Angles

1. Angle 0°

- $\sin 0^\circ = 0$
- $\cos 0^\circ = 1$
- $\tan 0^\circ = 0$
- $\sec 0^\circ = 1$

2. Angle 30° (or $\pi/6$)

- $\sin 30^\circ = \frac{1}{2}$
- $\cos 30^\circ = \frac{\sqrt{3}}{2}$
- $\tan 30^\circ = \frac{\sin 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$
- $\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{2}{\sqrt{3}}$

3. Angle 45° (or $\pi/4$)

- $\sin 45^\circ = \frac{\sqrt{2}}{2}$
- $\cos 45^\circ = \frac{\sqrt{2}}{2}$
- $\tan 45^\circ = 1$
- $\sec 45^\circ = \sqrt{2}$

4. Angle 60° (or $\pi/3$)

- $\sin 60^\circ = \frac{\sqrt{3}}{2}$
- $\cos 60^\circ = \frac{1}{2}$
- $\tan 60^\circ = \sqrt{3}$
- $\sec 60^\circ = 2$

5. Angle 90° (or $\pi/2$)

- $\sin 90^\circ = 1$
- $\cos 90^\circ = 0$
- $\tan 90^\circ$ is undefined (since $\cos 90^\circ = 0$)
- $\sec 90^\circ$ is undefined

Using Special Triangles for Exact Values

1. 30-60-90 Triangle

- Side ratios: $1 : \sqrt{3} : 2$
- Opposite 30°: $\frac{1}{2}$
- Opposite 60°: $\frac{\sqrt{3}}{2}$
- Hypotenuse: 1 or 2 depending on scale

2. 45-45-90 Triangle

- Side ratios: $1 : 1 : \sqrt{2}$
- Legs: $\frac{\sqrt{2}}{2}$
- Hypotenuse: 1 or scaled accordingly

Sample Problem: Finding Exact Values from a Figure

Suppose the figure shows a right triangle with:
- An angle $\theta = 45^\circ$
- Legs of equal length, say each of length 1

Step 1: Determine side lengths
- Since legs are 1, hypotenuse $c = \sqrt{1^2 + 1^2} = \sqrt{2}$

Step 2: Find sine, cosine, tangent, and secant
- $\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $\tan \theta = \frac{\sin \theta}{\cos \theta} = 1$
- $\sec \theta = \frac{1}{\cos \theta} = \sqrt{2}$

Important Identities and Tips

- Reciprocal identities:

    - $\sec \theta = \frac{1}{\cos \theta}$
    - $\csc \theta = \frac{1}{\sin \theta}$
    - $\cot \theta = \frac{1}{\tan \theta} = \frac{\cos \theta}{\sin \theta}$

- Using the Pythagorean identity:
$$
\sin^2 \theta + \cos^2 \theta = 1
$$

- For angles in different quadrants, consider the signs of the functions based on the quadrant.

Conclusion

Calculating the exact values of tangent, sine, and secant for a given angle involves understanding the relationships between the sides of right triangles, the unit circle, and special angles. By memorizing the key exact values for common angles and utilizing fundamental identities, one can determine these functions precisely without resorting to decimal approximations. Whether working with a known triangle

Frequently Asked Questions

Given a right triangle where the angle θ is 0°, find the values of sin θ, tan θ, and sec θ, providing exact values.

sin 0° = 0, tan 0° = 0, sec 0° = 1.

If the angle θ in a right triangle is 0°, what are the exact values of sin θ, tan θ, and sec θ?

sin 0° = 0, tan 0° = 0, sec 0° = 1.

For an angle of 0°, determine the exact values of sine, tangent, and secant functions.

sin 0° = 0, tan 0° = 0, sec 0° = 1.

At angle θ = 0°, what are the exact sine, tangent, and secant values?

sin 0° = 0, tan 0° = 0, sec 0° = 1.

In a right triangle with θ = 0°, what are the exact trigonometric ratios for sine, tangent, and secant?

sin 0° = 0, tan 0° = 0, sec 0° = 1.

Additional Resources

Find Tan, Sin, And Seco, Where 0 Is The Angle Shown In The Figure. Give Exact Values, Not Decimal Approximations.

Understanding how to find the sine, cosine, tangent, and secant of a given angle is fundamental in trigonometry, a core branch of mathematics that deals with the relationships between the angles and sides of triangles. When the problem specifies to find these trigonometric ratios for a particular angle, especially without resorting to decimal approximations, it emphasizes the importance of exact values—most often expressed in terms of square roots or simplified fractions. This article aims to thoroughly explore methods to determine sin, tan, and sec of an angle, given a figure with the angle labeled, and to illustrate how to derive these values precisely, along with practical insights into their significance in mathematical and real-world contexts.

Understanding the Basics of Trigonometric Ratios

Before diving into calculations, it’s essential to recall the fundamental definitions of the primary trigonometric functions for an angle θ in a right triangle:

- Sine ($\sin \theta$): The ratio of the length of the side opposite the angle to the hypotenuse.
- Cosine ($\cos \theta$): The ratio of the length of the adjacent side to the hypotenuse.
- Tangent ($\tan \theta$): The ratio of the length of the opposite side to the adjacent side.
- Secant ($\sec \theta$): The reciprocal of cosine, i.e., $\sec \theta = \frac{1}{\cos \theta}$.

In figures where the angle θ is not part of a right triangle, or where the figure involves a unit circle, these ratios can be computed based on coordinates or known lengths, often leveraging special angles or geometric properties.

Approach to Finding Exact Values

When the problem provides a figure with a specific angle θ, the key steps to find exact trigonometric ratios include:

1. Identify the right triangle or coordinate points involved.
2. Use known special angles where applicable (e.g., $30^\circ, 45^\circ, 60^\circ$) and their exact ratios.
3. Apply the Pythagorean theorem if side lengths are known or can be deduced.
4. Express the ratios in simplest radical form or as fractions without decimal approximations.
5. Use reciprocal identities for secant and cosecant when necessary.

Now, we will explore these steps in detail for each ratio.

Calculating $\sin \theta$

$\sin \theta$ is straightforward when the length of the opposite side and hypotenuse are known or can be deduced. When the figure involves a right triangle, the process is:

- Identify the side opposite θ.
- Find its length relative to the hypotenuse.

Example: Suppose the figure shows a right triangle with an angle θ, side opposite θ of length a, and hypotenuse of length c. If these are known or can be expressed in radical form, then:

$$
\sin \theta = \frac{a}{c}
$$

Special angles such as 30°, 45°, and 60° have well-known exact sine values:

- $\sin 30^\circ = \frac{1}{2}$
- $\sin 45^\circ = \frac{\sqrt{2}}{2}$
- $\sin 60^\circ = \frac{\sqrt{3}}{2}$

Pros:
- Easy to compute for special angles.
- Values are exact and widely tabulated.

Cons:
- Not directly applicable unless the angle matches special angles or the figure provides sufficient data.

Calculating $\tan \theta$

$\tan \theta$ involves the ratio of the opposite to adjacent sides:

$$
\tan \theta = \frac{\text{opposite}}{\text{adjacent}}
$$

In practice:

- Identify the lengths of the sides forming the angle.
- Simplify the ratio to its lowest terms.
- Express in radical form if necessary.

Example: If the adjacent side length is b, and the opposite side length is a, then:

$$
\tan \theta = \frac{a}{b}
$$

Special angles provide exact tangent values as well:

- $\tan 45^\circ = 1$
- $\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
- $\tan 60^\circ = \sqrt{3}$

These ratios are particularly convenient for calculations involving known angles.

Features:
- Useful when the triangle's side lengths are known.
- Simplifies calculations with special angles.

Limitations:
- If the triangle is not right-angled or given sides are not known, alternative methods are needed.

Calculating $\sec \theta$

Secant is the reciprocal of cosine:

$$
\sec \theta = \frac{1}{\cos \theta}
$$

Thus, once $\cos \theta$ is known, $\sec \theta$ can be directly obtained. When working with figures:

- Determine the length of the adjacent side and hypotenuse for $\cos \theta$.
- Take the reciprocal to find $\sec \theta$.

Exact Values for Special Angles:

- $\sec 30^\circ = \frac{1}{\cos 30^\circ} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$
- $\sec 45^\circ = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$
- $\sec 60^\circ = \frac{1}{\frac{1}{2}} = 2$

Advantages:
- Useful in solving complex equations involving secant.
- Exact radical forms provide precision.

Disadvantages:
- Requires a known cosine value.
- Less intuitive unless familiar with reciprocal relationships.

Application: Example Problem and Solution

Suppose the figure shows a right triangle with:

- An angle θ, with the side opposite θ of length $\sqrt{3}$,
- The hypotenuse of length 2,
- The adjacent side length can be deduced using the Pythagorean theorem.

Step 1: Find the adjacent side:

$$
\text{adjacent} = \sqrt{c^2 - a^2} = \sqrt{2^2 - (\sqrt{3})^2} = \sqrt{4 - 3} = \sqrt{1} = 1
$$

Step 2: Calculate $\sin \theta$:

$$
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}
$$

Step 3: Calculate $\tan \theta$:

$$
\tan \theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3}
$$

Step 4: Calculate $\cos \theta$:

$$

\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{2}
\]

Step 5: Calculate \(\sec \theta\):

\[
\sec \theta = \frac{1}{\cos \theta} = \frac{1}{\frac{1}{2}} = 2
\]

This example illustrates how geometric properties and the Pythagorean theorem facilitate deriving exact trigonometric ratios.

Special Cases and Common Angles

Many problems involve angles of special measures where ratios are memorized or easily derived:

Angle	\(\sin \theta\)	\(\cos \theta\)	\(\tan \theta\)	\(\sec \theta\)
30°	\(\frac{1}{2}\)	\(\frac{\sqrt{3}}{2}\)	\(\frac{1}{\sqrt{3}}\)	\(\frac{2}{\sqrt{3}}\)
45°	\(\frac{\sqrt{2}}{2}\)	\(\frac{\sqrt{2}}{2}\)	1	\(\sqrt{2}\)
60°	\(\frac{\sqrt{3}}{2}\)	\(\frac{1}{2}\)	\(\sqrt{3}\)	2

Using these values simplifies calculations significantly, especially in exam settings or when solving complex trigonometric identities.

Conclusion and Summary

Finding exact values of \(\sin\), \(\tan\), and \(\sec\

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