Find The Absolute Extrema Of The Given Function On The Indicated Closed And Bounded Set R.R. 344. F(x,y)=xyx3y;f(x,y)=xyx3y;

Find The Absolute Extrema Of The Given Function On The Indicated Closed And Bounded Set R.R. 344. F(x,y)=xyx3y;f(x,y)=xyx3y;

Understanding how to find the absolute extrema (maximum and minimum values) of a function on a closed and bounded set is a fundamental concept in calculus and optimization. This process involves analyzing the function within the interior of the domain and along its boundary, applying critical point analysis, and leveraging the Extreme Value Theorem. In this article, we will explore the step-by-step approach to determine the absolute extrema of the function $F(x,y) = xyx3y$ on the specified set $R.R. 344$.

Interpreting the Problem and Setting the Stage

Before diving into calculations, it is essential to clarify the problem's components:

- Function: $F(x,y) = xyx3y$

- Domain: The set $R.R. 344$, which appears to be a notation for a closed and bounded subset of $\mathbb{R}^2$. Since the specific description of the set isn't provided, we will assume it refers to a rectangular region, often denoted as $R = \{ (x,y) \mid a \leq x \leq b, c \leq y \leq d \}$.

- Objective: Find the absolute maximum and minimum values of $F(x,y)$ within this set.

Understanding the Function $F(x,y) = xyx3y$

The function notation appears to be somewhat ambiguous. It looks like the expression might be:

\[
F(x,y) = xy \times x^3 y

\]

which simplifies as follows:

\[
F(x,y) = xy \times x^3 y = (x y) \times (x^3 y)
\]

Multiplying the terms:

\[
F(x,y) = x y \times x^3 y = x \times y \times x^3 \times y = x^{1+3} \times y^{1+1} = x^4 y^2
\]

Therefore, the function simplifies to:

\[
\boxed{F(x,y) = x^4 y^2}
\]

This interpretation aligns with common algebraic structures and allows us to proceed with analysis.

Defining the Domain \(R.R. 344 \)

Given the lack of explicit boundaries, let's assume the domain is a rectangle:

\[
R = \{ (x,y) \mid a \leq x \leq b, c \leq y \leq d \}
\]

For the purposes of a comprehensive example, we might specify:

\[
a = -2, \quad b = 2, \quad c = -1, \quad d = 1
\]

This choice ensures the domain is a closed and bounded rectangle in \(\mathbb{R}^2 \).

Methodology for Finding Absolute Extrema

The process involves several steps:

1. Identify critical points inside the domain:

- Find points where the gradient of F is zero or undefined.

2. Evaluate the function along the boundary:

- Since the set is closed and bounded, the maximum and minimum can occur at interior critical points or on the boundary.

3. Compare all candidate values:

- Determine which points give the global maxima or minima.

Step 1: Find Critical Points in the Interior

The first step is to compute the partial derivatives of $F(x,y) = x^4 y^2$:

$$
\frac{\partial F}{\partial x} = 4x^3 y^2
$$
$$
\frac{\partial F}{\partial y} = 2x^4 y
$$

Set these derivatives equal to zero to locate critical points:

$$
4x^3 y^2 = 0
$$
$$
2x^4 y = 0
$$

From the first equation:

$$
4x^3 y^2 = 0 \implies x^3 y^2 = 0
$$

This implies either:

- $x^3 = 0 \implies x=0$, or

- $y^2 = 0 \implies y=0$

From the second equation:

$$
2x^4 y = 0 \implies x^4 y = 0
$$

Similarly, either:

- $x=0$, or
- $y=0$

Combining these:

- If $x=0$, then y can be any value in the domain.
- If $y=0$, then x can be any value.

Thus, the critical points in the interior are all points where:

$$
x=0, \quad y \in [c, d]
$$

and

$$
y=0, \quad x \in [a, b]
$$

In particular, the critical lines are:

- $x=0$, for $y \in [-1,1]$
- $y=0$, for $x \in [-2,2]$

Step 2: Analyze the Boundary

Since the critical points along the interior are along the lines $x=0$ and $y=0$, the absolute extrema may occur on these lines or at the corners of the rectangle:

$$
(-2, -1), \quad (-2, 1), \quad (2, -1), \quad (2, 1)
$$

and along the boundary edges:

- $x = a = -2$
- $x = b = 2$
- $y = c = -1$
- $y = d = 1$

Evaluating Along Boundaries

- Along $x = -2$:

$$
F(-2, y) = (-2)^4 y^2 = 16 y^2
$$

for $y \in [-1, 1]$.

- Along $x=2$:

$$
F(2, y) = 16 y^2
$$

for $y \in [-1,1]$.

- Along $y = -1$:

$$
F(x, -1) = x^4 \times 1 = x^4
$$

for $x \in [-2, 2]$.

- Along $y=1$:

$$
F(x, 1) = x^4
$$

for $x \in [-2, 2]$.

Evaluating at Corners

Calculate F at the four corners:

1. \((-2, -1) \):

\[
F(-2, -1) = 16 \times 1 = 16
\]

2. \((-2, 1) \):

\[
F(-2, 1) = 16 \times 1 = 16
\]

3. \((2, -1) \):

\[
F(2, -1) = 16 \times 1 = 16
\]

4. \((2, 1) \):

\[
F(2, 1) = 16 \times 1 = 16
\]

Step 3: Find Maximum and Minimum Values

Now, collect all candidate points and evaluate the function:

Point	\(F(x,y) \)	Notes
\((0, y) \), \(y \in [-1, 1] \)	\(0^4 y^2 = 0 \)	Critical line \(x=0 \)
\((x, 0) \), \(x \in [-2, 2] \)	\(x^4 \times 0 = 0 \)	Critical line \(y=0 \)
Corners: \((-2, -1), (-2, 1), (2, -1), (2, 1) \)	16	Maximum candidates
Along \(x = \pm 2 \), \(y \in [-1, 1] \):

\[
F = 16 y^2
\]

- Max at \(y= \pm 1 \): \(16 \times 1=16 \)

- Min at \(y= 0 \): \(0 \)

- Along \(y= \pm 1 \), \(x \in [-2, 2] \):

\[
F= x^4
\]

- Max at \(x= \pm 2 \): \(16 \)

- Min at \(x=0 \): \(0 \)

Summary of extrema:

- Absolute maximum: \(F=16 \) at points like \((2,1) \), \((-2,1) \), \((2,-1) \), \((-2,-1) \)

- Absolute minimum: \(F=

Frequently Asked Questions

What is the first step to find the absolute extrema of the function f(x, y) = xyx^3y on the set R.R. 344?

The first step is to identify the domain and the boundary of the set R.R. 344, then find the critical points inside the set by setting the partial derivatives of f(x, y) to zero.

How do you determine whether a critical point is an absolute maximum or minimum within the given closed and bounded set?

Evaluate the function at each critical point and compare their values, then also check the function values along the boundary to identify the absolute extrema.

What role does the boundary of the set R.R. 344 play in finding the absolute extrema of the function?

Since the set is closed and bounded, the absolute extrema can occur either at critical points inside the set or on its boundary. Therefore, analyzing the boundary is crucial to find the true global maximum and minimum.

Can Lagrange multipliers be used in this problem, and if so, how?

Yes, Lagrange multipliers can be used if the boundary is defined by an equation. They help find extrema along the boundary by solving the system involving the function's gradient and the boundary constraint.

What are common challenges faced when finding absolute extrema of functions like f(x, y) = xyx^3y on a set like R.R. 344?

Challenges include correctly identifying critical points, handling complex boundary conditions, and ensuring all potential extrema are checked, especially in nonlinear functions with multiple variables.

Why is it important to verify both interior critical points and boundary points when finding the absolute extrema on a closed and bounded set?

Because the absolute maximum and minimum can occur either at critical points inside the set or on the boundary, so comprehensive analysis ensures no potential extrema are overlooked.

Additional Resources

Extrema Analysis: Mastering the Absolute Maxima and Minima of \(f(x,y) = xyx^3 y\) on the Set \(R.R. 344\)

Introduction

When delving into the realm of multivariable calculus, one of the fundamental pursuits is to identify the absolute extrema—the highest and lowest points—of a function within a specified region. This task is not only academically enriching but also practically vital in fields like optimization, economics, engineering, and data science, where maximum efficiency or minimum cost often hinges on such critical points.

Today, we embark on an in-depth exploration of the function:

\[
f(x, y) = xyx^3 y
\]

on a particular closed and bounded set denoted as \(R.R. 344 \). While the notation may seem cryptic initially, our goal is to decode this set, analyze the function thoroughly, and identify its absolute extrema rigorously. Let's approach this as both an expert analysis and a detailed review, unraveling each step with clarity and precision.

Clarifying the Function and the Domain

Understanding the Function

At first glance, the function appears to be:

$$
f(x, y) = xyx^3 y
$$

which, upon closer inspection, seems redundant or possibly miswritten. Typically, in multivariable calculus, such expressions might be:

- $f(x, y) = xy \times x^3 y$, or
- $f(x, y) = x y x^3 y$, which simplifies algebraically.

Assuming the most logical interpretation, the function simplifies as:

$$
f(x, y) = xy \times x^3 y
$$

Let's expand this:

$$
f(x, y) = (x y) \times (x^3 y) = x y \times x^3 y
$$

Multiplying term-by-term:

$$
f(x, y) = x \times x^3 \times y \times y = x^{1+3} y^{1+1} = x^4 y^2
$$

Hence, the function simplifies to:

$$
\boxed{
f(x, y) = x^4 y^2
}
$$

This simplification is crucial for the subsequent analysis.

Deciphering the Set $R.R. 344$

The notation "R.R. 344" appears to be a placeholder or shorthand. In typical calculus contexts, the set where we analyze extrema is a closed and bounded subset of $\mathbb{R}^2$. Common examples include:

- A rectangle $[a, b] \times [c, d]$,

- A disk $\{ (x, y) \mid x^2 + y^2 \leq r^2 \}$,
- Or a more general compact region.

Given the context, "R.R. 344" likely corresponds to a rectangle or a specific region, perhaps:

$$
R = \{ (x, y) \mid a \leq x \leq b, \ c \leq y \leq d \}
$$

Suppose, for clarity, that:

$$
R = \{ (x, y) \mid 0 \leq x \leq 2, \ 0 \leq y \leq 3 \}
$$

This is a typical closed and bounded rectangle in $\mathbb{R}^2$. Alternatively, if the problem specifies a different set, the same methodology applies, provided the set is compact and closed.

For our detailed analysis, let's assume:

$$
\boxed{
\textbf{Set} \ R = [a, b] \times [c, d]
}
$$

with specific bounds:

$$
a = 0, \quad b = 2, \quad c = 0, \quad d = 3
$$

This assumption allows us to demonstrate the approach thoroughly. If you have specific bounds, replace these accordingly.

Step-by-Step Approach to Find the Absolute Extrema

Finding the absolute maximum and minimum of $f(x, y) = x^4 y^2$ on a closed, bounded rectangle involves several systematic steps:

1. Identify Critical Points inside the Region.
2. Evaluate f on the Boundary.
3. Compare Values to Find Global Extrema.

Analyzing Critical Points

Step 1: Compute Partial Derivatives

To find critical points inside the region, we set the partial derivatives to zero:

\[
f_x(x, y) = \frac{\partial}{\partial x} (x^4 y^2) = 4x^3 y^2
\]
\[
f_y(x, y) = \frac{\partial}{\partial y} (x^4 y^2) = 2 x^4 y
\]

Step 2: Solve for Critical Points

Set derivatives equal to zero:

\[
f_x(x, y) = 0 \implies 4x^3 y^2 = 0
\]
\[
f_y(x, y) = 0 \implies 2 x^4 y = 0
\]

From \(f_x = 0\):

\[
4 x^3 y^2 = 0 \implies x^3 y^2 = 0
\]

which implies either:

- \(x = 0\), or
- \(y = 0\)

Similarly, from \(f_y = 0\):

\[
2 x^4 y = 0 \implies x^4 y = 0
\]

which again implies:

- \(x = 0\), or
- \(y = 0\)

Critical points occur where either \(x=0\) or \(y=0\), within our domain.

- At \(x=0\): \(f(0, y) = 0^4 y^2 = 0\) for all \(y\).
- At \(y=0\): \(f(x, 0) = x^4 \times 0^2 = 0\) for all \(x\).

Thus, the entire lines \(x=0\) and \(y=0\) within the domain are critical points, with $f=0$.

Boundary Analysis

Since the domain is a rectangle, the boundary comprises four sides:

- Side 1: $x = a = 0$, with $c \leq y \leq d$.
- Side 2: $x = b = 2$, with $c \leq y \leq d$.
- Side 3: $y = c = 0$, with $a \leq x \leq b$.
- Side 4: $y = d = 3$, with $a \leq x \leq b$.

Let's evaluate f on each.

Side 1: $x=0$

$$
f(0, y) = 0^4 y^2 = 0
$$

for $y \in [0, 3]$. The function is zero along this entire side.

Side 2: $x=2$

$$
f(2, y) = (2)^4 y^2 = 16 y^2
$$

for $y \in [0, 3]$. The maximum value on this side occurs at the highest y:

$$
f(2, 3) = 16 \times 9 = 144
$$

and the minimum at $y=0$:

$$
f(2, 0) = 0
$$

Side 3: $y=0$

$$
f(x, 0) = x^4 \times 0 = 0
$$

for $x \in [0, 2]$.

Side 4: $y=3$

```
\[
f(x, 3) = x^4 \times 9 = 9 x^4
\]

for \( x \in [0, 2] \). The maximum occurs at \( x=2 \):

\[
f(2, 3) = 9 \times 16 = 144
\]

and the minimum at \( x=0 \):

\[
f(0, 3) = 0
\]

---

Summary of Critical and Boundary Values

| Point/Line | \(f(x, y)\) Value | Remarks |
|-------------|--------------------|---------|
| \(x=0\) (entire line) | \(0\) | Critical line |
| \(y=0\) (entire line) | \(0\) | Critical line |
| \((2, y)\), \(y \in [0, 3]\) | \(16 y^2\) | Max at \( y=3 \Rightarrow 144
\) |
| \((x, 3)\), \(x \in [0, 2]\) | \(9 x^4\) | Max at \( x=2 \Rightarrow 144 \)
|

From these, the absolute maximum value within the region is:

\[
\boxed{
f_{\text
```

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