

Find The Derivatives Of The Following Functions $(2x-5x+6x-3)/x$

$$Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$$

$$Y = 5^x - 6 \operatorname{cosec} \frac{1}{2}(x) + \frac{3}{7}(\cos 5x) - 2$$

Find The Derivatives Of The Following Functions $(2x-5x+6x-3)/x$ $Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$ $Y = 5^x - 6 \operatorname{cosec} \frac{1}{2}(x) + \frac{3}{7}(\cos 5x) - 2$

Understanding how to differentiate complex functions is a fundamental skill in calculus. Whether you're tackling problems in mathematics, engineering, or physics, knowing how to find derivatives helps analyze how functions change and behave. In this article, we will explore the process of finding derivatives for several intricate functions, including rational functions, inverse trigonometric functions, exponential functions, and trigonometric functions with fractional exponents. Let's dive into the step-by-step methods to differentiate each function effectively.

Differentiating the Rational Function: $(2x - 5x + 6x - 3) / x$

Simplify the Numerator

Before differentiating, simplify the expression:

- Combine like terms in the numerator:

$$\begin{aligned} 2x - 5x + 6x - 3 &= (2x - 5x + 6x) - 3 \\ &= (2x - 5x + 6x) - 3 \\ &= (3x) - 3 \end{aligned}$$

- The simplified function becomes:

$$f(x) = (3x - 3) / x$$

Rewrite as Separate Terms

Express the function as:

$$f(x) = (3x)/x - 3/x = 3 - 3/x$$

Differentiate Term-by-Term

Now, differentiate each term:

- Derivative of 3 is 0 (constant).

- Derivative of $-3/x$:

Recall that $1/x = x^{-1}$, so:

$$d/dx (-3/x) = -3 d/dx (x^{-1}) = -3 (-1) x^{-2} = 3 x^{-2} = 3 / x^2$$

Final Derivative of the Rational Function

Putting it all together:

$$f'(x) = 0 + 3 / x^2 = 3 / x^2$$

Differentiating $Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$

Recall Derivative Formulas for Inverse Trigonometric Functions

- $d/dx [\sin^{-1}(x)] = 1 / \sqrt{1 - x^2}$
- $d/dx [\cos^{-1}(x)] = -1 / \sqrt{1 - x^2}$
- $d/dx [\ln x] = 1 / x$

Differentiate the Sum of Inverse Trigonometric Functions

Applying the derivatives:

$$d/dx [\sin^{-1}(x) + \cos^{-1}(x)] = 1 / \sqrt{1 - x^2} - 1 / \sqrt{1 - x^2} = 0$$

This simplifies beautifully because these derivatives cancel each other out.

Combine with Derivative of $\ln x$

Therefore, the derivative becomes:

$$Y' = 0 - 1 / x = -1 / x$$

Result: The derivative of the function $Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$ is $-1 / x$.

Differentiating the Function: $Y = 5^x - 6 \csc^{\{1/2\}}(x) + (3/7) \cos(5x) - 2$

Breaking Down the Function

The function involves exponential, trigonometric, and constant terms:

- 5^x (exponential function)
- $\csc^{\{1/2\}}(x)$ (cosecant with fractional exponent)
- $\cos(5x)$
- Constant term (-2)

Derivative of 5^x

Using the exponential rule:

$$\frac{d}{dx} [a^x] = a^x \ln(a)$$

So,

$$\frac{d}{dx} [5^x] = 5^x \ln(5)$$

Derivative of $-6 \csc^{\{1/2\}}(x)$

First, write $\csc^{\{1/2\}}(x)$ as $(\csc x)^{\{1/2\}}$.

Apply the chain rule:

$$\frac{d}{dx} [(\csc x)^{\{1/2\}}] = (1/2) (\csc x)^{\{-1/2\}} \frac{d}{dx} [\csc x]$$

Recall:

$$\frac{d}{dx} [\csc x] = -\csc x \cot x$$

Therefore,

$$\frac{d}{dx} [(\csc x)^{\{1/2\}}] = (1/2) (\csc x)^{\{-1/2\}} (-\csc x \cot x) = - (1/2) \csc x \cot x / (\csc x)^{\{1/2\}}$$

Simplify:

$$- (1/2) \csc x \cot x / (\csc x)^{\{1/2\}} = - (1/2) \cot x (\csc x)^{\{1 - 1/2\}} = - (1/2) \cot x (\csc x)^{\{1/2\}}$$

Multiplying by the coefficient -6:

$$\frac{d}{dx} [-6 \csc^{\{1/2\}}(x)] = -6 [\text{derivative of } \csc^{\{1/2\}}(x)] = -6 [- (1/2) \csc x \cot x / (\csc x)^{\{1/2\}}] = 3 \csc x \cot x / (\csc x)^{\{1/2\}}$$

Expressed more simply:

$$= 3 \cot x (\csc x)^{1/2}$$

Derivative of $(3/7) \cos(5x)$

Apply chain rule:

$$d/dx [\cos(5x)] = -5 \sin(5x)$$

Thus,

$$d/dx [(3/7) \cos(5x)] = (3/7) (-5 \sin(5x)) = -(15/7) \sin(5x)$$

Derivative of constant (-2)

Derivative of a constant is zero.

Summarize the Derivative of Y

Putting all parts together:

$$Y' = 5^x \ln(5) + 3 \cot x (\csc x)^{1/2} - (15/7) \sin(5x)$$

Final derivative:

$$Y' = 5^x \ln(5) + 3 \cot x (\csc x)^{1/2} - (15/7) \sin(5x)$$

Conclusion

Differentiating complex functions involves applying fundamental rules such as the power rule, chain rule, product rule, and recognizing special derivatives for inverse trigonometric and exponential functions. In this article, we've demonstrated how to approach various types of functions step-by-step:

- Simplify expressions where possible before differentiating.
- Use known derivative formulas for inverse trig functions and exponentials.
- Apply chain rule carefully when dealing with composite functions like fractional powers or compositions with trigonometric functions.

Mastering these techniques will enable you to confidently differentiate a wide range of functions, essential for solving advanced calculus problems. Keep practicing with different functions to strengthen your understanding and proficiency in derivatives.

Keywords: derivatives, calculus, differentiate functions, inverse trigonometric derivatives, exponential derivatives, trigonometric functions, chain rule, quotient rule, derivatives of complex functions

Frequently Asked Questions

What is the derivative of the function $f(x) = (2x - 5x + 6x - 3) / x$?

First, simplify the numerator: $(2x - 5x + 6x - 3) = (3x - 3)$. So, $f(x) = (3x - 3)/x = 3 - 3/x$. The derivative is $f'(x) = 0 + 3/x^2 = 3/x^2$.

How do you find the derivative of $Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$?

Recall that $\sin^{-1}(x) + \cos^{-1}(x) = \pi/2$, which is constant. Therefore, its derivative is zero. The derivative of $-\ln x$ is $-1/x$. Hence, $Y' = 0 - 1/x = -1/x$.

What is the derivative of $Y = 5^x - 6\csc^{1/2}(x) + (3/7)\cos(5x) - 2$?

Derivative of 5^x is $5^x \ln 5$. For $-6\csc^{1/2}(x)$, rewrite as $-6[\csc(x)]^{1/2}$; using chain rule, derivative involves $-6(1/2)\csc^{-1/2}(x)(-\csc x \cot x) = 3\csc^{-1/2}(x)\csc x \cot x$. Derivative of $(3/7)\cos(5x)$ is $-(3/7)5\sin(5x)$. The derivative of -2 is 0. Summing up: $Y' = 5^x \ln 5 + 3\csc^{-1/2}(x)\csc x \cot x - (15/7)\sin(5x)$.

How do you differentiate the inverse trigonometric functions $\sin^{-1}(x)$ and $\cos^{-1}(x)$?

The derivatives are $d/dx [\sin^{-1}(x)] = 1 / \sqrt{1 - x^2}$ and $d/dx [\cos^{-1}(x)] = -1 / \sqrt{1 - x^2}$.

What is the derivative of a composite function involving powers of cosecant, such as $\csc^{1/2}(x)$?

Using chain rule, $d/dx [\csc^{1/2}(x)] = (1/2)\csc^{-1/2}(x)(-\csc x \cot x) = -(1/2)\csc^{-1/2}(x)\csc x \cot x$.

In the function $Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$, why does

the derivative simplify to $-1/x$?

Because $\sin^{-1}(x) + \cos^{-1}(x)$ equals $\pi/2$, a constant, whose derivative is zero. The derivative of $-\ln x$ is $-1/x$, so the entire derivative simplifies to $-1/x$.

Can you explain the derivative of functions involving exponential bases like 5^x ?

Yes. The derivative of a^x , where $a > 0$, is $a^x \ln a$. So, for 5^x , the derivative is $5^x \ln 5$.

Additional Resources

Mastering Derivatives: A Comprehensive Guide to Differentiating Complex Functions

Introduction

Calculus is a foundational pillar of advanced mathematics, physics, engineering, and numerous scientific disciplines. At the heart of calculus lies the concept of derivatives—tools that measure how functions change concerning variables. Derivatives enable us to analyze rates of change, slopes of curves, and optimize functions in real-world applications.

In this detailed review, we will explore the process of finding derivatives for two intricate functions, emphasizing the techniques, rules, and understandings essential for tackling such problems. These functions are:

- $$Y = \frac{2x - 5x + 6x - 3}{x}$$
- $$Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$$
- $$Y = 5^x - 6 \csc^{1/2}(x) + \frac{3}{7} \cos 5x - 2$$

(Note: The original prompt seems to include multiple functions; for clarity and depth, we will focus on these, elaborating on their derivatives with comprehensive explanations.)

Understanding the Fundamentals of Derivatives

Before delving into specific functions, it's crucial to review fundamental derivative rules and concepts:

- Power Rule: $\frac{d}{dx} [x^n] = n x^{n-1}$
- Constant Rule: Derivative of a constant is zero.
- Constant Multiple Rule: $\frac{d}{dx} [c \cdot f(x)] = c \cdot f'(x)$
- Sum/Difference Rule: $\frac{d}{dx} [f(x) \pm g(x)] = f'(x) \pm g'(x)$
- Product Rule: $\frac{d}{dx} [f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- Quotient Rule: $\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
- Chain Rule: $\frac{d}{dx} [f(g(x))] = f'(g(x)) \cdot g'(x)$
- Inverse Trigonometric Derivatives:
 - $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$

$$-\frac{d}{dx} \cos^{-1}(x) = -\frac{1}{\sqrt{1-x^2}}$$

- Exponential and Logarithmic Derivatives:

$$\frac{d}{dx} a^x = a^x \ln a$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

Analyzing and Deriving the First Function

Function 1: Simplification and Differentiation

$$Y = \frac{2x - 5x + 6x - 3}{x}$$

Step 1: Simplify the numerator

Combine like terms:

$$2x - 5x + 6x - 3 = (2x - 5x + 6x) - 3 = (2x - 5x + 6x) - 3$$

Calculate:

$$(2x - 5x + 6x) = (2 - 5 + 6)x = 3x$$

Thus, numerator simplifies to:

$$3x - 3$$

Step 2: Rewrite the function

$$Y = \frac{3x - 3}{x}$$

Express as:

$$Y = \frac{3x}{x} - \frac{3}{x} = 3 - 3x^{-1}$$

Step 3: Differentiate

Applying the derivatives:

$$\begin{aligned} Y' &= 0 - 3 \cdot (-1) x^{-2} = 3 x^{-2} = \frac{3}{x^2} \end{aligned}$$

Final derivative:

$$\boxed{\frac{dy}{dx} = \frac{3}{x^2}}$$

Note: The simplification process is crucial to make differentiation straightforward. Always look for algebraic simplifications before applying calculus rules.

Analyzing and Differentiating the Second Function

Function 2: $Y = \sin^{-1}(x) + \cos^{-1}(x) - \ln x$

This function combines inverse trigonometric functions and a logarithm, both of which have well-established derivatives.

Step 1: Recall derivatives

$$\begin{aligned} - \frac{d}{dx} \sin^{-1}(x) &= \frac{1}{\sqrt{1-x^2}} \\ - \frac{d}{dx} \cos^{-1}(x) &= -\frac{1}{\sqrt{1-x^2}} \\ - \frac{d}{dx} \ln x &= \frac{1}{x} \end{aligned}$$

Step 2: Differentiate term-by-term

$$Y' = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} - \frac{1}{x}$$

Notice that the first two terms cancel out:

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

Thus:

$$Y' = -\frac{1}{x}$$

Final derivative:

$$\boxed{\frac{dy}{dx} = -\frac{1}{x}}$$

Insight and interpretation:

The sum of the inverse sine and cosine functions simplifies to a constant over their domain, which is consistent with the identity:

$$\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$$

for all $x \in [-1, 1]$. Differentiating this constant yields zero, but the subtraction of $(\ln x)$ introduces the derivative $(-1/x)$.

Analyzing and Differentiating the Third Function

Function 3: $(Y = 5^x - 6 \csc^{1/2}(x) + \frac{3}{7} \cos 5x - 2)$

This function combines exponential, trigonometric, and algebraic components. Let's analyze each term independently.

Term 1: (5^x)

- Derivative:

$$\frac{d}{dx} 5^x = 5^x \ln 5$$

Term 2: $(-6 \csc^{1/2}(x))$

- Rewriting:

$$-6 \left(\csc x \right)^{1/2}$$

- Derivative:

Applying the chain rule:

$$\frac{d}{dx} \left(\csc x \right)^{1/2} = \frac{1}{2} \left(\csc x \right)^{-1/2} \cdot \frac{d}{dx} \csc x$$

Recall:

$$\frac{d}{dx} \csc x = -\csc x \cot x$$

Therefore,

$$\begin{aligned} \frac{d}{dx} \left(\csc x \right)^{1/2} &= \frac{1}{2} \left(\csc x \right)^{-1/2} \cdot (-\csc x \cot x) \\ &= -\frac{1}{2} \csc^{-1/2} x \cdot \cot x \end{aligned}$$

Multiplying by (-6) :

$$-6 \times \left(-\frac{1}{2} \csc^{-1/2} x \cdot \cot x \right) = 3 \csc^{-1/2} x \cot x$$

Term 3: $\left(\frac{3}{7} \cos 5x \right)$

- Derivative:

$$\frac{3}{7} \times (-5) \sin 5x = -\frac{15}{7} \sin 5x$$

Term 4: (-2)

- Derivative:

$$0$$

Combined derivative:

$$Y' = 5^x \ln 5 + 3 \csc^{-1/2} x \cot x - \frac{15}{7} \sin 5x$$

Final expression for the derivative:

$$\boxed{\frac{dy}{dx} = 5^x \ln 5 + 3 \csc^{-1/2} x \cot x - \frac{15}{7} \sin 5x}$$

Deep

Find The Derivatives Of The Following Functions $2x^5x^6x^3XY \sin 1X \cos 1X \ln x Y^5X^6 \operatorname{cosec} \frac{1}{2}X^{\frac{3}{7}} \cos 5x^2$

Find The Derivatives Of The Following Functions $2x^5x^6x^3XY \sin 1X \cos 1X \ln x Y^5X^6 \operatorname{cosec} \frac{1}{2}X^{\frac{3}{7}} \cos 5x^2$

Related Articles

- [When There Are Batch-level Or Product-level Costs, In Comparison To A Traditional Cost System, An Activity-based](#)
- [In the figure below, N lies between M and P. Find the location of N so that the ratio of MN to NP is 4 to 5.](#)
- [The Year Is 2186. In An Effort To Eliminate Discrimination and Competition, Humans Have Been Genetically engineered](#)

[Back to Home](#)