Find The Equation Of The Plane Tangent ToF(x,y) = 16-x - y At The Point (2, -1).Answer: Z=_________x+_____________y+

Find The Equation Of The Plane Tangent To F(x,y) = 16 - x - y At The Point (2, -1). Answer: Z=_________x+_____________y+

Understanding the Problem: Finding the Tangent Plane to a Surface

In multivariable calculus, a common problem involves finding the equation of a tangent plane to a surface defined by a function $z = f(x, y)$ at a specific point. Here, the given function is:

$$
f(x, y) = 16 - x - y
$$

and the point of interest is $(x, y) = (2, -1)$. The goal is to determine the explicit equation of the tangent plane at this point, which can be generally expressed as:

$$
z = a x + b y + c
$$

or in the standard form:

$$
z - z_0 = p (x - x_0) + q (y - y_0)
$$

where:

- (x_0, y_0, z_0) is the point of tangency,
- $p = \frac{\partial f}{\partial x}$ evaluated at the point,
- $q = \frac{\partial f}{\partial y}$ evaluated at the point.

Step-by-Step Approach to Find the Tangent Plane Equation

1. Confirm the Function and Point

Given:

$$
f(x, y) = 16 - x - y
$$

Point:

$$
(x_0, y_0) = (2, -1)
$$

Calculate the corresponding z_0:

$$
z_0 = F(2, -1) = 16 - 2 - (-1) = 16 - 2 + 1 = 15
$$

So, the point of tangency in 3D space is:

$$
(2, -1, 15)
$$

2. Find Partial Derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$

- Partial derivative with respect to x:

$$
\frac{\partial F}{\partial x} = -1
$$

- Partial derivative with respect to y:

$$
\frac{\partial F}{\partial y} = -1
$$

Since the function is linear, these derivatives are constant everywhere, including at the point of interest.

3. Write the Equation of the Tangent Plane

The general form of the tangent plane to $z = F(x, y)$ at (x_0, y_0, z_0) is:

$$
z = z_0 + \frac{\partial F}{\partial x} (x - x_0) + \frac{\partial F}{\partial y} (y - y_0)
$$

Substituting the known values:

$$
z = 15 + (-1)(x - 2) + (-1)(y + 1)
$$

Simplify:

$$
z = 15 - (x - 2) - (y + 1)
$$

\[

$$
z = 15 - x + 2 - y - 1
$$

Combine like terms:

$$
z = (15 + 2 - 1) - x - y = 16 - x - y
$$

However, this is the original function $f(x, y)$, which indicates the surface itself is a plane, and the tangent plane at any point is the surface itself.

Final Equation of the Tangent Plane

Given all the calculations, the tangent plane at $(2, -1)$ is:

$$
z = -1 \cdot x + (-1) \cdot y + 16
$$

or, better written as:

$$
z = -x - y + 16
$$

In the form requested:

$$
z = \underline{-1} x + \underline{-1} y + \underline{16}
$$

which simplifies to:

$$
z = -x - y + 16
$$

Answer:

$$
z = \boxed{-1x - 1y + 16}
$$

or, in the blank format:

$$
z = \underline{-1} x + \underline{-1} y + 16
$$

Additional Insights and Practical Applications

Why Is the Tangent Plane Important?

Understanding the tangent plane to a surface at a given point is fundamental in calculus because:

- It provides a linear approximation of the surface near that point.
- It helps in understanding the local behavior of the surface.
- It is essential in optimization problems, where tangent planes can be used to find maxima and minima.
- It plays a crucial role in differential geometry and in the computation of derivatives on manifolds.

Real-World Applications

- Engineering and Design: In manufacturing, tangent planes assist in understanding the surface's local geometry, which is essential for machining and surface finishing.
- Physics: Surface approximations help in modeling gravitational or electromagnetic fields near a surface.
- Computer Graphics: Rendering realistic surfaces relies on tangent planes for shading and lighting calculations.

Summary of Key Steps

To summarize, here are the main steps to find the tangent plane to a surface:

1. Identify the surface function $F(x, y)$ and the point of tangency (x_0, y_0).

2. Calculate the z-coordinate at that point: $z_0 = F(x_0, y_0)$.

3. Compute the partial derivatives $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$ at (x_0, y_0).

4. Write the tangent plane equation using the point-slope form of the plane:

$$
z = z_0 + \frac{\partial F}{\partial x}(x - x_0) + \frac{\partial F}{\partial y}(y - y_0)
$$

5. Simplify to get the explicit plane equation in the form $z = ax + by + c$.

Conclusion

In conclusion, the equation of the tangent plane to the surface $F(x, y) = 16 - x - y$ at the point $(2, -1)$ is:

$$
z = -x - y + 16
$$

This plane provides the best linear approximation of the surface at that point and is crucial for understanding local surface behavior. The process involves calculating the function value at the point, the partial derivatives, and then assembling these into the plane equation. This methodology is widely applicable across various disciplines where surface analysis is essential.

Remember: Mastering the process of finding tangent planes enhances your understanding of multivariable calculus and enables you to analyze complex surfaces with confidence and precision.

Frequently Asked Questions

What is the general method to find the tangent plane to a surface defined by F(x, y) at a given point?

The tangent plane can be found by computing the gradient vector of F at the point and using the point-normal form: $z - z_0 = \nabla F \cdot (x - x_0, y - y_0)$.

How do you determine the partial derivatives of F(x,y) = 16 - x - y?

The partial derivative with respect to x is $\partial F/\partial x = -1$, and with respect to y is $\partial F/\partial y = -1$.

What is the gradient vector of F(x,y) = 16 - x - y at the point (2, -1)?

The gradient vector is $\nabla F = (-1, -1)$.

How do you find the z-coordinate corresponding to the point (2, -1) on the surface?

Substitute x=2 and y=-1 into F(x,y): $F(2, -1) = 16 - 2 - (-1) = 15$.

What is the equation of the tangent plane to the surface at the point (2, -1)?

Using the gradient and point: $z - 15 = -1(x - 2) - 1(y + 1)$.

How do you simplify the tangent plane equation to the form Z=...x + ... y + ...?

Expanding and rearranging: $z - 15 = -1x + 2 - 1y - 1$, so $z = -x - y + 16$.

What is the final equation of the tangent plane in the requested format?

$Z = -1x - 1y + 16$.

Why is the tangent plane to the surface at (2, -1) given by Z = -x - y + 16?

Because the gradient vector at that point is (-1, -1), and the point's z-value is 15, leading to the tangent plane equation Z = -x - y + 16.

Additional Resources

Find The Equation Of The Plane Tangent To F(x,y) = 16 - x - y At The Point (2, -1). Answer: Z = __________x + __________ y + __________

Understanding how to find the tangent plane to a surface at a specific point is a fundamental skill in multivariable calculus, with applications ranging from engineering to physics. In this article, we will explore the process step-by-step to determine the equation of the tangent plane for the surface defined by the function $F(x,y) = 16 - x - y$, at the point $(2, -1)$. By the end, you'll grasp how to compute the tangent plane's equation in a clear, precise manner.

Introduction to the Problem

Given the function $F(x,y) = 16 - x - y$, which implicitly defines a surface in three-dimensional space as $z = F(x,y)$, our goal is to find the equation of the tangent plane to this surface at the point $(2, -1)$. The final form seeks to be expressed as:

$$z = a x + b y + c$$

where a, b, and c are constants to be determined.

The Significance of Tangent Planes in Multivariable Calculus

Before we delve into the calculations, it's essential to understand why tangent planes are significant:

- Local Linear Approximation: The tangent plane provides the best linear approximation of the surface near a given point.
- Surface Behavior: It helps visualize how the surface behaves in the immediate vicinity of the point.
- Applications: Used in optimization, 3D modeling, and understanding gradients.

Step 1: Understanding the Surface and the Point

Our surface is defined by:

$$z = F(x,y) = 16 - x - y$$

The point of interest is $(x_0, y_0) = (2, -1)$. To find the tangent plane at this point, we need to determine the corresponding z-coordinate:

$$z_0 = F(2, -1) = 16 - 2 - (-1) = 16 - 2 + 1 = 15$$

Thus, the point on the surface is $(2, -1, 15)$.

Step 2: Computing Partial Derivatives

The gradient vector, which is essential in defining the tangent plane, involves the partial derivatives of F:

- $F_x = \frac{\partial F}{\partial x}$
- $F_y = \frac{\partial F}{\partial y}$

For our function:

$$F_x = -1$$
$$F_y = -1$$

Both derivatives are constants, reflecting the linear nature of the surface.

Step 3: Formulating the Equation of the Tangent Plane

The general formula for the tangent plane to a surface $z = F(x,y)$ at (x_0, y_0, z_0) is:

$$z - z_0 = F_x(x_0, y_0)(x - x_0) + F_y(x_0, y_0)(y - y_0)$$

Since F_x and F_y are constants, the equation simplifies to:

$$z - 15 = -1 (x - 2) - 1(y + 1)$$

Breaking it down:

$$
z - 15 = -(x - 2) - (y + 1)
$$

$$
z - 15 = -x + 2 - y - 1
$$

$$
z - 15 = -x - y + 1
$$

Adding 15 to both sides:

$$
z = -x - y + 1 + 15
$$

$$
z = -x - y + 16
$$

Thus, the equation of the tangent plane in the form $z = a x + b y + c$ is:

$$z = -1 \times x + -1 \times y + 16$$

or more neatly:

$$\boxed{z = -x - y + 16}$$

Final Answer: The Equation of the Tangent Plane

In the form specified in the problem statement:

$$\boxed{\textbf{Z} = \underline{-1} \times \textbf{x} + \underline{-1} \times \textbf{y} + \underline{16}}$$

which simplifies to:

$$Z = -1x - 1y + 16$$

Additional Insights and Applications

Why Is This Important?

The process illustrated above is foundational in multivariable calculus. It allows engineers and scientists to approximate complex surfaces locally, aiding in:

- Optimization tasks: Finding maxima or minima on surfaces.
- Design and modeling: Creating accurate local linear models.
- Analysis of gradients: Understanding the direction of steepest ascent or descent.

Extending the Concept

While the example considers a straightforward linear function, the approach generalizes to more complex, nonlinear surfaces. In such cases, partial derivatives may vary with x and y, and the tangent plane's equation would involve actual point-dependent derivatives.

Summary

- The surface $z = 16 - x - y$ is linear, making the tangent plane straightforward to compute.
- The partial derivatives are constant: $f_x = -1$ and $f_y = -1$.
- The tangent plane at $(2, -1)$ passes through $(2, -1, 15)$.
- The resulting tangent plane equation is:

$$Z = -x - y + 16$$

This exercise underscores the elegance and practicality of differential calculus in understanding and approximating surfaces in three-dimensional space. Whether in academic settings or real-world applications, mastering the derivation of tangent planes is an essential step toward a deeper grasp of multivariable analysis.

Find The Equation Of The Plane Tangent Tof X Y 16 X Y At The Point 2 1 Answer Z X Y

Find The Equation Of The Plane Tangent Tof X Y 16 X Y At The Point 2 1 Answer Z X Y

Related Articles

- Find The Marginal Profit For Producing X Units. (The Profit Is Measured In Dollars.) P=0.55x ^{2} +3,000x1,150,000
- What Must People Provide In Order To Be Governed Under A Social Contract?rational Consent To The Governmentallegiance

- Identify The Incorrect Statement(s). A Solution _____ I. Can Be A Solid, Liquid, Or Gas. Ii. Can Be Heterogeneous