Find The Limit. Enter DNE If The Limit Does Not Exist.(x,y) Lim (0,0) X2y/ X2 +9y2 = ..."""

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Understanding limits in multivariable calculus is essential for analyzing the behavior of functions as they approach specific points. The problem at hand involves evaluating the limit:

$$
\lim_{(x,y) \to (0,0)} \frac{x^2 y}{x^2 + 9 y^2}
$$

This type of limit explores how the function behaves as both variables, x and y, tend toward zero simultaneously. Determining whether this limit exists requires examining the function's behavior along different paths toward the origin. If the limit values differ along different paths, the overall limit does not exist (DNE). Conversely, if all paths lead to the same value, that value is the limit.

In this article, we will explore various methods to evaluate this limit, including direct substitution, path testing, and using polar coordinates. We will also discuss the importance of understanding the behavior of functions in multiple variables and provide guidance on how to approach similar problems.

Understanding the Function and Its Behavior

Before diving into calculations, it's important to understand the structure of the given function:

$$
f(x,y) = \frac{x^2 y}{x^2 + 9 y^2}
$$

- The numerator, $x^2 y$, tends to zero as $(x,y) \to (0,0)$ because x^2 and y both approach zero.
- The denominator, $x^2 + 9 y^2$, is always non-negative and approaches zero as $(x,y) \to (0,0)$.

The key question is: does the ratio of these two expressions approach a single finite value as (x,y) approaches $(0,0)$? To answer this, we need to analyze the behavior along different paths approaching the origin.

Evaluating the Limit Along Different Paths

Path analysis is a straightforward way to detect potential discrepancies in the limit's value.

Path 1: $y = 0$

- Substituting $y=0$:

$$
f(x,0) = \frac{x^2 \cdot 0}{x^2 + 9 \cdot 0^2} = 0
$$

- As $x \to 0$, $f(x,0) \to 0$.

Result along Path 1: The limit approaches 0.

Path 2: $x = 0$

- Substituting $x=0$:

$$
f(0,y) = \frac{0 \cdot y}{0 + 9 y^2} = 0
$$

- As $y \to 0$, $f(0,y) \to 0$.

Result along Path 2: The limit also approaches 0.

Path 3: $y = m x$, where m is a constant

- Substituting $y = m x$:

$$
f(x, m x) = \frac{x^2 (m x)}{x^2 + 9 (m x)^2} = \frac{m x^3}{x^2 + 9 m^2 x^2}
$$

- Simplify numerator and denominator:

$$
= \frac{m x^3}{x^2 (1 + 9 m^2)} = \frac{m x}{1 + 9 m^2}
$$

- As $x \to 0$, $f(x, m x) \to 0$.

Result along Path 3: Approaches 0 regardless of m.

These initial path tests suggest the limit might be 0, but to confirm, we need to analyze the behavior more generally, especially considering more complex paths.

Using Polar Coordinates to Evaluate the Limit

Polar coordinates provide a systematic way to analyze limits approaching the origin, since they convert the two-variable approach into a single variable r:

$$
x = r \cos \theta,\quad y = r \sin \theta
$$

As $(x,y) \to (0,0)$, $r \to 0$. Substituting into the function:

$$
f(r,\theta) = \frac{(r \cos \theta)^2 \cdot r \sin \theta}{(r \cos \theta)^2 + 9 (r \sin \theta)^2}
$$

Simplify numerator and denominator:

$$
= \frac{r^3 \cos^2 \theta \sin \theta}{r^2 (\cos^2 \theta + 9 \sin^2 \theta)}
= \frac{r^3 \cos^2 \theta \sin \theta}{r^2 (\cos^2 \theta + 9 \sin^2 \theta)}
$$

Cancel r^2:

$$
= r \frac{\cos^2 \theta \sin \theta}{\cos^2 \theta + 9 \sin^2 \theta}
$$

Now, as $r \to 0$:

$$
f(r, \theta) \to 0 \times \frac{\cos^2 \theta \sin \theta}{\cos^2 \theta + 9 \sin^2 \theta} = 0
$$

since the remaining factor is bounded (since $\cos^2 \theta$ and $\sin \theta$ are bounded between -1 and 1).

Conclusion: The limit approaches 0 regardless of the path taken toward the origin.

Final Determination of the Limit

Based on the path analysis and the polar coordinate approach, the function approaches 0 along all paths toward $(0, 0)$. Therefore, the limit exists and is equal to 0.

Answer:

$$
\boxed{0}
$$

Summary and Key Takeaways

- When evaluating multivariable limits, start with simple paths such as $x=0$, $y=0$, and lines $y=mx$.
- Check the behavior along various paths to see if they yield different

limits; if they do, the overall limit does not exist.
- Polar coordinates are a powerful tool to analyze the behavior near the origin uniformly, especially when the limit is symmetric or involves quadratic forms.
- If all paths lead to the same limit, that value is the limit of the function at the point.

Common Mistakes and Tips for Limit Evaluation in Multiple Variables

- **Assuming the limit exists without testing multiple paths**: Always verify along different paths.
- **Ignoring paths that could lead to different limits**: Sometimes, certain paths reveal that the limit does not exist.
- **Using polar coordinates**: When the function involves quadratic forms, converting to polar coordinates simplifies the analysis.
- **Paying attention to indeterminate forms**: Just because substitution yields 0/0 doesn't mean the limit is 0; path analysis confirms the behavior.

Conclusion

The limit

$$
\lim_{(x,y) \to (0,0)} \frac{x^2 y}{x^2 + 9 y^2}
$$

exists and is equal to 0. This conclusion is supported by examining multiple paths and employing polar coordinates, which demonstrate that the function's value approaches zero regardless of the approach route. When tackling similar problems, always consider multiple paths and coordinate transformations to accurately determine the existence and value of multivariable limits.

Remember: If you find different limits along different paths, the overall limit does not exist (DNE). If all paths lead to the same value, that value is the limit.

Frequently Asked Questions

What is the limit of $(x,y) \to (0,0)$ for the function $f(x,y) = (x^2 y) / (x^2 + 9 y^2)$?

The limit is DNE (Does Not Exist) because the limit depends on the path taken towards (0,0).

How do you determine if the limit of a multivariable function exists at a point?

You evaluate the limit along different paths approaching the point; if the limits differ, the overall limit does not exist.

What are common paths to test for the limit of (x,y) → (0,0) in this problem?

Paths such as $y = 0$, $y = x$, and $y = kx$ are common to see if the limit varies with different approaches.

What is the limit of (x,y) → (0,0) along y = 0?

Along $y = 0$, the function becomes $(x^2\ 0) / (x^2 + 0) = 0 / x^2 = 0$, so the limit along this path is 0.

What is the limit along y = x?

Substituting $y = x$ gives $(x^2\ x) / (x^2 + 9\ x^2) = x^3 / (10\ x^2) = x / 10$, which approaches 0 as $x \to 0$.

Why does the limit not exist if different paths yield different limits?

Because the limit depends on the path taken, indicating the function's behavior is not consistent in all directions, so the overall limit does not exist.

Additional Resources

Find The Limit. Enter DNE If The Limit Does Not Exist. (x,y) Lim (0,0) $(x^2y) / (x^2 + 9y^2)$ = ...

When tackling multivariable calculus problems, especially those involving limits as (x, y) approaches a particular point, the phrase "Find The Limit. Enter DNE If The Limit Does Not Exist." often appears as a directive. These problems challenge students and practitioners to analyze the behavior of functions in two variables near a specific point—in this case, the origin (0,0). The particular function under consideration, $f(x, y) = (x^2y) / (x^2 + 9y^2)$, encapsulates the intricacies of multivariable limits, especially when the limit's existence depends on the path taken towards the point (0,0). Understanding whether this limit exists and, if so, what its value is, requires a systematic approach that combines algebraic manipulation, substitution, and perhaps even more advanced techniques like polar coordinates. This comprehensive review aims to dissect this problem thoroughly, providing clarity and insight into the methodologies involved.

Understanding Multivariable Limits

What Is a Multivariable Limit?

In single-variable calculus, limits explore the behavior of a function as the input approaches a specific point. For functions of two variables, (x, y), the concept extends naturally but introduces new complexities. The limit of a function $f(x, y)$ as (x, y) approaches (a, b) is the value L if, for every sequence of points (x_n, y_n) approaching (a, b), the function values $f(x_n, y_n)$ approach L.

Mathematically:
$$
\lim_{(x, y) \to (a, b)} f(x, y) = L
$$
if for every $\varepsilon > 0$, there exists $\delta > 0$ such that whenever $\sqrt{(x - a)^2 + (y - b)^2} < \delta$, it follows that $|f(x, y) - L| < \varepsilon$.

Key Point:
- The limit must be the same regardless of the path taken toward the point (a, b).
- If different paths yield different limits, the overall limit does not exist (DNE).

Why Is It More Complex Than Single-Variable Limits?

In single-variable calculus, the approach toward a point is straightforward along a single line. In two variables, multiple paths can be taken to approach the same point, and the function's behavior along these paths may differ. This multiplicity makes the analysis of limits more nuanced. For example, approaching $(0, 0)$ along $y = 0$ might give a different value than approaching along $y = x$, indicating the limit does not exist.

Analyzing the Function: $f(x, y) = (x^2 y) / (x^2 + 9y^2)$

Initial Observations and Domain

The function $f(x, y) = (x^2 y) / (x^2 + 9 y^2)$ is rational, with the denominator zero when $x = 0$ and $y = 0$. Since at $(0, 0)$, the denominator becomes zero, the function is not defined there, which prompts us to investigate the limit as (x, y) approaches $(0, 0)$. The key questions are:

- Does the limit exist at $(0, 0)$?
- If it does, what is its value?
- If not, how do different paths influence the limit?

Features:
- The numerator involves $x^2 y$, which suggests symmetry with respect to x^2.
- The denominator involves $x^2 + 9 y^2$, which is always non-negative and zero only at (0, 0).
- The structure hints that polar coordinates might simplify the analysis.

Methodologies for Limit Evaluation

Path Analysis

The first step often involves approaching (0, 0) along different paths to see if the limit is consistent. Common paths include:

- $y = 0$
- $x = 0$
- $y = mx$ (a straight line with slope m)
- $y = k x^n$ (power functions)

If different paths yield different limits, then the overall limit DNE.

Polar Coordinates

Converting to polar coordinates simplifies the approach by expressing x and y in terms of r and θ:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

As $r \to 0$, the point approaches (0, 0) regardless of θ. The function becomes:

$$
f(r, \theta) = \frac{(r \cos \theta)^2 \times r \sin \theta}{(r \cos \theta)^2 + 9 (r \sin \theta)^2}
= \frac{r^3 \cos^2 \theta \sin \theta}{r^2 (\cos^2 \theta + 9 \sin^2 \theta)}
$$

Simplifying:

$$
f(r, \theta) = \frac{r \cos^2 \theta \sin \theta}{\cos^2 \theta + 9 \sin^2 \theta}
$$

Now, as $r \to 0$, the numerator tends to zero, and the denominator is independent of r.

The limit becomes:

$$
\lim_{r \to 0} f(r, \theta) = 0
$$

since the numerator approaches zero for all θ, and the denominator is finite and non-zero (except at points where $\cos^2 \theta + 9 \sin^2 \theta = 0$), which doesn't happen for real θ).

This suggests the limit might be zero, but we must verify if the behavior is uniform across all θ.

Path-Based Analysis

Approach Along y = 0

Substituting $y = 0$ into $f(x, y)$:
$$
f(x, 0) = \frac{x^2 \times 0}{x^2 + 0} = 0
$$
As x approaches 0, $f(x, 0) \to 0$.

Approach Along x = 0

Substituting $x = 0$:
$$
f(0, y) = \frac{0 \times y}{0 + 9 y^2} = 0
$$
As y approaches 0, $f(0, y) \to 0$.

Approach Along y = m x

Substitute $y = m x$:
$$
f(x, m x) = \frac{x^2 \times m x}{x^2 + 9 m^2 x^2} = \frac{m x^3}{x^2 (1 + 9 m^2)} = \frac{m x}{1 + 9 m^2}
$$
As $x \to 0$, $f(x, m x) \to 0$, regardless of m.

Approach Along y = k x^n

Suppose $y = k x^n$, for some $n > 0$:
$$
f(x, k x^n) = \frac{x^2 \times k x^n}{x^2 + 9 k^2 x^{2 n}} = \frac{k x^{n+2}}{x^2 + 9 k^2 x^{2 n}}
$$
Divide numerator and denominator by x^2:
$$
= \frac{k x^{n}}{1 + 9 k^2 x^{2 n - 2}}
$$
As $x \to 0$, numerator $\to 0$. For the denominator:

- If $2 n - 2 > 0 \Rightarrow n > 1$, then $x^{2 n - 2} \to 0$, so denominator $\to 1$.
- If $n = 1$, denominator $\to 1 + 9 k^2$, finite and non-zero.
- If $0 < n < 1$, then $x^{2 n - 2} \to \infty$, and the denominator tends to infinity, making the entire expression tend to zero.

Thus, along these paths, the limit tends to zero.

Conclusion from Path Analysis:
All paths examined yield the same limit, zero, indicating the limit might exist and be zero.

Formal Verification Using Polar Coordinates

Using the earlier derived form in polar coordinates:

$$
f(r, \theta) = \frac{r \cos^2 \theta \sin \theta}{\cos^2 \theta + 9 \sin^2 \theta}
$$

As $r \to 0$, the entire expression approaches zero regardless of θ, since the numerator involves r, which tends to zero, and the denominator remains finite and positive for all θ (since $\cos^2 \theta + 9 \sin^2 \theta \geq 1 \times \sin^2 \theta \geq 0$), and only zero if both

Find The Limit Enter Dne If The Limit Does Not Exist X Y Lim 0 0 X2y X2 9y2

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