Find The Truth Set Of Each Of These Predicates Where The Domain Is The Set Of Integers. Show All Work.P(x):x31Q(x):x2=2R(x):x

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In the realm of formal logic and predicate calculus, determining the truth set of predicates over a specific domain is a fundamental task. When the domain is the set of integers, it involves analyzing each predicate carefully, understanding what conditions they impose on the elements of the domain, and identifying all integers that satisfy these conditions. This comprehensive guide aims to walk through each predicate—P(x), Q(x), and R(x)—step-by-step, showing all work involved in deriving their respective truth sets within the integers. Whether you're a student, a teacher, or a logic enthusiast, this article will clarify the process and provide a detailed explanation to enhance your understanding.

Understanding the Predicates and Domain

Before diving into the solutions, let's clarify the predicates and domain:

- Domain: The set of all integers, denoted as $\mathbb{Z}$.
- Predicates:
- P(x): $x \geq 31$
- Q(x): $x^2 = 2$
- R(x): x (the predicate description is incomplete in the provided prompt, but assuming it is "x" or a relation involving x, we will interpret it based on typical logical notation. For this analysis, we'll assume R(x) is a predicate involving x, but since the description is incomplete, we will clarify later.)

Note: The prompt appears to have some formatting issues, especially with R(x). It is likely that the original intended predicate for R(x) was incomplete or misformatted. For the purpose of this article, let's assume R(x): x is even, which is a common predicate involving integers. If you have a different predicate in mind, please specify, and we can analyze accordingly.

Analyzing Each Predicate

Let's analyze each predicate in detail, determine their truth sets, and explain the reasoning process.

$P(x)$: $x \geq 31$

This predicate states that x is greater than or equal to 31.

Step 1: Understand the predicate

- $P(x)$ is true when $x \geq 31$.

Step 2: Identify the truth set

- Since the domain is all integers, the set of all integers x such that $x \geq 31$.

Step 3: Write the truth set

- The truth set, denoted as T_P, is:

$$T_P = \{ x \in \mathbb{Z} \mid x \geq 31 \}$$

Step 4: Express the set

- All integers starting from 31 and increasing without bound:

$$T_P = \{ 31, 32, 33, 34, \ldots \}$$

Summary:

- The truth set of $P(x)$ over the integers is all integers greater than or equal to 31.

$Q(x)$: $x^2 = 2$

This predicate states that the square of x equals 2.

Step 1: Understand the predicate

- $Q(x)$ is true when $x^2 = 2$.

Step 2: Solve the equation

- To find x such that $x^2 = 2$, solve in the set of integers:

$$ x^2 = 2 $$

- The solutions over real numbers are $x = \pm \sqrt{2}$. But since the domain is integers, check whether $\sqrt{2}$ is an integer.

Step 3: Check for integer solutions

- $\sqrt{2}$ is approximately 1.4142, which is not an integer.

- Therefore, there are no integers x such that $x^2 = 2$.

Step 4: Determine the truth set

- Since no integer solutions exist, the truth set T_Q is empty.

$$ T_Q = \emptyset $$

Summary:

- The predicate Q(x): $x^2=2$ has no solutions in the integers; its truth set is empty.

R(x): x (Assuming R(x): x is even)

Given the incomplete predicate, we will assume R(x) states "x is even."

Step 1: Understand the predicate

- R(x): x is even, i.e., divisible by 2.

Step 2: Identify the truth set

- All integers x such that x mod 2 = 0.

Step 3: Write the truth set

$$ T_R = \{ x \in \mathbb{Z} \mid x \equiv 0 \ (\text{mod}\ 2) \} $$

- Alternatively, all integers of the form 2k where k is an integer:

$$ T_R = \{ 2k \mid k \in \mathbb{Z} \} $$

Step 4: Express the set

- This includes ..., -4, -2, 0, 2, 4, 6, 8, ..., and so on.

Summary:

- The truth set of $R(x)$ (assuming x is even) is all even integers.

Summary of All Truth Sets

Predicate	Description	Truth Set	Explanation
$P(x)$	$x \geq 31$	$\{ x \in \mathbb{Z} \mid x \geq 31 \}$	All integers greater than or equal to 31
$Q(x)$	$x^2 = 2$	$\emptyset$	No integer squares to 2
$R(x)$	x is even (assumed)	$\{ 2k \mid k \in \mathbb{Z} \}$	All even integers

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Conclusion

Determining the truth set of predicates over the integers involves understanding the conditions imposed by each predicate and solving the corresponding equations or inequalities within the domain. For $P(x)$, the set is straightforward: all integers greater than or equal to 31. For $Q(x)$, the equation $x^2 = 2$ has no integer solutions, resulting in an empty set. For $R(x)$, assuming it refers to "x is even," the truth set includes all even integers.

This process showcases the importance of translating logical predicates into mathematical conditions and solving them carefully within the domain constraints. Whether dealing with inequalities, equations, or properties like parity, methodically analyzing each predicate ensures accurate determination of their truth sets.

Additional Tips for Finding Truth Sets of Predicates

- Understand the Predicate: Clarify what the predicate states before solving.
- Translate into Mathematical Conditions: Convert predicates into equations or inequalities.
- Solve within the Domain: Be aware of the domain restrictions (here, integers).
- Check for Solutions: Verify whether solutions satisfy the predicate.
- Express the Set Clearly: Use set notation to precisely describe the truth set.

Final Remarks

Understanding how to find the truth set of predicates over the set of integers is fundamental in logical reasoning, computer science, and mathematics. It combines skills from algebra, inequalities, and logical analysis. Practice with various predicates to become proficient in translating logical statements into mathematical solutions, which is essential for more advanced topics like model theory, formal verification, and mathematical logic.

If you have further predicates or more complex conditions, approach each systematically—break down the problem, solve step-by-step, and clearly state your reasoning. This method ensures accurate results and enhances your logical problem-solving skills.

Frequently Asked Questions

What is the set of all integers x such that P(x): x > 3? Show your work.

P(x): $x > 3$. Since the domain is the set of integers, the set of all x satisfying P(x) is $\{x \mid x \in \mathbb{Z} \text{ and } x > 3\}$. Therefore, the set is $\{x \in \mathbb{Z} \mid x \geq 4\}$.

What is the set of all integers x such that Q(x): x^2 = 2? Show your work.

Q(x): $x^2 = 2$. The solutions to $x^2 = 2$ over the integers are $x = \pm\sqrt{2}$, but since $\sqrt{2}$ is not an integer, there are no integer solutions. Therefore, the set is $\varnothing$ (empty set).

What is the set of all integers x such that R(x): x? Show your work.

R(x): x. Assuming this means x is true or x satisfies some property, but as written, it appears incomplete. If R(x) is simply x (which doesn't specify a predicate), then the set of all x satisfying R(x) depends on the specific property. If R(x) is intended as the identity predicate, then the set is all integers, $\mathbb{Z}$.

Determine the set of integers satisfying all three predicates P(x): x > 3, Q(x): x^2 = 2, and R(x): x. Show your work.

From earlier, P(x): $x \geq 4$, Q(x): no solutions (empty set), R(x): all integers (assuming). Since Q(x) has no solutions, the intersection of all three predicates is empty. Therefore, the set is $\varnothing$.

Are there any integers x satisfying P(x): x > 3 and R(x): x, but not Q(x): x^2 = 2? Show your work.

Yes. Since Q(x): $x^2=2$ has no integer solutions, all integers satisfying P(x): x > 3 and R(x): x (assuming R(x): x is true) are simply all integers greater than 3. Therefore, the set is $\{x \in \mathbb{Z} \mid x > 3\}$.

Additional Resources

Predicate Logic Analysis: Determining the Truth Sets of P(x), Q(x), and R(x) Over the Set of Integers

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Introduction

Predicate logic serves as a foundational pillar in mathematical reasoning, allowing us to express properties and relations of elements within a domain—in this case, the set of integers. When analyzing predicates such as P(x), Q(x), and R(x), an integral task is to determine their truth sets: the collection of all elements in the domain for which the predicate is true.

In this comprehensive review, we will dissect each predicate carefully, delving into their definitions, the domain considerations, and the step-by-step reasoning necessary to identify their truth sets. By doing so, we aim to not only provide precise answers but also to illuminate the logical process involved, serving as a guide to anyone interested in formal logic or mathematical reasoning.

Understanding the Predicates and Their Domains

Before we analyze each predicate, it's crucial to clarify their definitions and the domain over which they are evaluated.

Domain: The set of all integers, denoted as $\mathbb{Z}$. This includes all positive integers, negative integers, and zero ($\dots, -3, -2, -1, 0, 1, 2, 3, \dots$).

Predicates:

1. $P(x): x \text{ is greater than } 31$ (interpreted as "x > 31")
2. $Q(x): x^2 = 2$
3. $R(x): x \text{ is true}$ (this appears to be a placeholder; more context is needed)

However, the third predicate $R(x): x$ seems incomplete or potentially misrepresented.

Commonly, in logic exercises, a predicate like $R(x)$ would be defined with some property. Since the prompt states $R(x): x$, perhaps it is shorthand or a typographical error. For clarity, let's assume it is:

- $R(x): x$ (which is not a predicate but an expression). Alternatively, perhaps the intended predicate is:

Possible correction:

Let's suppose the predicate is "x" meaning "x is true" (which is nonsensical), or perhaps it was meant as:

- $R(x): x \text{ is an integer}$

But since the domain is already the integers, this predicate would be trivially true for all integers.

Alternatively, perhaps the predicate is:

- $R(x): x \text{ is a real number}$, but that doesn't fit the domain.

Or possibly:

- $R(x): x$ (meaning only that the predicate is simply "x", which doesn't make sense).

Assumption for clarity: Given the pattern, perhaps the third predicate is:

"x" (meaning "x" or perhaps "x is an element satisfying some property"). Without additional context, perhaps the problem is incomplete or has a typo.

Conclusion: To proceed meaningfully, let's assume the third predicate is:

$R(x): x \text{ is an integer}$

which, over the domain $\mathbb{Z}$, is always true, hence its truth set is the entire set $\mathbb{Z}$.

Analyzing Each Predicate in Detail

Predicate 1: $P(x): x > 31$

Definition: The predicate $P(x)$ asserts that x is greater than 31.

Interpretation: We are looking for all integers x such that $x > 31$.

Step-by-step reasoning:

- Since the domain is $\mathbb{Z}$, the set of integers greater than 31 is all integers starting from 32 upwards.

- This includes: 32, 33, 34, ..., and so on, extending infinitely in the positive direction.

Truth set of $P(x)$:

$$
\boxed{
\{ x \in \mathbb{Z} \mid x > 31 \} = \{ x \in \mathbb{Z} \mid x \geq 32 \}
}
$$

Explicitly:

$$
\{ 32, 33, 34, 35, \dots \}
$$

Summary: The truth set of $P(x)$ comprises all integers greater than or equal to 32.

Predicate 2: $Q(x): x^2 = 2$

Definition: The predicate $Q(x)$ asserts that the square of x equals 2.

Interpretation: Find all integers x satisfying $x^2 = 2$.

Step-by-step reasoning:

- The equation $x^2 = 2$ has solutions over the real numbers $x = \pm \sqrt{2}$. Since $\sqrt{2}$ is irrational, the solutions are not integers.

- Within the domain of integers, the only candidate solutions are integers x such that $x^2 = 2$.

- Testing small integers:

- $x = 0 \Rightarrow x^2 = 0 \neq 2$

- $x = 1 \Rightarrow 1^2 = 1 \neq 2$

- $x = 2 \Rightarrow 4 \neq 2$

- $x = -1 \Rightarrow 1 \neq 2$

- $x = -2 \Rightarrow 4 \neq 2$

- For any integer x, $x^2 \geq 0$, and only $x = \pm 1$ satisfy $x^2 = 1$, which is not equal to 2.

- No integer satisfies $x^2 = 2$.

Conclusion:

$$
\boxed{
\text{The truth set of } Q(x): \emptyset
}
$$

Summary: The predicate $Q(x)$ is never true over the integers, so its truth set is empty.

Predicate 3: $R(x): x$

Clarification: As mentioned, this predicate appears ambiguous. Assuming the intended predicate is:

- $R(x): x \text{ is a particular property}$

Given the context, perhaps it is:

- $R(x): x \text{ is an integer}$

or simply the predicate "x" meaning "x is true" in propositional logic, which is nonsensical.

Assuming $R(x)$ is a predicate that is always true over the domain $\mathbb{Z}$.

Analysis:

- If $R(x)$ is interpreted as "x is an integer," then since the domain is $\mathbb{Z}$, every element in the domain satisfies $R(x)$.

- Therefore, the truth set:

$$
\boxed{
\{ x \in \mathbb{Z} \mid R(x) \text{ is true} \} = \mathbb{Z}
}
$$

- If $R(x)$ is some other property, more context would be necessary. Given the lack of further information, the most reasonable assumption is that $R(x)$ is an always-true predicate within the domain.

Summary: The truth set of \(R(x) \), under the assumption that it is always true, is the entire set of integers: \(\mathbb{Z}\).

Summary of All Truth Sets

Predicate	Description	Truth Set
\(P(x): x > 31 \)	All integers greater than 31	\(\{ x \in \mathbb{Z} \mid x \geq 32 \}\)
\(Q(x): x^2 = 2 \)	Integers whose square equals 2	\(\emptyset\)
\(R(x): x \) (assumed always true)	All integers, assuming always true	\(\mathbb{Z}\)

Conclusion & Final Remarks

Analyzing predicates over the set of integers requires meticulous examination of their definitions and logical implications. The process involves:

- Clarifying the predicate's meaning.
- Understanding the domain.
- Solving equations or inequalities as necessary.
- Considering the properties of integers, such as the absence of solutions for certain equations.

Key Takeaways:

- The predicate \(P(x): x > 31 \) has an infinite truth set starting from 32 upwards.
- The predicate \(Q(x): x^2 = 2 \) has an empty truth set in integers because no integer squares to 2.
- The predicate \(R(x) \), as interpreted, encompasses all integers if it's always true; otherwise, further context would be needed.

Final note: When dealing with logical predicates, always ensure the precise definitions and domain considerations are clear—this clarity is critical for accurate analysis and reasoning.

End of Article

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