For Each Of The Following, Compute The Integral Or Show It Doesn't Exist: (3a)C(x2+y2)2x2dAwhereC={(x,y):x2+y21}(3b)Sxy1dAwhereS={(x,y):1x,0yx1}

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When tackling multivariable calculus problems, particularly double integrals, it’s essential to carefully analyze the region of integration and the integrand's behavior. In this article, we will explore two such integrals:

- (3a) Compute $\iint_C (x^2 + y^2) 2x^2 \, dA$ where $C = \{(x, y) : x^2 + y^2 \leq 1 \}$.
- (3b) Compute $\iint_S xy \, dA$ where $S = \{ (x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1 \}$.

We will analyze each integral step-by-step, including checking for convergence or divergence, and providing detailed explanations suitable for students and enthusiasts seeking clarity in advanced calculus topics.

Understanding the Regions of Integration

Before delving into the integrals, it's vital to understand the regions:

Region C: The Unit Disk

- Defined as $C = \{ (x, y) : x^2 + y^2 \leq 1 \}$.
- A circle centered at the origin with radius 1.
- Symmetric about both axes.

Region S: The Square in the First Quadrant

- Defined as $S = \{ (x, y) : 0 \leq x \leq 1, 0 \leq y \leq 1 \}$.
- A unit square in the first quadrant.

Understanding these regions helps determine the most convenient coordinate system and whether the integrals are finite or divergent.

Part 1: Computing $\iint_C (x^2 + y^2) 2x^2 \, dA$

This integral involves a circular region C, which suggests that polar coordinates are an optimal choice. The integrand involves $x^2 + y^2$, which simplifies nicely in polar coordinates.

Step 1: Converting to Polar Coordinates

- Recall that $x = r \cos \theta$, $y = r \sin \theta$.
- $x^2 + y^2 = r^2$.
- The differential area element: $dA = r \, dr \, d\theta$.

The integrand becomes:

$$
(x^2 + y^2) 2x^2 = r^2 \times 2 (r \cos \theta)^2 = r^2 \times 2 r^2 \cos^2 \theta = 2 r^4 \cos^2 \theta.
$$

Step 2: Setting up the Polar Integral

Since C is the unit disk:
- r varies from 0 to 1.
- θ varies from 0 to 2π.

Therefore,

$$
\iint_C (x^2 + y^2) 2x^2 \, dA = \int_0^{2\pi} \int_0^1 2 r^4 \cos^2 \theta \times r \, dr \, d\theta.
$$

Simplify the integrand:

$$
= \int_0^{2\pi} \int_0^1 2 r^5 \cos^2 \theta \, dr \, d\theta.
$$

Step 3: Computing the Integral over r and θ

Integral over r:

$$
\int_0^1 2 r^5 \, dr = 2 \times \frac{r^6}{6} \bigg|_0^1 = 2 \times \frac{1}{6} = \frac{1}{3}.
$$

Integral over θ:

$$

\int_0^{2\pi} \cos^2 \theta \, d\theta.
\]

Recall the identity:
\[
\cos^2 \theta = \frac{1 + \cos 2\theta}{2}.
\]

Thus,
\[
\int_0^{2\pi} \cos^2 \theta \, d\theta = \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta = \frac{1}{2} \int_0^{2\pi} 1 \, d\theta + \frac{1}{2} \int_0^{2\pi} \cos 2\theta \, d\theta.
\]

Calculating each:
\[
\frac{1}{2} \times 2\pi + \frac{1}{2} \times 0 = \pi.
\]

Final Calculation:
\[
\iint_C (x^2 + y^2) 2x^2 \, dA = \frac{1}{3} \times \pi = \frac{\pi}{3}.
\]

Conclusion:
The integral converges to \(\boxed{\frac{\pi}{3}}\).

Part 2: Computing \(\iint_S xy \, dA\)

This integral is over a square region \(S = \{(x, y): 0 \leq x \leq 1, 0 \leq y \leq 1\}\). Since the region is simple and bounded, and the integrand is continuous, the integral should exist and be straightforward to compute.

Step 1: Setting up the Integral

\[
\iint_S xy \, dA = \int_0^1 \int_0^1 xy \, dy \, dx.
\]

Note: Integration order can be switched if desired, but here it’s straightforward.

Step 2: Computing the Inner Integral over y

For a fixed x,

$$
\int_0^1 xy \, dy = x \int_0^1 y \, dy = x \times \frac{y^2}{2} \bigg|_0^1 = x \times \frac{1}{2} = \frac{x}{2}.
$$

Step 3: Computing the Outer Integral over x

$$
\int_0^1 \frac{x}{2} \, dx = \frac{1}{2} \int_0^1 x \, dx = \frac{1}{2} \times \frac{x^2}{2} \bigg|_0^1 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.
$$

Final Result:

$$
\boxed{\frac{1}{4}}.
$$

Conclusion:
The integral over the square S exists and evaluates to $\frac{1}{4}$.

Summary and Final Remarks

- The double integral over the unit disk C of the function $(x^2 + y^2) 2x^2$ converges and equals $\boxed{\frac{\pi}{3}}$. Converting to polar coordinates simplifies the process, taking advantage of the symmetry and geometry of the region.

- The double integral over the unit square S of the function xy also converges, with the value $\boxed{\frac{1}{4}}$. Its straightforward evaluation highlights the ease of integrating polynomial functions over rectangular regions.

Important Takeaways:

- For regions with circular symmetry, polar coordinates are often the best choice.
- Always check the region of integration and the behavior of the integrand to assess convergence.
- Continuity of the integrand over a bounded region guarantees the existence of the integral.
- Simplify integrands using algebraic identities, especially when dealing with trigonometric functions in polar coordinates.

By carefully analyzing the regions and the integrand's properties, you can efficiently compute or determine the divergence of multivariable integrals, which is fundamental in advanced calculus and mathematical analysis.

Keywords for SEO:
Double integrals, multivariable calculus, polar coordinates, unit disk integral, square region integral, convergence of integrals, integrand analysis, calculus problems, advanced calculus solutions, integrating over circular and rectangular regions.

Frequently Asked Questions

How do you set up the integral for the region $C = \{(x,y): x^2 + y^2 \leq 1\}$ when computing $\iint_C C(x^2 + y^2)^2 x^2 \, dA$?

Since C is the unit disk, it's best to convert to polar coordinates: $x = r \cos \theta$, $y = r \sin \theta$, with r from 0 to 1 and θ from 0 to 2π. The integrand becomes $C r^4 (r \cos \theta)^2 = C r^6 \cos^2 \theta$. The differential area element dA is $r \, dr \, d\theta$, so the integral setup is $\int_0^{2\pi} \int_0^1 C r^6 \cos^2 \theta \, dr \, d\theta$.

What is the value of the integral over the region C for the function $C(x^2 + y^2)^2 x^2$, and does it converge?

Yes, the integral converges. Computing the integral in polar coordinates yields a finite value: the angular part involving $\cos^2 \theta$ integrates to π, and the radial part $\int_0^1 r^6 r \, dr = \int_0^1 r^7 \, dr = 1/8$. Combining these, the integral equals C π 1/8, which is finite.

How should we approach the integral over the region $S = \{(x,y): 0 \leq x \leq 1, 0 \leq y \leq 1\}$ for the function xy?

Since the region S is a square with x and y from 0 to 1, set up a double integral: $\int_0^1 \int_0^1 xy \, dy \, dx$. The integral is straightforward to evaluate as it factors into the product of two single integrals: $(\int_0^1 x \, dx) (\int_0^1 y \, dy)$.

What is the value of the double integral of xy over the square region S, and does the integral exist?

Yes, the integral exists and is finite. Calculating, $\int_0^1 x \, dx = 1/2$ and $\int_0^1 y \, dy = 1/2$, so the total integral is (1/2) (1/2) = 1/4.

Could the integral over the region S of xy be divergent, and under what circumstances?

In this case, since the region is finite and the integrand xy is continuous and bounded on S, the integral converges. If the region were unbounded or the integrand had a non-integrable singularity, divergence could occur. But here, the integral converges.

What are the key steps in determining whether these integrals exist or diverge?

The key steps include: converting to appropriate coordinate systems if necessary, analyzing the bounds of integration, checking for singularities within the region, and evaluating the integral to see if it results in a finite number. Bounded regions and continuous integrands guarantee convergence; unbounded regions or singularities may lead to divergence.

Additional Resources

Investigate and Compute the Given Double Integrals: A Detailed Analysis

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In the realm of multivariable calculus, especially when dealing with double integrals, the challenge often lies not just in performing the computation but in understanding the nature and properties of the functions and regions involved. The problems under consideration—integrals over specific regions with given functions—serve as excellent exemplars for illustrating key concepts such as parameterization, region analysis, convergence, and the application of fundamental theorems.

This article provides an in-depth investigation into the evaluation of two particular double integrals:

1. (3a) $\iint_{C} (x^2 + y^2)^2 x^2 \, dA$, where $C = \{(x,y) : x^2 + y^2 \le 1 \}$

2. (3b) $\iint_{S} xy \, dA$, where $S = \{(x,y) : 0 \le x \le 1,\, x \le y \le 1 \}$

We will analyze each integral thoroughly, examining their definitions, regions, potential for convergence, and methods for computation. The discussion aims to not only find explicit solutions where possible but also to clarify cases where the integrals do not exist due to divergence or ill-defined regions.

Understanding the Regions and Functions: Foundations for Integration

Before tackling the integrals themselves, it is pivotal to analyze the regions over which these integrals are defined and the functions to be integrated.

Region C: The Unit Disk

The region $C = \{ (x,y) : x^2 + y^2 \le 1 \}$ is the closed unit disk centered at the origin. This is a standard region in polar coordinates, making it highly suitable for transformations:

- Polar Coordinates: $x = r \cos \theta$, $y = r \sin \theta$, with $r \in [0,1]$, $\theta \in [0, 2\pi]$.

- Area Element: $dA = r dr d\theta$.

The symmetry of the region simplifies many integrals, especially those involving functions depending on $x^2 + y^2$.

Region S: The Triangular Region

The region $S = \{ (x,y) : 0 \le x \le 1,\, x \le y \le 1 \}$ is a triangle bounded by the lines:

- $x = 0$

- $y = 1$

- $y = x$

This region can be visualized as a right triangle with vertices at $(0,0)$, $(1,1)$, and $(0,1)$.

- Order of Integration: It is natural to integrate with respect to y first, then x:

$$\int_{x=0}^1 \int_{y=x}^1 xy \, dy \, dx.$$

Part 1: Computing $\iint_{C} (x^2 + y^2)^2 x^2 \, dA$

This integral involves a function symmetric in x and y, with a polynomial structure, over a circular region.

Step 1: Recognizing the symmetry and choosing coordinates

Given the region C is a circle and the integrand involves $x^2 + y^2$, polar coordinates are ideal:

\[

$$
x = r \cos \theta, \quad y = r \sin \theta, \quad dA = r dr d\theta.
$$

Expressing the integrand:

$$
(x^2 + y^2)^2 x^2 = r^4 (\cos^2 \theta + \sin^2 \theta)^2 \times r^2 \cos^2 \theta = r^6 \times 1 \times \cos^2 \theta,
$$

since $(\cos^2 \theta + \sin^2 \theta) = 1$.

Therefore, the integral simplifies to:

$$
\iint_{C} (x^2 + y^2)^2 x^2 \, dA = \int_{0}^{2\pi} \int_{0}^{1} r^6 \cos^2 \theta \times r dr d\theta = \int_{0}^{2\pi} \int_{0}^{1} r^7 \cos^2 \theta \, dr d\theta.
$$

Step 2: Separating variables and integrating

The integral becomes:

$$
\left(\int_{0}^{2\pi} \cos^2 \theta \, d\theta \right) \times \left(\int_{0}^{1} r^7 \, dr \right).
$$

- Angular integral:

$$
\int_{0}^{2\pi} \cos^2 \theta \, d\theta = \pi,
$$

because:

$$
\int_{0}^{2\pi} \cos^2 \theta \, d\theta = \int_{0}^{2\pi} \frac{1 + \cos 2\theta}{2} d\theta = \frac{1}{2} \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{2\pi} = \frac{1}{2} \left(2\pi + 0 \right) = \pi.
$$

- Radial integral:

$$
\int_{0}^{1} r^7 \, dr = \frac{1}{8}.
$$

Step 3: Final result for integral (3a)

Multiplying the results:

$$
\pi \times \frac{1}{8} = \frac{\pi}{8}.
$$

Conclusion: The integral over the unit disk converges and evaluates to:

$$
\boxed{\frac{\pi}{8}}.
$$

Part 2: Analysis of $\iint_{S} xy \, dA$

The second integral involves a simple polynomial function over a triangular region.

Step 1: Set up the integral

Given the region $S = \{ (x,y) : 0 \le x \le 1,\, x \le y \le 1 \}$, the integral becomes:

$$
\int_{x=0}^{1} \int_{y=x}^{1} xy \, dy \, dx.
$$

Step 2: Integrate with respect to y first

Fix x, then:

$$
\int_{y=x}^{1} xy \, dy = x \int_{x}^{1} y \, dy = x \left[\frac{y^2}{2} \right]_x^1 = x \left(\frac{1^2}{2} - \frac{x^2}{2} \right) = x \times \frac{1 - x^2}{2}.
$$

Thus, the integral reduces to:

$$
\int_{0}^{1} \frac{x (1 - x^2)}{2} \, dx = \frac{1}{2} \int_{0}^{1} x (1 - x^2) \, dx.
$$

\]

Step 3: Final integration with respect to x

Compute:

$$
\int_{0}^{1} x (1 - x^2) \, dx = \int_{0}^{1} (x - x^3) \, dx = \left[\frac{x^2}{2} - \frac{x^4}{4} \right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}.
$$

Therefore, the entire integral is:

$$
\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}.
$$

Conclusion: The integral over the triangular region converges and equals:

$$
\boxed{\frac{1}{8}}.
$$

Discussion: Convergence, Divergence, and Implications

Both integrals, as analyzed, converge and yield finite results. However, this is not always guaranteed in double integrals, especially when dealing with unbounded regions or functions with singularities.

- For (3a): The integrand $(x^2 + y^2)^2 x^2$ is continuous and bounded over the closed disk C. The integral converges, as shown, with a finite value $\pi/8$.

- For (3b): The function xy is continuous over the bounded triangle S, and the integral converges, with value $1/8$.

Note: If the regions were unbounded or the integrand had singularities (e.g., division by zero), divergence could occur, making the integral undefined or infinite. In such cases

For Each Of The Following Compute The Integral Or Show It

Doesn T Exist 3a C X2 Y2 2x2dawherec X Y X2 Y21 3b Sxy1dawheres X Y 1x 0yx1

For Each Of The Following Compute The Integral Or Show It Doesn T Exist 3a C X2 Y2 2x2dawherec X Y X2 Y21 3b Sxy1dawheres X Y 1x 0yx1

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