For The Given Expression, Find The Quotient And The Remainder. Check Your Work By Verifying That (Quotient)(Divisor)

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Introduction to Polynomial Division

Polynomial division is a fundamental concept in algebra, analogous to long division with numbers. It involves dividing one polynomial (called the dividend) by another polynomial (called the divisor) to obtain a quotient and a remainder. This process is essential for simplifying algebraic expressions, solving polynomial equations, and understanding the structure of polynomials.

In this article, we will explore how to perform polynomial division, how to identify the quotient and the remainder, and how to verify the accuracy of your division by checking that the product of the quotient and divisor, plus the remainder, equals the original dividend.

Understanding the Division Process

What Is Polynomial Division?

Polynomial division seeks to express a dividend polynomial $P(x)$ as:

$$
P(x) = Q(x) \times D(x) + R(x)
$$

where:
- $Q(x)$ is the quotient polynomial,
- $D(x)$ is the divisor polynomial,
- $R(x)$ is the remainder polynomial, with degree less than that of $D(x)$.

When Do You Stop Dividing?

The division process continues until the degree of the remainder $R(x)$ is less than the degree of the divisor $D(x)$. At this point, the division is complete, and the quotient and remainder are determined.

Step-by-Step Method for Polynomial Division

Long Division Method

The long division process for polynomials is similar to numerical long division:

1. Arrange the Polynomials: Write the dividend and divisor in standard form, with descending powers of x.

2. Divide Leading Terms: Divide the leading term of the dividend by the leading term of the divisor to get the first term of the quotient.

3. Multiply and Subtract: Multiply the entire divisor by this term, subtract the result from the current dividend, and write the remainder.

4. Repeat: Use the new remainder as the dividend and repeat the process until the degree of the remainder is less than the degree of the divisor.

Synthetic Division (When Applicable)

Synthetic division is a shortcut method used when dividing by a linear binomial of the form $x - a$. It simplifies calculations but is limited to specific cases.

Example: Polynomial Division

Let's consider an example to illustrate the process:

Dividing $P(x) = 2x^3 + 3x^2 - x + 5$ by $D(x) = x + 1$.

Step 1: Write in Standard Form

Dividend: $2x^3 + 3x^2 - x + 5$

Divisor: $x + 1$

Step 2: Divide Leading Terms

Divide $2x^3$ by x:

$$
\frac{2x^3}{x} = 2x^2
$$

This is the first term of the quotient.

Step 3: Multiply and Subtract

Multiply $x + 1$ by $2x^2$:

$$
(2x^2)(x + 1) = 2x^3 + 2x^2
$$

Subtract this from the dividend:

$$
(2x^3 + 3x^2 - x + 5) - (2x^3 + 2x^2) = (0x^3 + (3x^2 - 2x^2) - x + 5) = x^2 - x + 5
$$

Step 4: Repeat the Process

Divide x^2 by x:

$$
\frac{x^2}{x} = x
$$

Multiply divisor by x:

$$
(x + 1)(x) = x^2 + x
$$

Subtract:

$$
(x^2 - x + 5) - (x^2 + x) = 0x^2 + (-x - x) + 5 = -2x + 5
$$

Next, divide $-2x$ by x:

$$
\frac{-2x}{x} = -2
$$

Multiply divisor by -2:

$$
(x + 1)(-2) = -2x - 2
$$

Subtract:

\[

$$
(-2x + 5) - (-2x - 2) = 0x + (5 + 2) = 7
$$

Since the degree of the remainder 7 (a constant) is less than the degree of divisor $x + 1$ (degree 1), the division ends.

Final Result for the Example

- Quotient: $Q(x) = 2x^2 + x - 2$
- Remainder: $R(x) = 7$

So,

$$
\boxed{
2x^3 + 3x^2 - x + 5 = (2x^2 + x - 2)(x + 1) + 7
}
$$

Verifying Your Work

Verification ensures that the division was performed correctly. To do this:

1. Multiply the quotient $Q(x)$ by the divisor $D(x)$.
2. Add the remainder $R(x)$.
3. The result should match the original dividend $P(x)$.

Applying this to our example:

$$
(2x^2 + x - 2)(x + 1) + 7
$$

First, expand the product:

$$
(2x^2)(x + 1) + x(x + 1) - 2(x + 1)
$$

which simplifies to:

$$
2x^3 + 2x^2 + x^2 + x - 2x - 2
$$

Combine like terms:

$$
2x^3 + (2x^2 + x^2) + (x - 2x) - 2 = 2x^3 + 3x^2 - x - 2
$$

Now, add the remainder 7:

$$
2x^3 + 3x^2 - x - 2 + 7 = 2x^3 + 3x^2 - x + 5
$$

This matches the original dividend, confirming the correctness of the division.

Key Points to Remember

- The degree of the remainder must be less than the degree of the divisor.
- Always perform polynomial division in standard form with descending powers of x.
- Verification is crucial to ensure the accuracy of the quotient and remainder.
- Long division is the most versatile method, but synthetic division can be faster for linear divisors.

Practice Problems

To solidify your understanding, try dividing the following polynomials and verifying your results:

1. Divide $x^4 - 3x^3 + 2x - 5$ by $x^2 - 1$.
2. Divide $5x^3 + 4x^2 - x + 6$ by $2x + 3$.
3. Divide $3x^2 - 2x + 7$ by $x - 4$.

For each problem, perform the division step-by-step, find the quotient and the remainder, and verify your work by checking that $(\text{Quotient}) \times (\text{Divisor}) + \text{Remainder} =$ original dividend.

Conclusion

Polynomial division is a vital skill in algebra, enabling the simplification of complex expressions and solving polynomial equations. By understanding the step-by-step process, practicing dividing various polynomials, and verifying your work, you'll develop confidence in handling polynomial division problems. Remember, the key is to ensure that the degree of the remainder is always less than the degree of the divisor and that your verification confirms the correctness of your results. With consistent practice, polynomial division will become an intuitive and essential tool in your mathematical toolkit.

Frequently Asked Questions

Given the division problem 125 ÷ 25, what is the quotient and remainder? How can you verify your answer?

The quotient is 5 and the remainder is 0. To verify, multiply the quotient by the divisor: 5 × 25 = 125, which matches the dividend, confirming the division is correct.

How do you find the quotient and remainder when dividing 78 by 10, and how can you verify the result?

Divide 78 by 10: the quotient is 7 and the remainder is 8. Verify by calculating 7 × 10 + 8 = 78, which equals the dividend, confirming the answer.

If you divide 144 by 12, what are the quotient and remainder? How do you check your work?

The quotient is 12 and the remainder is 0. Check by multiplying the quotient by the divisor: 12 × 12 = 144, matching the dividend, which confirms the division is correct.

Divide 95 by 7 and find the quotient and remainder. How can you verify your division?

The quotient is 13 and the remainder is 4. Verify by computing 13 × 7 + 4 = 95, which matches the dividend, confirming the division.

For the expression 200 ÷ 33, what are the quotient and remainder? How do you check your answer?

The quotient is 6 and the remainder is 2 since 6 × 33 + 2 = 200. To verify, multiply the quotient by the divisor and add the remainder to ensure it equals the dividend.

When dividing 50 by 8, what is the quotient and remainder, and how can you verify the calculation?

The quotient is 6 and the remainder is 2 because 6 × 8 + 2 = 50. Verify by performing this calculation to ensure correctness.

Additional Resources

For The Given Expression, Find The Quotient And The Remainder. Check Your Work By Verifying That (Quotient)(Divisor)

When tackling division problems, particularly in algebra and arithmetic, understanding the process of dividing a polynomial or a number into parts is essential. The task of finding the quotient and the remainder, then verifying your work, forms the core of many mathematical problems. This approach not only enhances your computational skills but also deepens your understanding of the division algorithm. In this comprehensive review, we will explore the step-by-step process involved in dividing an expression, how to accurately find the quotient and the remainder, and methods to verify your results effectively.

Understanding the Division Process

Division, at its core, is about partitioning a number or an expression into equal parts. Whether you're dividing simple integers or more complex algebraic expressions, the fundamental principle remains the same. The division process involves determining how many times the divisor fits into the dividend and identifying what is left over after that fitting—the remainder.

Division of Numbers vs. Division of Expressions

While dividing numbers (e.g., $20 \div 3$) is straightforward, dividing algebraic expressions involves more steps, including polynomial long division or synthetic division techniques.

Key features of dividing expressions:

- Polynomial division follows similar principles to numerical division but involves variables and coefficients.
- The degree of the divisor influences the division process.
- Remainders in algebraic division are often expressed as algebraic expressions with degrees less than the divisor.

Step-by-Step Procedure for Finding the Quotient and Remainder

The typical process involves dividing the leading term of the dividend by the leading term of the divisor, then multiplying and subtracting repeatedly until the degree of the remaining expression is less than that of the divisor.

Long Division Method

1. Arrange the dividend and divisor properly, ensuring all terms are in descending order of degree.
2. Divide the leading term of the dividend by the leading term of the divisor to obtain the first term of the quotient.
3. Multiply the entire divisor by this term and subtract the result from the current dividend segment.
4. Repeat the process with the new polynomial (the remainder), dividing its leading term by the divisor's leading term.
5. Continue until the degree of the remaining polynomial (the new remainder) is less than that of the divisor.

Example:

Divide $6x^3 + 11x^2 + 3x + 2$ by $2x + 1$.

- Leading term division: $6x^3 \div 2x = 3x^2$
- Multiply: $(2x + 1)(3x^2) = 6x^3 + 3x^2$
- Subtract: $(6x^3 + 11x^2 + 3x + 2) - (6x^3 + 3x^2) = 8x^2 + 3x + 2$
- Repeat:
- $8x^2 \div 2x = 4x$
- Multiply: $(2x + 1)(4x) = 8x^2 + 4x$
- Subtract: $(8x^2 + 3x + 2) - (8x^2 + 4x) = -x + 2$
- Continue:
- $-x \div 2x = -\frac{1}{2}$
- Multiply: $(2x + 1)(-\frac{1}{2}) = -x - \frac{1}{2}$
- Subtract: $(-x + 2) - (-x - \frac{1}{2}) = 2 + \frac{1}{2} = \frac{5}{2}$

Result:

- Quotient: $3x^2 + 4x - \frac{1}{2}$
- Remainder: $\frac{5}{2}$

Verifying the Work: Checking the Quotient and Remainder

Verification is a critical step to ensure accuracy in division problems. The fundamental principle is that:

$$\text{Dividend} = (\text{Divisor})(\text{Quotient}) + \text{Remainder}$$

By multiplying the quotient by the divisor and adding the remainder, you should recover the original dividend.

Method for Verification

1. Multiply the quotient by the divisor:
- Use algebraic multiplication, distributing each term carefully.
2. Add the remainder to this product.
3. Compare the resulting expression with the original dividend.

Continuing the example:

- Multiply $(3x^2 + 4x - \frac{1}{2})$ by $2x + 1$:

$$
(3x^2)(2x + 1) + (4x)(2x + 1) + \left(-\frac{1}{2}\right)(2x + 1)
$$

Calculations:

- $3x^2 \times 2x = 6x^3$
- $3x^2 \times 1 = 3x^2$
- $4x \times 2x = 8x^2$
- $4x \times 1 = 4x$
- $-\frac{1}{2} \times 2x = -x$
- $-\frac{1}{2} \times 1 = -\frac{1}{2}$

Sum all parts:

$$
6x^3 + 3x^2 + 8x^2 + 4x - x - \frac{1}{2} = 6x^3 + (3x^2 + 8x^2) + (4x - x) - \frac{1}{2}
$$

Simplify:

$$
6x^3 + 11x^2 + 3x - \frac{1}{2}
$$

Now, add the remainder $\frac{5}{2}$:

$$
6x^3 + 11x^2 + 3x - \frac{1}{2} + \frac{5}{2} = 6x^3 + 11x^2 + 3x + \left(-\frac{1}{2} +

\frac{5}{2}\right) = 6x^3 + 11x^2 + 3x + 2
\]

This matches the original dividend, confirming the correctness of the quotient and the remainder.

Advantages and Disadvantages of the Division Method

Understanding the strengths and limitations of polynomial division techniques helps in choosing the right approach for different problems.

Pros:

- Systematic process: Polynomial long division provides a clear, step-by-step method.
- Applicable to all degrees: Capable of dividing complex polynomials.
- Verification: The process allows for straightforward verification of results.
- Foundation for advanced topics: Essential for understanding concepts like polynomial factorization, synthetic division, and partial fractions.

Cons:

- Complexity for higher degrees: Long division can become cumbersome with high-degree polynomials.
- Time-consuming: Manual calculations may be lengthy for complicated expressions.
- Requires careful attention: Mistakes in each step can propagate, so precision is vital.

Features and Practical Tips

- Organize your work: Always write expressions in descending order of degree.
- Check your work at each step: To prevent compounding errors.
- Use synthetic division when applicable: For divisors of the form \(x - c \), synthetic division simplifies calculations.
- Practice with diverse examples: To gain confidence in applying division techniques across different types of expressions.

Conclusion

Mastering the process of dividing expressions to find the quotient and remainder, then verifying your work, is fundamental in algebra and higher mathematics. This process reinforces your understanding of polynomial relationships and enhances problem-solving skills. By following systematic steps—dividing leading terms, multiplying, subtracting, and repeating—you can accurately perform divisions of complex expressions. Remember to verify your results by reconstructing the original dividend, ensuring your calculations are correct. While the process may sometimes be tedious, especially with higher-degree polynomials, the clarity it provides is invaluable. With consistent practice and attention to detail, you will develop proficiency that will serve you well in advanced mathematical studies and real-world applications.

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