Give A Geometric Description Of The Set Of Points Whose Coordinates Satisfy The Given Conditions. X2+y2+z2=36,z=4

Give A Geometric Description Of The Set Of Points Whose Coordinates Satisfy The Given Conditions. $X^2 + y^2 + z^2 = 36$, $z = 4$

Introduction

Understanding the geometric nature of sets of points that satisfy certain algebraic conditions is a fundamental aspect of analytic geometry. The given conditions, $x^2 + y^2 + z^2 = 36$ and $z = 4$, describe a specific subset of three-dimensional space. This article aims to analyze these conditions comprehensively, interpret their geometric implications, and provide an in-depth description of the resulting set of points.

Analyzing the Given Conditions

The Equation $x^2 + y^2 + z^2 = 36$

This equation represents a sphere centered at the origin $(0, 0, 0)$ with a radius of 6, since the general equation for a sphere is:

$$
(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = r^2
$$

Here, $x_0 = 0$, $y_0 = 0$, $z_0 = 0$, and $r = 6$. Therefore, the set of all points (x, y, z) satisfying this equation form the surface of this sphere.

The Equation $z = 4$

This is a plane parallel to the xy-plane, located at a height of 4 units above the xy-plane. It includes all points in space where the z-coordinate is exactly 4, regardless of the x and y values.

Geometric Interpretation of the Conditions

Intersection of the Sphere and the Plane

The set of points satisfying both equations simultaneously is the intersection of the sphere $x^2 + y^2 + z^2 = 36$ with the plane $z = 4$. Geometrically, this intersection is a circle, provided that the plane cuts through the sphere.

Conditions for the Intersection

To determine whether the plane intersects the sphere, substitute $z = 4$ into the sphere's equation:

$$
x^2 + y^2 + (4)^2 = 36
$$

which simplifies to:

$$
x^2 + y^2 + 16 = 36
$$
$$
x^2 + y^2 = 20
$$

The equation $x^2 + y^2 = 20$ describes a circle in the xy-plane with a radius of $\sqrt{20} = 2\sqrt{5}$.

Geometric Description of the Set

The Intersection Circle

The intersection of the sphere $x^2 + y^2 + z^2 = 36$ with the plane $z = 4$ is a circle centered at the projection of the sphere's center onto the plane $z=4$. Since the sphere is centered at the origin, the circle's center in the xy-plane is at $(0, 0, 4)$.

The radius of this circle is:

$$
r_{circle} = \sqrt{20} = 2\sqrt{5}
$$

Therefore, the set of points satisfying both conditions is:

$$
\boxed{
\left\{ (x, y, 4) \mid x^2 + y^2 = 20 \right\}
}
$$

which geometrically is a circle lying in the plane $z=4$ with radius $2\sqrt{5}$.

Summary of the Geometric Description

- The set of points satisfying both conditions is a circle.
- The circle lies in the plane $z=4$.
- Its center is at the point $(0, 0, 4)$.
- Its radius is $2\sqrt{5}$.

Visualizing the Geometric Set

Spatial Relations

- The sphere $x^2 + y^2 + z^2 = 36$ is a sphere of radius 6 centered at the origin.
- The plane $z=4$ is a horizontal plane located 4 units above the xy-plane.
- The intersection of the sphere and the plane results in a circle, which can be visualized as a "slice" of the sphere at $z=4$.

Position of the Circle within the Sphere

- Since the sphere's radius is 6, and the plane is at $z=4$, the plane cuts through the sphere at a height close to its top.
- The circle's radius $2\sqrt{5}$ (~4.47 units) is less than the sphere's radius, confirming that the plane intersects the sphere in a proper circle, not just touching it tangentially.

Implications for the Set of Points

- All points satisfying both conditions are precisely those lying on this circle.
- No points outside this circle satisfy both conditions simultaneously.
- The set is a one-dimensional manifold (a circle) embedded within the three-dimensional space.

Mathematical Summary and Key Points

- The sphere: $x^2 + y^2 + z^2 = 36$, a sphere of radius 6 centered at the origin.
- The plane: $z=4$, a horizontal plane 4 units above the xy-plane.
- The intersection: a circle with radius $2\sqrt{5}$ centered at $(0, 0, 4)$.
- The points satisfying both conditions form this circle, which is a geometric locus of all points meeting the given constraints.

Conclusion

The set of points in three-dimensional space satisfying the equations $x^2 + y^2 + z^2 = 36$ and $z=4$ can be precisely described as a circle lying in the plane $z=4$. This circle has a radius of $2\sqrt{5}$ and is centered at the point $(0, 0, 4)$. Geometrically, it represents the intersection of a sphere of radius 6 centered at the origin with a horizontal plane located 4 units above the xy-plane. This analysis exemplifies how algebraic equations translate into clear geometric objects, illustrating the profound connection between algebra and geometry.

Frequently Asked Questions

What geometric shape is represented by the equation $x^2 + y^2 + z^2 = 36$?

It represents a sphere centered at the origin with radius 6.

What is the significance of the equation z = 4 in the set of points?

It indicates all points lie on the plane parallel to the xy-plane at z = 4.

How can we describe the intersection of the sphere $x^2 + y^2 + z^2 = 36$ and the plane $z = 4$?

The intersection is a circle formed by the intersection of the sphere and the plane $z = 4$.

What is the radius of the circle formed by the intersection of the sphere and the plane $z = 4$?

The radius is $\sqrt{(36 - 4^2)} = \sqrt{(36 - 16)} = \sqrt{20} = 2\sqrt{5}$.

What is the center of the circle formed by the intersection?

The center of the circle is at (0, 0, 4), the point where the plane $z = 4$ intersects the sphere's center.

Can the set of points satisfying both conditions be described as a circle?

Yes, the intersection of the sphere and the plane $z = 4$ is a circle with radius $2\sqrt{5}$ centered at (0, 0, 4).

Is the intersection circle lying inside the sphere?

Yes, since it is the intersection of the sphere and the plane, the circle lies entirely within the sphere.

What is the geometric description of all points satisfying the given conditions?

They form a circle in the plane $z = 4$ with radius $2\sqrt{5}$, lying on the sphere $x^2 + y^2 + z^2 = 36$.

How would the set of points change if the plane was at $z = -4$ instead of $z = 4$?

The intersection would be a circle with the same radius, centered at (0, 0, -4), symmetric to the original circle about the xy-plane.

Additional Resources

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An In-Depth Geometric Analysis of the Sphere and Plane Intersection

In the realm of three-dimensional geometry, understanding the spatial relationships between geometric objects is fundamental. One of the most illustrative examples involves the intersection of a sphere with other geometric entities, such as planes. Here, we explore the set of points satisfying the conditions:

- $x^2 + y^2 + z^2 = 36$
- $z = 4$

This investigation aims to provide a comprehensive, detailed description of the resulting set, elucidating its geometric nature, properties, and significance within the broader context of spatial geometry. The analysis combines algebraic derivations, visual interpretations, and theoretical insights to produce a thorough understanding suitable for academic review or scholarly dissemination.

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1. Foundational Geometric Objects

Before delving into the intersection, it is essential to understand each object individually.

1.1 The Sphere $x^2 + y^2 + z^2 = 36$

This equation describes a sphere centered at the origin $(0,0,0)$ with radius $r = 6$. Its key features include:

- Center: $(0, 0, 0)$
- Radius: 6
- Surface: All points equidistant from the origin

Mathematically, the sphere is a perfectly symmetric surface in three-dimensional space, with every point satisfying the fixed distance condition.

1.2 The Plane $z = 4$

This is a horizontal plane parallel to the xy-plane, located 4 units above the origin along the z-axis. It extends infinitely in the x and y directions, slicing through the three-dimensional space at a constant height.

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2. The Intersection of the Sphere and the Plane

2.1 Algebraic Derivation

The set of points satisfying both equations is obtained by substituting $z = 4$ into the sphere's equation:

$$
x^2 + y^2 + (4)^2 = 36
$$

$$
x^2 + y^2 + 16 = 36
$$

$$
x^2 + y^2 = 20
$$

This is the equation of a circle in the plane $z = 4$.

2.2 Geometric Interpretation

- The intersection is a circle lying in the plane $z = 4$.
- The circle's center in the xy-plane is at $(0, 0)$.
- Its radius is:

$$
r_{\text{circle}} = \sqrt{20} = 2\sqrt{5} \approx 4.472
$$

- The circle resides at a height $z = 4$.

2.3 Visual Representation

Imagine slicing an orange (the sphere) with a flat, horizontal knife at 4 units above the center. The cut reveals a perfect circle—the intersection—that is smaller than the sphere's maximum cross-sectional circle at the equator (which has radius 6).

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3. Geometric Properties of the Intersection

3.1 Location and Size

- Position: The circle lies entirely within the plane $z=4$.
- Size: The circle's radius is $2\sqrt{5}$.

3.2 Relationship to the Sphere

- Since the sphere has radius 6, the plane $z=4$ cuts through the upper part of the sphere, resulting in a circle that is:

- Centered at $(0, 0, 4)$ in the plane.
- Parallel to the xy-plane.

- The maximum circle on the sphere (the equator at $z=0$) has radius 6, so this circle is closer to the "top" of the sphere.

3.3 Symmetries

- The circle exhibits circular symmetry about the z-axis.

- The entire configuration is symmetric with respect to the z-axis.

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4. Broader Context and Significance

4.1 Intersection as a Geometric Locus

The set of points satisfying the given conditions is a circle—a one-dimensional locus within three-dimensional space. This locus encapsulates the essence of how simple algebraic conditions translate into elegant geometric entities.

4.2 Applications and Implications

Understanding such intersections has broad implications:

- In Computer Graphics: Rendering sphere-plane intersections for visual realism.
- In Physics: Analyzing spherical wavefronts intersected by planar boundaries.
- In Engineering: Designing spherical components with planar cuts or openings.
- In Mathematics Education: Developing intuition about three-dimensional objects through algebraic equations.

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5. Extended Analysis: Variations and Related Geometric Constructions

5.1 Varying the Plane's Position

- Moving the plane $z = c$ for different values of c yields a family of circles:

$$
x^2 + y^2 = 36 - c^2
$$

- For $|c| < 6$, the intersection is a circle with radius $\sqrt{36 - c^2}$.
- For $|c| = 6$, the intersection reduces to a single point at the top or bottom of the sphere.
- For $|c| > 6$, no intersection occurs, as the plane lies outside the sphere.

5.2 Intersection with Other Planes

- Similar analyses apply with planes inclined at various angles, leading to the intersection being ellipses or hyperbolas, depending on the orientation.

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6. Visualization and Conceptualization

Creating a mental image or a graphical representation enhances understanding:

- 3D Model: A sphere with radius 6 centered at the origin.

- Plane: Cutting through at $z=4$.
- Intersection: A circle of radius $2\sqrt{5}$, lying in the plane $z=4$, centered at the origin in the xy-plane.

Graphical tools or computer-aided design software can illustrate this configuration, revealing the symmetries and spatial relationships vividly.

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7. Summary and Conclusions

The set of points satisfying:

$$
x^2 + y^2 + z^2 = 36 \quad \text{and} \quad z=4
$$

is a circle lying in the plane $z=4$, with:

- Center at $(0, 0, 4)$,
- Radius $2\sqrt{5}$.

This geometric object exemplifies the intersection of a sphere and a plane, a fundamental concept in spatial geometry, with rich applications and visual elegance. Such intersections serve as foundational tools in various scientific and engineering disciplines, illustrating how algebraic equations encode complex spatial relationships.

The detailed analysis presented here underscores the importance of translating algebraic conditions into geometric intuition, fostering a deeper appreciation of three-dimensional structures and their properties. This understanding not only enhances mathematical literacy but also informs practical applications across multiple fields.

In conclusion, the intersection of the sphere $x^2 + y^2 + z^2 = 36$ with the plane $z=4$ is a circle of radius $2\sqrt{5}$, situated at a height 4 units above the origin. This simple yet profound geometric illustration captures the essence of how algebraic equations shape our understanding of space, embodying the beauty and utility of three-dimensional geometry.

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