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If a skier of mass 45kg is skiing down a 15m slope of 45 degrees, what is the time to reach the bottom of the slope assuming there is no friction. This question combines principles of physics, particularly kinematics and dynamics, to analyze the motion of a skier descending an inclined plane. By understanding the forces at play and applying fundamental equations of motion, we can accurately calculate the time it takes for the skier to reach the bottom of the slope in an idealized scenario where friction and air resistance are neglected. In this article, we will explore the physics involved, perform the necessary calculations, and discuss the implications of the results.

Understanding the Physics of a Skier Descending an Inclined Plane

The Components of Gravitational Force on an Incline

When an object, such as a skier, is placed on an inclined plane, gravity acts vertically downward with a force equal to the object's weight, which is calculated as:

- **Weight (W):** W = m g

where:
- m = mass of the skier = 45 kg
- g = acceleration due to gravity ≈ 9.81 m/s²

Calculating the weight:
W = 45 kg 9.81 m/s² ≈ 441.45 N

The component of gravitational force pulling the skier down the slope is given by:
Fparallel = W sin(θ)

where θ is the angle of the slope (45 degrees).

Decomposition of Forces and Motion

Since we're assuming no friction, the only force causing acceleration along the slope is the component of gravity parallel to the incline. The perpendicular component contributes to the normal force but doesn't affect acceleration in this idealized model.

The acceleration of the skier down the slope (a) can be derived from Newton's second law:
$a = F_{parallel} / m$

Substituting $F_{parallel}$:
$a = (W \sin(\theta)) / m$

But since $W = m g$, this simplifies to:
$a = g \sin(\theta)$

This is a critical insight: in the absence of friction and air resistance, the acceleration of a mass sliding down an inclined plane depends only on g and θ.

Calculating the Acceleration

Given:
- $g = 9.81 \text{ m/s}^2$
- $\theta = 45$ degrees

Calculate sin(45°):
$\sin(45°) = \sqrt{2} / 2 \approx 0.7071$

Hence:
$a = 9.81 \text{ m/s}^2 \; 0.7071 \approx 6.93 \text{ m/s}^2$

This means the skier accelerates down the slope at approximately 6.93 m/s^2.

Determining the Time to Reach the Bottom

Using Kinematic Equations

The primary kinematic equation relates initial velocity, acceleration, displacement, and time:
$d = v_i t + (1/2) a t^2$

Assuming the skier starts from rest ($v_i = 0$):
$d = (1/2) a t^2$

Where:
- d = length of the slope = 15 meters
- $a = 6.93 \text{ m/s}^2$ (from previous calculation)
- t = time to reach the bottom

Rearranged for t:
$t = \sqrt{2 d / a}$

Plugging in the numbers:
$t = \sqrt{(2\ 15\ m / 6.93\ m/s^2)} \approx \sqrt{(30 / 6.93)} \approx \sqrt{4.33} \approx 2.08$ seconds

Therefore, the skier will reach the bottom of the slope in approximately 2.08 seconds in this frictionless scenario.

Implications and Real-World Considerations

Idealized vs. Real-World Conditions

While this calculation provides a clear theoretical answer, real-world skiing involves several factors that slow down the descent:
- Friction between skis and snow
- Air resistance
- Loss of energy due to bumps or uneven terrain
- Initial velocity if the skier starts with a push

In practical situations, these factors increase the time to reach the bottom.

The Effect of Friction and Air Resistance

Including friction and air resistance would:
- Reduce acceleration
- Increase the total time to reach the bottom
- Potentially change the acceleration from the idealized $g \sin(\theta)$ value

However, for the purpose of this analysis, neglecting these factors helps to understand the fundamental physics involved.

Summary of Key Points

- The acceleration of a skier on a frictionless slope depends solely on gravity and the slope angle: $a = g \sin(\theta)$.
- For a slope of 15 meters at 45 degrees, the acceleration is approximately 6.93 m/s^2.
- The time to reach the bottom, starting from rest, is approximately 2.08 seconds.
- Real-world factors such as friction and air resistance would increase this time, but the ideal calculation provides a foundational understanding of the physics involved.

Conclusion

Calculating the time it takes for a skier to reach the bottom of a slope involves understanding the physics of inclined plane motion. In the idealized case where there is no friction, the acceleration is determined solely by gravity and the slope angle. Applying kinematic equations yields a descent time of approximately 2.08 seconds for a 15-meter slope at 45 degrees. This simplified model serves as a valuable educational tool, illustrating core physics principles and providing a baseline for more complex real-world analyses.

Whether you're a physics enthusiast or a skiing hobbyist curious about the science behind descent times, understanding these fundamental concepts enhances appreciation for both the sport and the physics that govern motion on inclined planes.

Frequently Asked Questions

What is the acceleration of the skier while descending the slope without friction?

The acceleration is given by $g \sin(\theta)$, where $g = 9.8\ m/s^2$ and $\theta = 45°$. So, acceleration = $9.8 \sin(45°) \approx 9.8\ 0.7071 \approx 6.93\ m/s^2$.

How can we calculate the time it takes for the skier to reach the bottom of the slope?

Using the equation $s = 0.5\ a\ t^2$, where $s = 15$ m, and $a \approx 6.93\ m/s^2$, solving for t gives $t = \text{sqrt}(2s / a) \approx \text{sqrt}(2\ 15 / 6.93) \approx \text{sqrt}(4.33) \approx 2.08$ seconds.

Does the mass of the skier affect the time taken to reach the bottom in this scenario?

No, since there is no friction and only gravitational acceleration along the slope, the mass cancels out and does not affect the time.

What assumptions are made in calculating the time for the skier to reach the bottom?

Assumptions include negligible air resistance, no friction, constant acceleration along the slope due to gravity, and that the slope angle remains constant.

How would the time change if the slope angle increased to 60 degrees?

With $\theta = 60°$, acceleration becomes $g \sin(60°) \approx 9.8\ 0.866 \approx 8.48\ m/s^2$. Then, time $t = \text{sqrt}(2\ 15 / 8.48) \approx \text{sqrt}(3.53) \approx 1.88$ seconds, so the skier would reach the bottom faster.

Additional Resources

If a skier of mass 45kg is skiing down a 15-meter slope inclined at 45 degrees, what is the time to reach the bottom of the slope assuming there is no friction? This seemingly straightforward question opens the door to a fascinating exploration of physics principles, including gravity, kinematics, and the calculation of motion along inclined planes. Understanding how to approach such a problem not only enhances our grasp of classical mechanics but also provides insights into real-world applications ranging from sports science to engineering.

In this article, we will analyze the problem step by step, applying fundamental physics concepts to determine the skier's acceleration and ultimately, the time it takes to reach the bottom of the slope. While the question appears simple, unpacking it involves understanding the forces at play, the equations governing motion, and assumptions like the absence of friction, which simplifies the analysis but also highlights the idealized nature of such calculations.

Understanding the Physical Scenario

The Setup

The problem involves a skier descending a slope characterized by the following parameters:

- Mass of the skier (m): 45 kg
- Length of the slope (L): 15 meters
- Incline angle (θ): 45 degrees
- Friction: Assumed to be zero (frictionless surface)

This scenario is a classic physics problem involving motion on an inclined plane, which is fundamental in understanding how gravity influences objects in various orientations.

Key Assumptions

To simplify calculations, several assumptions are made:

- No friction or air resistance: This idealization allows us to focus solely on gravitational acceleration.
- Rigid slope: The slope remains constant, with no deformation or irregularities.
- Point mass: The skier is treated as a point mass, ignoring rotational effects or internal mass distribution.
- Constant acceleration: The acceleration remains uniform along the slope, which holds true in the absence of friction and air resistance.

Analyzing the Forces and Motion

Gravitational Force Components

Gravity acts vertically downward with a force $F_g = mg$, where g is the acceleration due to gravity ($\approx 9.81\, \text{m/s}^2$). When a mass is on an inclined plane, this force can be decomposed into two components:

- Parallel component ($F_{parallel}$): Drives the skier down the slope.
- Perpendicular component (F_{perp}): Acts perpendicular to the slope surface, influencing normal force but not directly contributing to acceleration along the slope.

Mathematically:

$$
F_{parallel} = mg \sin \theta
$$

$$
F_{perp} = mg \cos \theta
$$

Given the angle $\theta = 45^{\circ}$, we have:

$$
F_{parallel} = 45\, \text{kg} \times 9.81\, \text{m/s}^2 \times \sin 45^{\circ}
$$

Calculating:

$$
\sin 45^{\circ} = \frac{\sqrt{2}}{2} \approx 0.7071
$$

$$
F_{parallel} = 45 \times 9.81 \times 0.7071 \approx 45 \times 6.936 \approx 311.9\, \text{N}
$$

This force is responsible for accelerating the skier down the slope.

Calculating Acceleration

Since the only force causing acceleration along the slope is $F_{parallel}$, Newton's second law provides:

$$
F_{net} = ma
$$

$$
a = \frac{F_{parallel}}{m} = \frac{mg \sin \theta}{m} = g \sin \theta
$$

Notice that the mass cancels out, indicating that in a frictionless environment, the acceleration is independent of the skier's mass.

Thus:

$$
a = 9.81\, \text{m/s}^2 \times 0.7071 \approx 6.936\, \text{m/s}^2
$$

This is the constant acceleration the skier experiences along the slope.

Calculating the Time to Reach the Bottom

Kinematic Equation for Uniform Acceleration

Given initial velocity $v_0 = 0$ (assuming the skier starts from rest), the displacement $s = 15\, \text{m}$, and acceleration a, the time t can be found using the equation:

$$
s = v_0 t + \frac{1}{2} a t^2
$$

Since $v_0 = 0$:

$$
s = \frac{1}{2} a t^2
$$

Rearranged to solve for t:

$$
t = \sqrt{\frac{2s}{a}}
$$

Plugging in values:

$$
t = \sqrt{\frac{2 \times 15}{6.936}} = \sqrt{\frac{30}{6.936}} \approx \sqrt{4.324} \approx 2.08\, \text{seconds}
$$

Therefore, the skier will reach the bottom of the slope in approximately 2.08 seconds.

Broader Implications and Real-World Considerations

The Significance of the Result

This calculation exemplifies how physics can predict motion in idealized scenarios. The key takeaway is that, in the absence of friction and air resistance, the time to descend a slope depends primarily on the slope's length and the component of gravitational acceleration along it.

In real-world skiing, factors such as friction, air resistance, skier technique, and slope irregularities significantly influence actual times. Nonetheless, this model provides a fundamental baseline for understanding the physics of descent.

Factors That Would Alter the Result

While the idealized calculation yields approximately 2.08 seconds, actual skiing times are typically longer due to:

- Friction between skis and snow: Acts opposite to motion, reducing acceleration.
- Air resistance: Especially significant at higher speeds.
- Initial velocity: Skier's starting movement or push-off.
- Slope irregularities: Changes in slope angle or surface conditions.
- Rotational effects: If considering the skier's body rotation or equipment, the dynamics become more complex.

In professional downhill racing, for instance, these factors are meticulously managed to optimize speed, but the core physics principle remains rooted in the same fundamental laws.

Summary and Key Takeaways

- The problem involves analyzing motion along an inclined plane with gravity as the driving force.
- The acceleration of a frictionless skier on a 45-degree slope is $g \sin \theta$, approximately 6.936 m/s^2.
- The time to reach the bottom can be calculated using basic kinematic equations, resulting in roughly 2.08 seconds for a 15-meter descent.
- The problem illustrates the elegance of physics in simplifying complex motions and highlights how idealized models serve as foundational tools for understanding real-world phenomena.

Final Thoughts

Understanding the physics of skiing down slopes is not just an academic exercise; it exemplifies the broader applications of mechanics in engineering, safety design, and sports science. By dissecting such problems, students and enthusiasts alike can appreciate how fundamental principles govern everyday experiences, turning complex motions into manageable calculations. Whether for designing safer ski slopes or optimizing athletic performance, the insights derived from physics remain invaluable.

In conclusion, a frictionless skier of 45 kg descending a 15-meter slope inclined at 45 degrees will reach the bottom in approximately 2.08 seconds, demonstrating how gravity and simple kinematic equations can accurately predict motion in ideal conditions.

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