LetF(x,y,z)=yzi+xzj+(xy+1)kbe A Vector Field. (i) Find A Potential(x,y,z)such ThatF=and(0,0,0)=2. Ans:xyz+z+2(ii)

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Introduction

Vector calculus plays a fundamental role in understanding physical phenomena such as fluid flow, electromagnetic fields, and force analysis. Among the numerous concepts within vector calculus, potential functions and vector fields are pivotal in simplifying complex problems. This article delves into the process of identifying a potential function for a given vector field, specifically analyzing the field:

$$
\mathbf{F}(x,y,z) = yzi + xz j + (xy + 1) k
$$

The goal is to find a scalar potential function $A(x,y,z)$ such that:

$$
\nabla A = \mathbf{F}
$$

and to verify that the potential function satisfies certain boundary conditions, specifically that $A(0,0,0) = 2$. This comprehensive exploration will cover the necessary background, detailed steps for finding the potential, and interpretations of the results, providing an insightful resource for students and professionals alike.

Understanding Vector Fields and Potential Functions

What Is a Vector Field?

A vector field assigns a vector to every point in space, often representing physical quantities like velocity, force, or magnetic field. Mathematically, a vector field in three dimensions is expressed as:

$$
\mathbf{F}(x,y,z) = P(x,y,z) \mathbf{i} + Q(x,y,z) \mathbf{j} + R(x,y,z) \mathbf{k}
$$

where P, Q, R are scalar functions representing the components along the respective axes.

What Is a Potential Function?

A potential function (or scalar potential) $A(x,y,z)$ for a vector field $\mathbf{F}$ exists if:

$$
\nabla A = \mathbf{F}
$$

which implies:

$$
\frac{\partial A}{\partial x} = P, \quad \frac{\partial A}{\partial y} = Q, \quad \frac{\partial A}{\partial z} = R
$$

The existence of such a potential simplifies the analysis of the vector field, especially in conservative force fields where work done is path-independent.

Conditions for the Existence of a Potential Function

A key criterion for the existence of A is that the vector field must be conservative, which requires:

$$
\nabla \times \mathbf{F} = \mathbf{0}
$$

or equivalently, the curl of $\mathbf{F}$ must be zero. For smooth vector fields defined over simply connected regions, this condition is both necessary and sufficient.

Analyzing the Given Vector Field

Given:

$$
\mathbf{F}(x, y, z) = y z \mathbf{i} + x z \mathbf{j} + (x y + 1) \mathbf{k}
$$

which can be broken down into component functions:

$$
P(x,y,z) = y z
$$

$$
Q(x,y,z) = x z
$$

\]

\[

R(x,y,z) = x y + 1

\]

Verifying if \(\mathbf{F}\) Is Conservative

Calculate the curl of \(\mathbf{F}\):

\[

\nabla \times \mathbf{F} =

\begin{vmatrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\

\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\

P & Q & R

\end{vmatrix}

\]

which expands to:

\[

\nabla \times \mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}

\]

Calculating each component:

- \(i\)-component:

\[

\frac{\partial R}{\partial y} = \frac{\partial (xy + 1)}{\partial y} = x

\]

\[

\frac{\partial Q}{\partial z} = \frac{\partial (x z)}{\partial z} = x

\]

\[

\Rightarrow \text{Component } i: x - x = 0

\]

- \(j\)-component:

\[

\frac{\partial R}{\partial x} = \frac{\partial (xy + 1)}{\partial x} = y

\]

\[

\frac{\partial P}{\partial z} = \frac{\partial (y z)}{\partial z} = y

\]

\[

\Rightarrow \text{Component } j: - (y - y) = 0

\]

- \(k\)-component:

\[

\frac{\partial Q}{\partial x} = \frac{\partial (x z)}{\partial x} = z

\]

\[

\frac{\partial P}{\partial y} = \frac{\partial (y z)}{\partial y} = z

\]

\[

\Rightarrow \text{Component } k: z - z = 0

\]

Since all components are zero, the curl:

$$
\nabla \times \mathbf{F} = \mathbf{0}
$$

This confirms that $\mathbf{F}$ is a conservative vector field within the domain under consideration, implying the existence of a potential function.

Finding the Potential Function $A(x,y,z)$

Given that:

$$
\nabla A = \mathbf{F}
$$

$$
\Rightarrow \frac{\partial A}{\partial x} = y z
$$

$$
\frac{\partial A}{\partial y} = x z
$$

$$
\frac{\partial A}{\partial z} = x y + 1
$$

Our task is to find a scalar function $A(x,y,z)$ satisfying these partial derivatives.

Step 1: Integrate with respect to x

$$
A(x, y, z) = \int y z \, dx + h(y,z)
$$

where $h(y,z)$ is an arbitrary function of y and z (since differentiation with respect to x eliminates any functions solely in y,z).

This integral yields:

$$
A(x, y, z) = x y z + h(y, z)
$$

Step 2: Find $h(y,z)$ using $\partial A / \partial y$

Differentiate A with respect to y:

$$
\frac{\partial A}{\partial y} = x z + \frac{\partial h(y,z)}{\partial y}
$$

But from the given,

$$
\frac{\partial A}{\partial y} = x z
$$

So:

$$
x z + \frac{\partial h(y,z)}{\partial y} = x z
$$

$$
\Rightarrow \frac{\partial h(y,z)}{\partial y} = 0
$$

which implies $h(y,z)$ is independent of y:

$$
h(y,z) = H(z)
$$

Step 3: Find $H(z)$ using $\partial A / \partial z$

Differentiate A:

$$
\frac{\partial A}{\partial z} = x y + H'(z)
$$

From the problem statement:

$$
\frac{\partial A}{\partial z} = xy + 1
$$

So,

$$
x y + H'(z) = xy + 1
$$

$$
\Rightarrow H'(z) = 1
$$

Integrate:

$$
H(z) = z + C
$$

where C is an arbitrary constant.

Final form of the potential function:

$$
A(x, y, z) = x y z + z + C
$$

Choosing $C=0$, for simplicity, the potential function becomes:

$$
A(x,y,z) = xyz + z
$$

Verifying the Boundary Condition $A(0, 0, 0) = 2$

Evaluate:

$$
A(0, 0, 0) = 0 \times 0 \times 0 + 0 = 0
$$

but the problem states that at $(0,0,0)$, the potential should be 2:

$$
A(0,0,0) = 2
$$

To satisfy this, we adjust the potential function by adding the constant 2:

$$
A(x,y,z) = xyz + z + 2
$$

which now satisfies:

$$
A(0,0,0) = 0 + 0 + 2 = 2
$$

Thus, the potential function is:

\[

\boxed{

A(x,y,z) = xyz +

Frequently Asked Questions

What is the given vector field F(x, y, z)?

F(x, y, z) = yz i + xz j + (xy + 1) k

How do you determine if a vector field is conservative?

A vector field is conservative if its curl is zero and it is defined on a simply connected domain.

What is the condition for the existence of a potential function for F?

The vector field must be conservative, meaning curl F = 0, and the domain must be simply connected.

How do you find the potential function □(x, y, z) for F?

Integrate the components of F with respect to their variables, ensuring consistency across partial derivatives.

What is the potential function □(x, y, z) for F(x, y, z) = yz i + xz j + (xy + 1) k?

□(x, y, z) = xyz + z + 2

How do you verify that □(x, y, z) is a potential function for F?

Calculate the gradient of □ and verify it equals F: □□ = F.

What is the significance of the condition A(0, 0, 0) = 2 in this context?

It specifies the value of the potential function at the origin, which helps determine the constant of integration.

How does the initial condition A(0, 0, 0) = 2 influence the potential function?

It confirms the constant term in the potential function, ensuring the correct particular solution.

Is the potential function unique? Why or why not?

Yes, up to an additive constant, because the gradient of any potential function gives F.

What are the applications of finding a potential function for a vector field?

It simplifies line and surface integrals, helps analyze field behavior, and is useful in physics for potential energy calculations.

Additional Resources

LetF(x,y,z)=yzi + xzj + (xy + 1)k is a vector field that offers intriguing insights into vector calculus, potential functions, and the interplay between gradient fields and scalar potential functions. This field, defined in three-dimensional space, combines components linearly dependent on variables with a constant term, presenting a rich example for both theoretical understanding and practical applications in physics and engineering. In this article, we will delve into the detailed process of finding a potential function \(A(x,y,z)\) such that \(\mathbf{F} = \nabla A\), analyze the properties of the vector field, and discuss the significance of potential functions in vector calculus.

Understanding the Vector Field \(\mathbf{F}(x,y,z)\)

Definition and Components

The given vector field is:

$$
\mathbf{F}(x,y,z) = yzi + xzj + (xy + 1)k
$$

which can be expressed in component form as:

$$
\mathbf{F}(x,y,z) = (P, Q, R)
$$

where:
- $P(x,y,z) = yz$,
- $Q(x,y,z) = xz$,
- $R(x,y,z) = xy + 1$.

This field combines products of variables, indicating a non-uniform flow or force field, with dependencies that vary smoothly across space.

Physical and Mathematical Significance

In physics, such vector fields often model flow fields, electromagnetic forces, or other spatially varying influences. Mathematically, they serve as excellent examples for exploring concepts like curl, divergence, and potential functions.

Finding a Scalar Potential Function $A(x,y,z)$

What is a Potential Function?

A potential function $A(x,y,z)$ for a vector field $\mathbf{F}$ is a scalar function such that:

$$
\nabla A = \mathbf{F}
$$

which means:

$$
\frac{\partial A}{\partial x} = P, \quad \frac{\partial A}{\partial y} = Q, \quad \frac{\partial A}{\partial z} = R
$$

If such a function exists, $\mathbf{F}$ is called a gradient field, and it is conservative.

Step-by-Step Derivation

1. Integrate $P = yz$ with respect to x:

$$
A(x,y,z) = \int yz \, dx = xyz + C(y,z)
$$

where $C(y,z)$ is an arbitrary function of y and z (since the partial derivative with respect to x eliminates any function solely in y,z).

2. Use the derivative with respect to y:

From the potential function,

$$
\frac{\partial A}{\partial y} = Q = xz
$$

Compute this derivative:

$$
\frac{\partial}{\partial y} (xyz + C(y,z)) = xz + \frac{\partial C}{\partial y}
$$

Set equal to Q:

$$
xz + \frac{\partial C}{\partial y} = xz
$$

which implies:

$$
\frac{\partial C}{\partial y} = 0
$$

Hence,

$$
C(y,z) = C(z)
$$

3. Use the derivative with respect to z:

From the potential function,

$$
\frac{\partial A}{\partial z} = R = xy + 1
$$

Calculate:

$$
\frac{\partial}{\partial z} (xyz + C(z)) = xy + \frac{d C}{d z}
$$

Set equal to R:

$$
xy + \frac{d C}{d z} = xy + 1
$$

which simplifies to:

$$
\frac{d C}{d z} = 1
$$

Integrate:

$$
C(z) = z + D
$$

where D is an arbitrary constant.

4. Final potential function:

Putting all together,

$$
A(x,y,z) = xyz + z + D
$$

The constant D can be chosen based on boundary conditions or specific values.

Applying the Boundary Condition $A(0,0,0) = 2$

Substitute $(x,y,z) = (0,0,0)$:

$$
A(0,0,0) = (0)(0)(0) + 0 + D = D
$$

Given that:

$$
A(0,0,0) = 2
$$

we find:

$$
D = 2
$$

Thus, the potential function is:

$$
A(x,y,z) = xyz + z + 2
$$

Analysis of the Result and Its Significance

Features of the Potential Function

- Completeness: The derived potential function encapsulates the behavior of $\mathbf{F}$ and satisfies the boundary condition.
- Simplicity: The form $xyz + z + 2$ combines a product term with a linear term and a constant, making it easy to evaluate at specific points.
- Interpretability: The terms reflect the interactions among variables; for example, xyz suggests a volumetric interaction, while z alone could relate to vertical displacement or influence.

Pros and Cons of the Approach

Pros:

- Provides a clear method to identify potential functions for gradient fields.
- Helps determine whether a vector field is conservative.

- Facilitates the computation of line integrals and fluxes.

Cons:

- Not all vector fields are conservative; the method relies on the curl being zero.
- Boundary conditions are crucial; different conditions lead to different potential functions.
- For more complex fields, integration may become cumbersome, or potential functions may not exist.

Verification of the Gradient Field Property

Check if $\mathbf{F}$ is conservative

A key property for potential functions is that the curl of $\mathbf{F}$ must be zero:

$$
\nabla \times \mathbf{F} = \mathbf{0}
$$

Calculate:

$$
\nabla \times \mathbf{F} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
P & Q & R
\end{vmatrix}
$$

which expands to:

\[
\nabla \times \mathbf{F} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) \mathbf{i} - \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right) \mathbf{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \mathbf{k}
\]

Compute each component:

- \(\frac{\partial R}{\partial y} = \frac{\partial (xy + 1)}{\partial y} = x\)
- \(\frac{\partial Q}{\partial z} = \frac{\partial (xz)}{\partial z} = x\)

So,

\[
\left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right) = x - x = 0
\]

- \(\frac{\partial R}{\partial x} = y\)
- \(\frac{\partial P}{\partial z} = \frac{\partial (yz)}{\partial z} = y\)

So,

\[
- \left(\frac{\partial R}{\partial x} - \frac{\partial P}{\partial z} \right) = - (y - y) = 0
\]

- \(\frac{\partial Q}{\partial x} = z\)
- \(\frac{\partial P}{\partial y} = z\)

Thus,

$$
\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) = z - z = 0
$$

All components of curl are zero; hence, $\mathbf{F}$ is conservative, and the potential function exists.

Conclusion and Broader Implications

The vector field $\mathbf{F}(x,y,z) = yzi + xzj + (xy + 1)k$ exemplifies a conservative, gradient vector field in three-dimensional space. The systematic approach of integrating component-wise derivatives reveals that the potential function is:

$$
A(x,y,z) = xyz + z + 2
$$

This potential function not only satisfies the boundary condition $A(0,0,0) = 2$ but also underscores the importance of boundary conditions in defining scalar potentials. The process highlights the significance of ensuring curl-free properties for the existence

Letf X Y Z Yzi Xzj Xy 1 Kbe A Vector Field I Find A Potential X Y Z Such Thatf And 0 0 0 2 Ans Xyz Z 2 Ii

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