

"NUMERICAL ANALYSIS3.a) Apply The Simpson's Rule, With $H = 1/4$, To Approximate The Integral $21 E Dx B$)

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Introduction to Numerical Integration and Simpson's Rule

Numerical integration is a fundamental technique in calculus used to approximate the value of definite integrals, especially when an antiderivative is difficult to obtain analytically or when dealing with empirical data. Among various methods, Simpson's Rule stands out for its accuracy and simplicity, especially for smooth functions.

Simpson's Rule approximates the integral of a function over an interval by fitting quadratic polynomials through the data points and integrating these polynomials. It provides a better approximation than the trapezoidal rule by considering the curvature of the function.

In this article, we will explore the application of Simpson's Rule with a specified step size and evaluate a specific integral, providing a detailed step-by-step solution.

Understanding the Problem Statement

Before applying Simpson's Rule, let's understand the problem clearly:

- The integral to approximate is: $\int_a^b f(x) \, dx$.
- The given step size is $H = \frac{1}{4}$.
- The integral is denoted as $(21 E Dx B)$, which appears to be a notation placeholder. Assuming the typical context of numerical analysis, this likely refers to a specific integral with limits (a) and (b) , and a function $(f(x))$.

Note: Since the problem statement as provided is somewhat ambiguous, we'll interpret it as a general instruction to approximate the integral of a function $f(x)$ over an interval $[a, b]$ using Simpson's Rule with $H = \frac{1}{4}$.

Step 1: Clarifying the Integral and Interval

To proceed, we need to specify:

- The function $f(x)$.
- The limits of integration a and b .

Suppose, for illustration, that:

- $f(x) = \text{a given function, e.g., } x^2$.
- The limits are from $a = 0$ to $b = 1$.

This choice simplifies calculations, but the method applies to any continuous function over the interval.

Note: If the original problem provides specific function and limits, substitute accordingly.

Step 2: Determining the Number of Subintervals

Given:

- Step size $H = \frac{1}{4}$.
- The interval length $b - a$.

For example, with $a = 0$ and $b = 1$:

- Total interval length: $1 - 0 = 1$.
- Number of subintervals n :

$$n = \frac{b - a}{H} = \frac{1}{\frac{1}{4}} = 4$$

Since Simpson's Rule requires an even number of subintervals, and $(n=4)$ is even, the method is applicable.

Step 3: Partitioning the Interval

The points dividing the interval are:

```
\[
x_0 = a = 0
\]
\[
x_1 = a + H = 0 + \frac{1}{4} = 0.25
\]
\[
x_2 = a + 2H = 0 + \frac{1}{2} = 0.5
\]
\[
x_3 = a + 3H = 0 + \frac{3}{4} = 0.75
\]
\[
x_4 = a + 4H = 0 + 1 = 1
\]
```

Corresponding function values:

```
\[
f(x_0), f(x_1), f(x_2), f(x_3), f(x_4)
\]
```

Step 4: Calculating Function Values

Calculate $(f(x))$ at each point:

- For $(f(x) = x^2)$:

```
\[
f(0) = 0^2 = 0
\]
\[
f(0.25) = (0.25)^2 = 0.0625
\]
\[
```

```

f(0.5) = (0.5)^2 = 0.25
\]
\[
f(0.75) = (0.75)^2 = 0.5625
\]
\[
f(1) = 1^2 = 1
\]
---

```

Step 5: Applying Simpson's Rule Formula

The Simpson's Rule approximation for n subintervals (where n is even) is:

```

\[
\int_a^b f(x) \, dx \approx \frac{H}{3} \left[ f(x_0) + 4 \sum_{i=1, 3, 5, \dots}^{n-1} f(x_i) + 2 \sum_{i=2, 4, 6, \dots}^{n-2} f(x_i) + f(x_n) \right]
\]

```

In our case:

```

\[
\int_0^1 f(x) \, dx \approx \frac{1/4}{3} \left[ f(0) + 4(f(0.25) + f(0.75)) + 2f(0.5) + f(1) \right]
\]

```

Calculating:

```

\[
\frac{H}{3} = \frac{1/4}{3} = \frac{1}{12}
\]

```

Sum the function values:

```

\[
f(0) = 0
\]
\[
f(0.25) = 0.0625
\]
\[
f(0.75) = 0.5625
\]
\[
f(0.5) = 0.25
\]

```

$$\begin{aligned} & \backslash[\\ & f(1) = 1 \\ & \backslash] \end{aligned}$$

Plug into the formula:

$$\begin{aligned} & \backslash[\\ & \int_0^1 x^2 dx \approx \frac{1}{12} \left[0 + 4(0.0625 + 0.5625) + \right. \\ & \left. 2(0.25) + 1 \right] \\ & \backslash] \end{aligned}$$

Compute the sums inside brackets:

$$\begin{aligned} & \backslash[\\ & 4(0.625) = 4 \times 0.625 = 2.5 \\ & \backslash] \\ & \backslash[\\ & 2 \times 0.25 = 0.5 \\ & \backslash] \end{aligned}$$

Now sum all:

$$\begin{aligned} & \backslash[\\ & 0 + 2.5 + 0.5 + 1 = 4 \\ & \backslash] \end{aligned}$$

Finally, multiply by $\left(\frac{1}{12}\right)$:

$$\begin{aligned} & \backslash[\\ & \text{Approximate integral} \approx \frac{1}{12} \times 4 = \frac{4}{12} = \\ & \frac{1}{3} \approx 0.3333 \\ & \backslash] \end{aligned}$$

Step 6: Interpreting the Result

The exact value of $\left(\int_0^1 x^2 dx\right)$ is:

$$\begin{aligned} & \backslash[\\ & \left[\frac{x^3}{3} \right]_0^1 = \frac{1^3}{3} - 0 = \frac{1}{3} \approx \\ & 0.3333 \\ & \backslash] \end{aligned}$$

Our Simpson's Rule approximation perfectly matches the exact integral in this case because the function (x^2) is a quadratic polynomial, and Simpson's Rule integrates quadratics exactly when the number of intervals is even.

Generalization and Considerations

- Choice of Step Size (H) : Smaller step sizes generally lead to more accurate approximations. Here, $(H = 1/4)$ was used, but smaller values like $(1/8)$ could improve precision for more complex functions.
- Number of Subintervals: The method requires an even number of subintervals; otherwise, the formula is not directly applicable.
- Function Behavior: For functions with high curvature or discontinuities, more refined methods or smaller step sizes are necessary.
- Error Analysis: The error term in Simpson's Rule depends on the fourth derivative of the function, and understanding this helps in estimating the approximation's accuracy.

Conclusion

Applying Simpson's Rule with the specified step size allows for efficient and accurate approximation of definite integrals, especially for smooth functions. In the illustrative example of $(f(x) = x^2)$ over $([0, 1])$, the approximation aligns perfectly with the analytical solution, highlighting the method's effectiveness.

When dealing with complex functions or different limits, the same procedure applies: partition the interval, evaluate the function at the partition points, and apply Simpson's formula accordingly. This method remains a cornerstone in numerical analysis due to its balance of simplicity and accuracy.

References and Further Reading

- "Numerical Analysis" by Richard L. Burden and J. Douglas Faires.
- "An Introduction to Numerical Methods" by Samuel S. Wagstaff.
- Online resources such as Khan Academy and MIT OpenCourseWare offer tutorials on numerical integration and Simpson's Rule.

Note: Replace the specific function and limits with the actual problem data for precise calculations.

Frequently Asked Questions

What is Simpson's Rule and how is it applied to approximate integrals?

Simpson's Rule approximates the definite integral by dividing the interval into even subintervals, applying quadratic polynomials over each, and summing their areas. It uses the formula: $(h/3) [f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 4f(x_{n-1}) + f(x_n)]$, where h is the subinterval width.

Given $H = 1/4$, how do you determine the number of subintervals for Simpson's Rule?

The number of subintervals n is determined by dividing the total interval length by H . For example, if the integral is over $[a, b]$, then $n = (b - a) / H$. Since $H = 1/4$, n should be chosen so that $(b - a)$ is divisible by $1/4$.

How do you set up the points for applying Simpson's Rule with $H = 1/4$?

You identify the endpoints a and b of the integral and generate equally spaced points $x_0, x_1, \dots, x_n$, where each $x_i = a + i H$. These points are used to evaluate the function at each subinterval.

What is the integral expression for $21 \int_D^B E \, dx$, and how does Simpson's Rule approximate it?

The integral is expressed as the sum of the function $E(x)$ over the interval from D to B . Simpson's Rule approximates this integral by evaluating E at equally spaced points between D and B with step size H , then applying the quadratic approximation formula.

How do you evaluate the function $E(x)$ at the necessary points for Simpson's Rule?

You substitute each x value into the function $E(x)$, calculating the corresponding y -values. These are then used in the Simpson's Rule formula to approximate the integral.

What are common errors to watch out for when applying Simpson's Rule with $H = 1/4$?

Common errors include incorrect calculation of the number of subintervals, misalignment of evaluation points, improper application of the Simpson's formula, and neglecting to evaluate the function at all required points.

How does choosing $H = 1/4$ influence the accuracy of the Simpson's Rule approximation?

A smaller step size H generally increases the accuracy of Simpson's Rule because it reduces the error associated with quadratic approximation. $H = 1/4$ strikes a balance between computational effort and precision.

Can Simpson's Rule be used for any function $E(x)$, and what are its limitations?

Simpson's Rule is most accurate for functions that are smooth and well-behaved over the interval. It may produce less accurate results for functions with high oscillations, discontinuities, or sharp corners.

Additional Resources

Numerical Analysis3.a) Apply The Simpson's Rule, With $H = 1/4$, To Approximate The Integral $\int_a^b E(x) dx$ B is a classic problem that exemplifies the power and elegance of numerical methods in calculus. When dealing with integrals that are difficult or impossible to evaluate analytically, techniques like Simpson's Rule offer practical approximations that are both accurate and computationally efficient. In this comprehensive guide, we will walk through the process of applying Simpson's Rule step by step, using the specified parameters, and explore the underlying principles that make this method a staple in numerical analysis.

Introduction to Numerical Integration and Simpson's Rule

Numerical integration involves estimating the value of a definite integral, especially for functions that are complex, non-elementary, or given through data points. Among various techniques, Simpson's Rule is renowned for providing high accuracy with relatively simple calculations.

Simpson's Rule approximates the integral of a function over a closed interval by fitting quadratic polynomials (parabolas) through the function's points and summing their areas. This approach often yields better approximations than other methods like the Trapezoidal Rule, particularly for functions that are smooth and well-behaved.

Understanding the Problem

Let's clarify the specific problem:
Find an approximate value for the integral

$$I = \int_a^b E(x) \, dx$$

using Simpson's Rule with a step size $(H = \frac{1}{4})$.

While the problem statement contains some cryptic notation ("21 E Dx B"), it is common in numerical analysis exercises to specify the integral bounds and the function involved. Based on typical problem structures, we can interpret this as:

- The integral is over an interval $([a, b])$.
- The function to integrate is $(E(x))$.
- The step size $(H = 1/4)$.

In practice, you would be given specific limits (a) and (b) , and a function $(E(x))$. For this guide, we'll assume the interval is from $(a = 0)$ to $(b = 1)$, a common choice for demonstration, unless specified otherwise.

Step 1: Setting Up the Problem

Determine the interval and number of subintervals

Given:

- $(a = 0)$
- $(b = 1)$
- Step size $(H = \frac{1}{4})$

The number of subintervals (n) is:

$$n = \frac{b - a}{H} = \frac{1 - 0}{1/4} = 4$$

Since Simpson's Rule requires an even number of subintervals (which we have here), we can proceed.

Calculate the nodes

The points (or nodes) at which the function will be evaluated are:

$$\begin{aligned} x_0 &= a = 0 \\ x_1 &= a + H = 0 + \frac{1}{4} = 0.25 \\ x_2 &= 0.5 \\ x_3 &= 0.75 \\ x_4 &= 1 \end{aligned}$$

Step 2: Evaluating the Function $E(x)$ at the Nodes

In practice, the function $E(x)$ could be any function, such as e^x , $\sin x$, etc. For demonstration, let's assume:

$$E(x) = e^x$$

Calculating $E(x)$ at each node:

x	$E(x) = e^x$	Numeric Value
0	$e^0 = 1$	1
0.25	$e^{0.25} \approx 1.2840$	1.2840
0.5	$e^{0.5} \approx 1.6487$	1.6487
0.75	$e^{0.75} \approx 2.1170$	2.1170
1	$e^1 \approx 2.7183$	2.7183

Step 3: Applying Simpson's Rule Formula

The Simpson's Rule formula:

$$\int_a^b E(x) \, dx \approx \frac{H}{3} \left[E(x_0) + 4 \sum_{\text{odd } i} E(x_i) + 2 \sum_{\text{even } i, i \neq 0, n} E(x_i) + E(x_n) \right]$$

Where:

- $E(x_0)$ and $E(x_n)$ are the function values at the endpoints.
- The sum over odd i includes x_1, x_3 .
- The sum over even $i \neq 0, n$ includes x_2 .

Substituting our values:

$$I \approx \frac{H}{3} \left[E(0) + 4 E(0.25) + 2 E(0.5) + 4 E(0.75) + E(1) \right]$$

Plugging in the numbers:

$$I \approx \frac{1/4}{3} \left[1 + 4(1.2840) + 2(1.6487) + 4(2.1170) + 2.7183 \right]$$

Simplify the coefficients:

$$\frac{1}{4^3} \left[1 + 5.1360 + 3.2974 + 8.4680 + 2.7183 \right]$$

Calculate the sum inside the brackets:

$$\text{Sum} = 1 + 5.1360 + 3.2974 + 8.4680 + 2.7183 \approx 20.6197$$

Calculate the prefactor:

$$\frac{1}{4^3} = \frac{0.25^3}{1} \approx 0.08333$$

Finally, compute the approximation:

$$\approx 0.08333 \times 20.6197 \approx 1.7183$$

Final Approximate Value

$$\boxed{\approx 1.7183}$$

This is the approximate value of the integral $\int_0^1 e^x dx$ using Simpson's Rule with $H = 1/4$.

Extending the Analysis: Variations and Considerations

When the interval or function changes:

- Different interval: Adjust a and b accordingly, recalculate the nodes, and evaluate $E(x)$ at each point.
- Different step size H : Ensure n is even; if not, you may need to choose a different H or subdivide further.
- Complex functions: For functions that are not elementary, numerical evaluation at each node is necessary, often via computational tools.

Error estimation:

Simpson's Rule has an error term proportional to the fourth derivative of the function:

$$\left| E_s \right| \leq \frac{(b-a)^5}{180 n^4} \max_{x \in [a, b]} |E^{(4)}(x)|$$

Knowing the behavior of $E^{(4)}(x)$ can help assess the accuracy of the approximation.

Practical Tips for Applying Simpson's Rule

1. Ensure an even number of subintervals: Simpson's Rule requires (n) even.
2. Choose appropriate (H) : Smaller (H) increases accuracy but requires more evaluations.
3. Use computational tools: For complex functions or multiple intervals, software like MATLAB, Python (SciPy), or graphing calculators can automate the process.
4. Verify with other methods: Cross-check results with Trapezoidal Rule or higher-order methods for consistency.

Conclusion

Applying Simpson's Rule with a step size of $(H = 1/4)$ provides an efficient and accurate approximation to definite integrals, especially when the function is smooth and well-behaved. This method exemplifies how numerical analysis bridges the gap between theoretical calculus and practical computation, enabling us to evaluate integrals that are otherwise analytically intractable.

By carefully setting up the problem, evaluating the function at the appropriate nodes, and applying the formula meticulously, you can confidently approximate integrals in diverse scientific and engineering applications. Whether you are dealing with complex data sets, experimental measurements, or theoretical functions, Simpson's Rule remains an essential tool in your numerical analysis toolkit.

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